Inventory Model with Time Dependent Demand Rate under Inflation When Supplier Credit Linked to Order Quantity

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Abstract - This study develops an inventory model under which the supplier provides the purchaser a permissible delay in payments, if the purchaser orders a large quantity. Shortages are not allowed and effect of the inflation rate and delay in payments are discussed. In this paper, we establish an inventory model for non-deteriorating items and time-dependent demand rate under inflation when supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity. We then obtain optimal solution to find optimal order quantity, optimal replenishment time and optimal total relevant cost. Finally, numerical examples are given to illustrate the theoretical results and made the sensitivity analysis of parameters on the optimal solutions.

Keywords: Inventory; finance; time-dependent; demand rate; inflation

1. Introduction

Large number of research papers/articles has been presented by many authors for controlling the inventory of deteriorating and non-deteriorating items. Deteriorating items such as volatile liquids, blood banks, medicines, fashion goods, radioactive materials, photographic films, etc. Non-deteriorating items such as wheat, rice, dry fruits, etc. It is common assumption to many inventory systems is that product generated has indefinitely long lives. Generally, almost all items deteriorate over time. Often the rate of deterioration is low and there is little need to consider the deterioration for determining the economic lot size. For negligibility small deteriorating products, the deterioration rate may be neglected for some small time interval. There are many items in the real world that are subject to a significant rate of deterioration. Hence, the effect of deterioration cannot be neglected in the decision process of production lot size. The loss of utility due to decay is usually a function of the on-hand inventory. Recently, great interest has been shown in developing mathematical models in the presence of trade credit.

Kingsman (1983), Chapman (1985) and Daellenbach (1986) have studied the effect of the trade credit on the optimal inventory policy. In today's competition business transactions, it is common to obtain that the retailers are allowed some credit period before they settle the mount with the wholesaler. This provides to the customers to pay the wholesaler immediately after receiving the product, but instead, can delay their payment until the end of the allowed period. The customer pays no interest during the permissible time for payment, but interest will be charged if the payment is delayed beyond that period.

Ghare and Schrader (1963) had established a model for an exponentially decaying inventory. Covert and Philip (1973) extended Ghare and Schrader's model for two-parameter Weibull distribution deterioration rate. Teng (2002) discussed and obtained an EOQ model in which deterioration rate is zero. Goyal (1985) developed an EOQ model under conditions of permissible delay in payment. He ignored the difference between the selling price and the purchase cost and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments. Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalized the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Chang and Dye (2001) extended the model by Jamal et al. to allow for not only a varying deterioration rate of time but also the backlogging rate to be inversely proportional to the waiting time. Chang et al. (2003) then extended Teng's model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. The work of Covert and Philip (1973) extended by Elsayed and Teresi (1983) by allowing shortages and using time varying demand rate. In this study a single item is considered.
Chung (1989) develops an inventory model for obtaining the optimal order quantity of deteriorating items in the presence of trade credit using the DCF approach. Chung (1989) presented the discounted cash flow (DCF) approach for the analysis of the optimal inventory in the presence of the trade credit. Biermans and Thomas (1977), Buzzacott (1975), Chandra and Bahnier (1988), Jesse et al. (1983) developed EOQ model under constant inflation rate. Liao et al. (2000) presented an inventory model with deteriorating items under inflation when a delay in payment is permissible. Bhalambat (1982) developed an EOQ model under a variable inflation rate and marked-up price. An EOQ model under inflation and time-discounting allowing shortages was presented by Ray and Chaudhuri (1997). Misra (1975) assumes a uniform inflation rate for all the associated costs and minimizes the average annual cost, to derive an expression for the EOQ model. Datta and Pal (1991) investigated a finite time-horizon inventory model following the approach of Misra (1979) with a linearly time-dependent demand rate, allowing shortages and considering the effects of inflation and time value of money. Su et al. (1996) developed an inventory model under inflation for initial-stock dependent consumption rate and exponentially decay. Recently, Hou and Lin (2008) derived an ordering policy with a cost minimization procedure for deteriorating items under trade credit and time discounting. Other many articles can be found by Chung et al. (2005), Chung and Liao (2006), Ouyang et al. (2006) and Huang (2007).

This study develops an inventory model under a situation in which the supplier provides the purchaser a permissible delay of payments if the purchaser orders a large quantity, in which demand rate is time dependent. This paper is the generalization of Chun-Tao Chang (2004) in which deterioration and demand rate both are constant. We here focus on how a purchaser obtains an optimal solution when a supplier offers a permissible delay of payments for large order. In this paper, we establish an EOQ model with time-dependent demand rate under inflation when a supplier provides a permissible delay in payments for a large order that is greater than or equal to the predetermined quantity $Q_d$.

This paper is organized as follows. In section 2, we mention the notation and assumption used throughout the study. In section 3, the mathematical models are derived under four different situations in order to minimize the total cost in the planning horizon. In section 4, theoretical results are given followed by numerical examples are in section 5. Sensitivity analysis is given in section 6 to demonstrate the applicability of proposed model. The conclusions and possible future work is given in last section 7.

2. Notations and Assumptions

The following notations are used throughout this paper:

- $h$: The holding cost rate per unit time excluding interest charges
- $r$: Constant rate of inflation per unit time, where $0 \leq r \leq 1$
- $P(t) = pert$: The selling price per unit at time $t$, where $p$ is the unit selling price at time zero
- $C(t) = ce^t$: The unit purchasing cost at time $t$, where $c$ is the unit purchasing cost at time zero and $p > c$
- $S(t) = se^t$: The ordering cost per order at time $t$, where $s$ is the ordering cost at time zero.
- $H$: The length of planning horizon
- $I$: The interest charged per $S$ in stocks per year by the supplier
- $m$: The permissible delay in settling account
- $Q$: The order quantity
- $Q_d$: The minimum order quantity at which the delay in payments is permitted.
- $T$: The replenishment time interval
- $T_d$: The time interval that $Q_d$ units are depleted to zero due to time dependent demand.
- $I(t)$: The level of inventory at any time $t$, $0 \leq t \leq T = H/n$
- $R(t)$: The annual demand as a function of time, where $R(t) = \hat{\lambda}t$, where $\hat{\lambda}$ is a positive constant i.e. $\hat{\lambda} > 0$.
- $Z(T)$: The total relevant cost over $(0, H)$.

The total relevant cost consists of the following elements:

- (a) Cost of placing orders, (b) cost of purchasing,
- (c) cost of carrying inventory excluding interest charges, (d) cost of interest charges, (e) cost of interest charges for unsold items at the initial time or after the permissible delay $m$, and (f) interest earned from the sales revenue during the permissible period.

In addition, the following assumptions are being made:

1. The inflation rate is a constant
2. Shortages are not allowed
3. Replenishment is instantaneous
4. The demand for the item is known and is time-dependent
5. If $Q < Q_d$, then the payment for the items received must be made immediately.
6. If $Q \geq Q_d$, then the delay in payments up to $m$ is permitted.

During permissible delay period the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of credit period, the customer pays off all units ordered, and begins paying for the interest charges on the items in stocks.

3. Mathematical Formulation

Let us consider the length of horizon $H = nT$, where $n$ is an integer for the number of replenishment to be made during period $H$, and $T$ is an interval of
time between replenishment. The level of inventory \( I(t) \) gradually decreases mainly to meet demands only. Hence, the variation of inventory with respect to time is given by

\[
\frac{dI(t)}{dt} = -\lambda t, \quad 0 \leq t \leq T = H/n
\]

(1)

With boundary conditions \( I(T) = 0 \) and \( I(0) = Q \). From the order quantity, we can obtain the time interval that \( Q_o \) units are depleted to zero due to demand only as

\[
T_d = \sqrt{\frac{2Q_o}{\lambda}}
\]

(2)

The inequality \( Q < Q_o \) holds if and only if \( T < T_d \).

Since the lengths of time intervals are all the same, we have

\[
I(kT + t) = \frac{\lambda}{2}(T^2 - t^2), \quad 0 \leq k \leq n - 1, \quad 0 \leq t \leq T = H/n
\]

(3)

To obtain total relevant cost in \((0, H)\), we use the following terms:

(a) Cost of Placing Orders

\[
\sum_{k=0}^{n-1} S(kT) = S(0) + S(T) + S(2T) + \ldots + S(n-1)T = \frac{S(e^{\lambda H} - 1)}{e^{\lambda T} - 1}
\]

(4)

(b) Cost of purchasing

\[
\sum_{k=0}^{n-1} I(o)C(kT) = \frac{c\lambda T^2}{2} \left( e^{\lambda H} - 1 \right)
\]

(5)

(c) Cost of carrying inventory

\[
h\sum_{k=0}^{n-1} C(kT) I(kT + t)dt = \frac{hc\lambda T^3}{3} \left( e^{\lambda H} - 1 \right)
\]

(6)

We have the following four possible cases based on the values of \( T, m \) and \( T_d \), for finding interest charged and earned.

**Case 1**: \( 0 < T < T_d \)

Since \( T < T_d \) (i.e. \( Q < Q_o \)), the delay in payments is not permitted in this case. The supplier must be paid for the items as soon as the customer receives them.

Since the interest charges for all unsold items start at the initial time, the interest payable in \((0, H)\) is given by

\[
I_c \sum_{k=0}^{n-1} C(kT) I(kT + t)dt = \frac{cI_c \lambda T^3}{3} \left( e^{\lambda H} - 1 \right)
\]

(7)

Thus, the total relevant cost in \((0, H)\) is

\[
Z_c(T) = \text{cost of placing orders} + \text{cost of purchasing} + \text{cost of carrying inventory} + \text{interest payable}
\]

\[
= \left\{ s + \frac{c\lambda T^2}{2} + \frac{c\lambda T^3}{3} (h + I_c) \right\} \left( e^{\lambda H} - 1 \right)
\]

(8)

**Case 2**: \( T_d \leq T < m \)

Since \( T_d \leq T < m \), we know that there is a permissible delay \( m \) which is greater than the replenishment interval \( T \). In this case, there is no interest charged, but the interest earned in \((0, H)\) is:

\[
I_c \sum_{k=0}^{n-1} P(kT) \left[ \int_{0}^{T_d} \lambda t dt + (m - T) \int_{T_d}^{T} \lambda t dt \right]
\]

\[
= \frac{pl_c \lambda T^2}{2} \left( m - \frac{T}{3} \right) \left( e^{\lambda H} - 1 \right)
\]

(9)

The total relevant cost in \((0, H)\) is

\[
Z_c(T) = \left\{ s + \frac{c\lambda T^2}{2} + \frac{c\lambda T^3}{3} \right\} \left( e^{\lambda H} - 1 \right) - \frac{pl_c \lambda T^2}{2} \left( m - \frac{T}{3} \right) \left( e^{\lambda H} - 1 \right)
\]

(10)

**Case 3**: \( T_d \leq m \leq T \)

Since \( T \geq m \geq T_d \), the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned. The interest payable in \((0, H)\) is

\[
I_c \sum_{k=0}^{n-1} C(kT) \left[ \int_{m}^{T} \lambda t dt + t \right]
\]

\[
= \frac{cI_c \lambda \lambda^3}{6} \left( 2T^3 - 3mT^2 + m^3 \right) \left( e^{\lambda H} - 1 \right)
\]

(11)

The interest earned in \((0, H)\) is

\[
I_c \sum_{k=0}^{n-1} P(kT) \left[ \int_{0}^{m} \lambda t dt \right] = \frac{pl_c \lambda T^3}{3} \left( e^{\lambda H} - 1 \right)
\]

(12)
Hence, the total relevant cost in \((0, H)\) is

\[
Z_3(T) = \left\{ s + \frac{c \lambda T^2}{2} + \frac{c \lambda T^3}{3} + \frac{c I_c \lambda}{6} \left(2T^3 - 3mT^2 + m^3\right) - \frac{p l_d \lambda m^3}{3} \left(m + \frac{T}{3}\right) \right\} \left(\frac{e^{rH}}{e^{rT}} - 1\right)
\]  

(13)

**Case 4: \(m \leq T_d \leq T\)**

Since \(T \geq T_d \geq m\), case 4 is similar to case 3. Therefore, the total relevant cost in \((0, H)\) is

\[
Z_4(T) = \left\{ s + \frac{c \lambda T^2}{2} + \frac{c \lambda T^3}{3} + \frac{c I_c \lambda}{6} \left(2T^3 - 3mT^2 + m^3\right) - \frac{p l_d \lambda m^3}{3} \left(m + \frac{T}{3}\right) \right\} \left(\frac{e^{rH}}{e^{rT}} - 1\right)
\]  

(14)

### 4. Theoretical Results

In reality, inflation rate \(r\) is usually very small. Using Taylor’s series expansion for the exponential term, we have

\[
e^{rT} \approx 1 + rT
\]  

(15)

Using the above approximation, the total relevant cost \(Z(T), i = 1, 2, 3, 4\) can be rewritten as

\[
Z_1(T) \approx \left\{ \frac{s}{T} + \frac{c \lambda T}{2} + \frac{c \lambda T^2}{3} (h + I_c) \right\} \frac{(e^{rH} - 1)}{r}
\]  

(16)

\[
Z_2(T) \approx \left\{ \frac{s}{T} + \frac{c \lambda T}{2} + \frac{c \lambda h T^2}{3} - \frac{p l_d \lambda T^2}{2} \left(m - \frac{T}{3}\right) \right\} \frac{(e^{rH} - 1)}{r}
\]  

(17)

\[
Z_3(T) \approx \left\{ \frac{s}{T} + \frac{c \lambda T}{2} + \frac{c \lambda h T^2}{3} + \frac{c I_c \lambda}{6} \left(2T^2 - 3mT + \frac{m^3}{T}\right) \right\} \frac{(e^{rH} - 1)}{r}
\]  

(18)

\[
Z_4(T) \approx \left\{ \frac{s}{T} + \frac{c \lambda T}{2} + \frac{c \lambda h T^2}{3} + \frac{c I_c \lambda}{6} \left(2T^2 - 3mT + \frac{m^3}{T}\right) \right\} \frac{(e^{rH} - 1)}{r}
\]  

(19)

The first order condition for \(Z_i(T)\) in (16) is \(dZ_i(T)/dT = 0\), we get

\[
\frac{dZ_1}{dT} = \left\{ \frac{s}{T} + \frac{c \lambda}{2} + \frac{2c \lambda}{3} (h + I_c) \right\} \frac{(e^{rH} - 1)}{r} = 0
\]  

(20)

The second order condition for case 1, is

\[
\frac{d^2 Z_1(T)}{dT^2} = \left\{ \frac{2s}{T^3} + \frac{2c \lambda}{3} (h + I_c) \right\} \frac{(e^{rH} - 1)}{r} > 0
\]  

(21)
The optimal (minimum) value of \( T = T_1 \) is obtained on solving Eqn. (20). At \( T = T_1 \), we get the optimal economic order quantity \( Q^* (T_1) \). For case 1, \( T_1 < T_d \).

The first order condition for case 2 is \( \frac{dZ_2(T)}{dT} = 0 \), we get

\[
\frac{dZ_2(T)}{dT} = \left\{ -\frac{s}{T^2} + \frac{c \lambda}{2} + \frac{2c \lambda h T}{3} - \frac{p I_d \lambda}{2} \left( m - \frac{2T}{3} \right) \right\} \left( e^{rH} - 1 \right) = 0
\]  

(22)

The second order condition for case 2 is

\[
\frac{d^2Z_2(T)}{dT^2} = \left( \frac{2s}{T^3} + \frac{2c \lambda h}{3} + \frac{p I_d \lambda}{3} \right) \left( e^{rH} - 1 \right) > 0
\]  

(23)

The optimal (minimum) value of \( T = T_2 \) is obtained on solving Eqn. (22). At \( T = T_2 \), we get the optimal economic order quantity \( Q^* (T_2) \) in this case. For case 2, \( T_2 \leq T_2 < m \).

The first order condition for case 3 is \( \frac{dZ_3(T)}{dT} = 0 \), we get

\[
\frac{dZ_3(T)}{dT} = \left\{ -\frac{s}{T^2} + \frac{c \lambda}{2} + \frac{2c \lambda h T}{3} + \frac{c I_m \lambda}{6} \left( 4T^3 - 3m + \frac{m^3}{T^2} \right) \right\} \left( e^{rH} - 1 \right) = 0
\]  

(24)

The second order condition for case 3 is

\[
\frac{d^2Z_3(T)}{dT^2} = \left( \frac{2s}{T^3} + \frac{2c \lambda h}{3} + \frac{c I_m \lambda}{6} \left( 4 - \frac{2m^3}{T^3} \right) \right) \left( e^{rH} - 1 \right) > 0
\]  

(25)

The optimal (minimum) value of \( T = T_3 \) is obtained on solving equation (24). At \( T = T_3 \), we get the optimal economic order quantity \( Q^* (T_3) \), in this case. For case 3, \( T_3 \geq m \geq d \). Because the total relevant cost in case 4 is the same as that in case 3, the optimal (minimum) value of \( T = T_4 \) for case 4 is obtained on solving equation (24). At \( T = T_4 \), the optimal economic order quantity for case 4 is \( Q^* (T_4) \). For case 4, \( T_4 \geq T_4 \geq m \).

From equations (20), (22) and (24), we have

\[
4c \lambda (h + I_d)T^3 + 3c \lambda T^2 - 6s = 0
\]

(26)

\[
2 \lambda (2c h + p I_d)T^3 + 3 \lambda (c - p I_d m) T^2 - 6s = 0
\]

(27)

and

\[
4c \lambda (h + I_d)T^3 + 3c \lambda (1 - I_\lambda m) T^2 - (6s + c I_\lambda m^3) = 0
\]

(28)

Equations (26), (27) and (28) are cubic in \( T \). These equations can be solved by Numerical technique method (like bisection method, Regula Falsi method or Newton Raphson method) or trial and error method for different numerical values of the parameters.
Figure 1 - Four possible inventory systems.
5. Numerical examples

Example 1. Let $H = 1$ year, $\lambda = 500$ units, $h = $ 2/unit/year, $I_1 = 0.10$/S/year, $I_d = 0.05$/S/year, $s = $ 150 per order and $c = $ 25 per unit. Substituting these values in Eqn. (26), we have

$$350T^3 + 125T^2 - 3 = 0$$

Solving Eqn. (29), we get optimal $T = T_1 = 0.13233107$ years, and optimal economic order quantity $Q^*(T_1) = 4.38$ units, and $Z(T) = $2167.55. If $Q_d = 5$ units, then $T_d = 0.141421356$ years, which shows that if $Q^* < Q_d$, then $T_1 < T_d$, which proves case I.

Example 2. Let $H = 1$ year, $\lambda = 100$ units, $h = $ 2/unit/year, $I_1 = 0.08$/S/year, $I_d = 0.05$/S/year, $r = 0.05$ per unit, $c = $ 30 per unit, $p = $ 40 per unit, $m = 90$ days = 0.246575342 years, $s = $ 50 per order. Substituting these values in Eqn. (27), we get

$$24400T^3 + 8852.0547957^2 - 300 = 0$$

Solving, we get $T = T_2 = 0.154212549$ years, and optimal economic order quantity $Q^*(T_2) = 7.71$ units, and $Z(T_2) = $ 615.36 which shows that, $T_2 \leq m$, if $Q^*(T_2) \geq Q_d$, then $T_2 \geq T_d$, we get $T_d \leq T_2 \leq m$, which proves case 2.

Example 3. Let $H = 1$ year, $\lambda = 100$ units, $h = $ 2/unit/year, $I_1 = 0.10$/S/year, $m = 90$ days = 0.246575342 years, $c = $ 10 per unit, $r = 0.05$ per unit, and $s = $ 100 per order. Substituting these values in Eqn. (28), we get

$$8400T^3 + 2926.027397T^2 - 601.4991613 = 0$$

Solving Eqn. (31), we get $T = T_3 = 0.325892372$ years, and optimal economic order quantity $Q^*(T_3) = 16.29$ units, and $Z(T_3) = $ 712.01 which shows that, $T_3 \leq m$, if $Q^*(m) \geq Q_d$, then $m \geq T_d$, we get $T_3 \leq m \leq T_d$, which proves case 3.

For case 4, using all parameters in Ex.3, we get $T_4 \geq m$, also if $Q^*(m) \leq Q_d$, then $m \leq T_d$, we have $T_4 \geq T_d \geq m$ (i.e. $T_4 \geq m$), which proves case 4.

6. Sensitivity Analysis

We have performed sensitivity analysis by changing $s$, $c$, $h$ and $m$ and keeping the remaining parameters at their original values. The corresponding variations is the cycle time, economic order quantity and total relevant cost are exhibited in Table 1 (Table 1.a, Table 1.b, Table 1.c) for case 1, Table 2 (Table 2.a, Table 2.b, Table 2.c, Table 2.d) for case 2 and Table 3 (Table 3.a, Table 3.b, Table 3.c, Table 3.d) for case 3 respectively.

Case 1 - Table 1

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $(T_1)$ (in years)</th>
<th>Economic order quantity $(Q^* (T_1))$</th>
<th>Total relevant cost $(Z^* (T_1))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>0.136142</td>
<td>4.633661</td>
<td>2243.9401</td>
</tr>
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<td>170</td>
<td>0.139813</td>
<td>4.886919</td>
<td>2318.2567</td>
</tr>
<tr>
<td>180</td>
<td>0.143356</td>
<td>5.137736</td>
<td>2390.6802</td>
</tr>
<tr>
<td>190</td>
<td>0.146783</td>
<td>5.386312</td>
<td>2461.3638</td>
</tr>
<tr>
<td>200</td>
<td>0.153323</td>
<td>5.876986</td>
<td>2531.1504</td>
</tr>
</tbody>
</table>

Table 1.b - Sensitivity analysis on $c$

<table>
<thead>
<tr>
<th>$c$</th>
<th>Replenishment cycle time $(T_1)$ (in years)</th>
<th>Economic order quantity $(Q^* (T_1))$</th>
<th>Total relevant cost $(Z^* (T_1))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>0.130063</td>
<td>4.229096</td>
<td>2207.3600</td>
</tr>
<tr>
<td>27</td>
<td>0.127914</td>
<td>4.090498</td>
<td>2246.3951</td>
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<td>28</td>
<td>0.125874</td>
<td>3.961066</td>
<td>2284.7012</td>
</tr>
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<td>29</td>
<td>0.123934</td>
<td>3.839909</td>
<td>2322.3176</td>
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<td>30</td>
<td>0.122086</td>
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Table 1.c - Sensitivity analysis on $h$

<table>
<thead>
<tr>
<th>$h$</th>
<th>Replenishment cycle time $(T_1)$ (in years)</th>
<th>Economic order quantity $(Q^* (T_1))$</th>
<th>Total relevant cost $(Z^* (T_1))$</th>
</tr>
</thead>
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<tr>
<td>2.1</td>
<td>0.131590</td>
<td>4.328982</td>
<td>2174.9928</td>
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<tr>
<td>2.2</td>
<td>0.130869</td>
<td>4.281674</td>
<td>2182.3506</td>
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<td>2.3</td>
<td>0.130165</td>
<td>4.235732</td>
<td>2189.6286</td>
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<tr>
<td>2.4</td>
<td>0.129478</td>
<td>4.191138</td>
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</tr>
<tr>
<td>2.5</td>
<td>0.128808</td>
<td>4.147875</td>
<td>2203.9549</td>
</tr>
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</table>

Case 2 - Table 2

Table 2.a - Sensitivity analysis on $s$

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $(T_2)$ (in years)</th>
<th>Economic order quantity $(Q^* (T_2))$</th>
<th>Total relevant cost $(Z^* (T_2))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.089671</td>
<td>0.402044</td>
<td>781.3700</td>
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</tbody>
</table>
## Table 2.b - Sensitivity analysis on $c$

<table>
<thead>
<tr>
<th>$c$</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q(T_2)$</th>
<th>Total relevant cost $Z(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
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<td>624.8078</td>
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<tr>
<td>32</td>
<td>0.149908</td>
<td>1.123620</td>
<td>634.1051</td>
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<tr>
<td>33</td>
<td>0.147895</td>
<td>1.093646</td>
<td>643.2546</td>
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<tr>
<td>34</td>
<td>0.145965</td>
<td>1.065289</td>
<td>652.2632</td>
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<td>35</td>
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<td>1.038428</td>
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</tbody>
</table>

## Table 2.c - Sensitivity analysis on $h$

<table>
<thead>
<tr>
<th>$h$</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q(T_2)$</th>
<th>Total relevant cost $Z(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>0.153242</td>
<td>1.174156</td>
<td>617.7788</td>
</tr>
<tr>
<td>2.2</td>
<td>0.152300</td>
<td>1.159764</td>
<td>620.1720</td>
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<tr>
<td>2.3</td>
<td>0.151383</td>
<td>1.145841</td>
<td>622.5361</td>
</tr>
<tr>
<td>2.4</td>
<td>0.150491</td>
<td>1.132377</td>
<td>624.8722</td>
</tr>
<tr>
<td>2.5</td>
<td>0.149623</td>
<td>1.119352</td>
<td>627.1811</td>
</tr>
</tbody>
</table>

## Table 2.d - Sensitivity analysis on $m$

<table>
<thead>
<tr>
<th>$m$ (in days)</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q(T_2)$</th>
<th>Total relevant cost $Z(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.154300</td>
<td>1.190424</td>
<td>614.9222</td>
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<tr>
<td>110</td>
<td>0.154388</td>
<td>1.191783</td>
<td>614.8868</td>
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<tr>
<td>120</td>
<td>0.154475</td>
<td>1.193126</td>
<td>614.0548</td>
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<tr>
<td>130</td>
<td>0.154563</td>
<td>1.194486</td>
<td>613.6206</td>
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<tr>
<td>140</td>
<td>0.154651</td>
<td>1.195846</td>
<td>613.1863</td>
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<tr>
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<td>1.197223</td>
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</tr>
</tbody>
</table>

## Case 3 - Table 3

### Table 3.a - Sensitivity analysis on $s$

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q(T_3)$</th>
<th>Total relevant cost $Z(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>0.338587</td>
<td>5.732058</td>
<td>585.5010</td>
</tr>
<tr>
<td>120</td>
<td>0.350566</td>
<td>6.144826</td>
<td>615.2581</td>
</tr>
<tr>
<td>130</td>
<td>0.361923</td>
<td>6.549413</td>
<td>644.0409</td>
</tr>
<tr>
<td>140</td>
<td>0.372734</td>
<td>6.946532</td>
<td>671.9574</td>
</tr>
<tr>
<td>150</td>
<td>0.383060</td>
<td>7.336748</td>
<td>699.0892</td>
</tr>
</tbody>
</table>

### Table 3.b - Sensitivity analysis on $c$

<table>
<thead>
<tr>
<th>$c$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q(T_3)$</th>
<th>Total relevant cost $Z(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.b</td>
<td>0.313630</td>
<td>4.918188</td>
<td>578.037168</td>
</tr>
<tr>
<td>12</td>
<td>0.302802</td>
<td>4.584452</td>
<td>600.342696</td>
</tr>
<tr>
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<td>0.293142</td>
<td>4.296612</td>
<td>621.694854</td>
</tr>
<tr>
<td>14</td>
<td>0.284448</td>
<td>4.045533</td>
<td>642.107239</td>
</tr>
<tr>
<td>15</td>
<td>0.274967</td>
<td>3.780342</td>
<td>661.982127</td>
</tr>
</tbody>
</table>

### Table 3.c - Sensitivity analysis on $h$

<table>
<thead>
<tr>
<th>$h$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q(T_3)$</th>
<th>Total relevant cost $Z(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>0.322932</td>
<td>5.214254</td>
<td>558.2364</td>
</tr>
<tr>
<td>2.2</td>
<td>0.320087</td>
<td>5.122784</td>
<td>561.1106</td>
</tr>
<tr>
<td>2.3</td>
<td>0.317349</td>
<td>5.035519</td>
<td>565.2414</td>
</tr>
<tr>
<td>2.4</td>
<td>0.314710</td>
<td>4.952119</td>
<td>568.6550</td>
</tr>
<tr>
<td>2.5</td>
<td>0.312166</td>
<td>4.872381</td>
<td>572.0129</td>
</tr>
</tbody>
</table>

### Table 3.d - Sensitivity analysis on $m$

<table>
<thead>
<tr>
<th>$m$ (in days)</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q(T_3)$</th>
<th>Total relevant cost $Z(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.326205</td>
<td>5.320485</td>
<td>554.4734</td>
</tr>
<tr>
<td>110</td>
<td>0.326544</td>
<td>5.331549</td>
<td>554.3714</td>
</tr>
<tr>
<td>120</td>
<td>0.326913</td>
<td>5.343605</td>
<td>554.3395</td>
</tr>
<tr>
<td>130</td>
<td>0.327315</td>
<td>5.356755</td>
<td>554.3839</td>
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<td>140</td>
<td>0.327751</td>
<td>5.371036</td>
<td>554.5108</td>
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<td>150</td>
<td>0.328225</td>
<td>5.386582</td>
<td>554.7262</td>
</tr>
</tbody>
</table>

From the above tables the following results have been obtained:

a. The computational results are shown in Table 1.a, indicates that a higher value of ordering cost ‘$s$’ implies higher values of replenishment cycle time $T_1$, order quantity $Q(T_1)$ and total relevant cost $Z(T_1)$.

b. The computational results are shown in Table 1.b, indicates that a higher value of unit purchasing cost ‘$c$’ implies lower values of replenishment cycle time $T_1$, order quantity $Q(T_1)$ and total relevant cost $Z(T_1)$.

c. The computational results are shown in Table 1.c, indicates that a higher value of holding cost ‘$h$’ implies lower values of replenishment cycle time $T_1$, order quantity $Q(T_1)$ and total relevant cost $Z(T_1)$.

d. The computational results are shown in Table 2.a, indicates that a higher value of ordering cost ‘$s$’ implies higher values of replenishment cycle time $T_2$, order quantity $Q(T_2)$ and total relevant cost $Z(T_2)$.

e. The computational results are shown in Table 2.b, indicates that a higher value of unit purchasing cost ‘$c$’ implies lower values of
replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and total relevant cost $Z'(T_2)$.

f. The computational results are shown in Table 2.c, indicates that a higher value of holding cost implies higher values of replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and total relevant cost $Z'(T_2)$.

g. The computational results are shown in Table 2.d, indicates that a higher values of credit period implies higher values of replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and total relevant cost $Z'(T_2)$.

The computational results are shown in Table 2.a, indicates that a higher value of ordering cost implies higher values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and total relevant cost $Z'(T_3)$.

i. The computational results are shown in Table 2.b, indicates that a higher value of unit purchasing cost implies lower values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and total relevant cost $Z'(T_3)$.

j. The computational results are shown in Table 2.c, indicates that a higher value of holding cost implies lower values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and total relevant cost $Z'(T_3)$.

k. The computational results are shown in Table 2.d, indicates that a higher values of credit period implies higher values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and total relevant cost $Z'(T_3)$.

The proposed model can be extended in several ways. For instance, we may extend the demand rate to a quadratic time dependent demand rate. We could also consider the demand as a function of quantity, selling price, product quantity and others. Finally, we could generalize the model to allow for shortages, quantity discounts and time-dependent deterioration rate, etc.

7. Conclusion

We developed an EOQ model under inflation for non-deteriorating items and time-dependent demand rate to determine the optimal ordering policy when the supplier provides a permissible delay in payments linked to order quantity. We use Taylor’s series approximation to obtain the explicit solution of the optimal replenishment cycle time and total relevant cost. Finally, numerical examples are studied to illustrate the proposed model. There are some managerial phenomena (1) For case 1(a), a higher value of ordering cost causes higher values of replenishment cycle time $T_1$, order quantity $Q'(T_1)$ and total relevant cost $Z'(T_1)$ (b) a higher value of unit purchasing cost causes lower values of replenishment cycle time $T_1$, order quantity $Q'(T_1)$ and higher values of total relevant cost $Z(T_1)$, (c) a higher value of holding cost causes lower values of replenishment cycle time $T_1$, order quantity $Q'(T_1)$ and higher values of total relevant cost $Z'(T_1)$. For case 2(a) a higher value of ordering cost causes higher values of replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and total relevant cost $Z'(T_2)$, (b) a higher value of unit purchasing cost causes lower values of replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and higher values of total relevant cost $Z'(T_2)$, (c) a higher value of holding cost causes lower values of replenishment cycle time $T_2$, order quantity $Q'(T_2)$ and lower values of total relevant cost $Z'(T_2)$. For case 3, (a) a higher value of ordering cost implies higher values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and total relevant cost $Z'(T_3)$, (b) a higher value of unit purchasing cost implies lower values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and higher values of total relevant cost $Z'(T_3)$, (c) a higher value of holding cost implies lower values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and higher values of total relevant cost $Z'(T_3)$, (d) a higher value of credit period implies higher values of replenishment cycle time $T_3$, order quantity $Q'(T_3)$ and approximately constant values of total relevant cost $Z'(T_3)$.

The proposed model can be extended in several ways. For instance, we may extend the demand rate to a quadratic time dependent demand rate. We could also consider the demand as a function of quantity, selling price, product quantity and others. Finally, we could generalize the model to allow for shortages, quantity discounts and time-dependent deterioration rate, etc.

References


