Quick Simplex Algorithm for Optimal Solution to the Linear Programming Problem along with Theoretical Proof of Formulae.

#1 Mrs. N. V. Vaidya, #2 Dr. Mrs. N. N. Kasturiwale

#1 Assistant professor, Dr. Babasaheb Ambedkar College of Engg and Research, Wanadongari, Nagpur 441110, INDIA.

#2 Professor, Department of Statistics, Institute of Science, Nagpur 440 001, INDIA.

nkasturiwale@rediffmail.com, nalinivaidya1968@yahoo.com

Abstract—In this paper, a new approach is suggested while solving linear programming problems using simplex method. The method sometimes involves less iteration than in the simplex method or at the most an equal number because the method attempts to replace more than one basic variable simultaneously.

Keywords: basic feasible solution, optimum solution, simplex method, key determinant.

1. Introduction

The linear programming has its own importance in obtaining the solution of a problem where two or more activities complete for limited resources.

Mathematically we have to maximize the objective function $cx$, subject to $Ax = b$, $x \geq 0$

Where

- $x = n \times 1$ column vector
- $A = m \times n$ coefficient matrix
- $b = m \times 1$ Column vector
- $C = 1 \times n$ row vector

and the columns of $A$ are denoted by $P_1, \ldots, P_n$.

There are two methods to obtain the solution of the above problem. These methods can be classified as:

(i) The graphical method
(ii) Simplex method.

The simplex method is the most general and powerful. We now give a brief account of the simplex method as below:

Consider a non-degenerate basic feasible solution

$$x_0 = (x_{10}, x_{20}, \ldots, x_{m0}, 0 \ldots 0)$$

The corresponding value of the objective function is

$$x_{10}c_1 + x_{20}c_2 + \cdots + x_{m0}c_m = z_0$$

It follows from the study of linear programming that for any fixed $j$, a set of feasible solutions can be constructed such that $z < z_0$ for any member of the set where net evaluation $z_j - c_j > 0$. The condition imposed on $\theta$ is

$$\theta = \min \frac{x_{10}}{x_{1j}} > 0, x_{ij} > 0 \text{ for fixed } j.$$
2. Simultaneous replacement of \( n \) variables is possible only when **pivotal** elements in the entering vectors are in different rows.

3. We define key determinant \( R \), which is of order \( n \).

4. \( R \) is the determinant of sub matrix of matrix \( A \).

5. This sub matrix is obtained by using rows and columns containing pivotal elements.

6. We are giving formula to obtain simplex table after replacement of such \( n \) variables.

7. We shall call elements in the new simplex table as * elements.

8. Star elements are obtained by ratio of two determinants.

9. Denominator is nothing but the determinant \( R \).

10. In the rows containing a pivotal element numerator is of order \( n \).

11. Numerators are obtained as follows:

Here column of pivotal element is replaced by the column for which * elements are to be obtained.

Numerator of star elements in the rows in which no pivotal element is there is a determinant of order \((n+1)\)

12. It is obtained by adding the corresponding column in the current simplex table and the row of the element to \( R \).

13. In previous case, simplex table after replacing \( P_5, P_6, P_7 \) by \( P_1, P_2, P_3 \) will be as follows.

---

### Table 1

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
<th>( P_7 )</th>
<th>( P_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pivot ( a_1 )</td>
<td>( b_1 )</td>
<td>( c_1 )</td>
<td>( d_1 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>Pivot ( b_2 )</td>
<td>( c_2 )</td>
<td>( d_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>( b_3 )</td>
<td>Pivot ( c_3 )</td>
<td>( d_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>( b_4 )</td>
<td>( c_4 )</td>
<td>( d_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Here \( a_1, b_2 \) and \( c_3 \) are the pivotal elements when \( P_1, P_2, P_3 \) are the entering vectors in initial simplex table, then \( R = \) **Table 2**

<table>
<thead>
<tr>
<th>Pivot ( a_1 )</th>
<th>( b_1 )</th>
<th>( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_2 )</td>
<td>Pivot ( b_2 )</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>( b_3 )</td>
<td>Pivot ( c_3 )</td>
</tr>
</tbody>
</table>

is of order 3 as 3 variables are entered simultaneously.

---
$d_4^{***} = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix}$

14. One must evaluate $X_B$ Column first and nth iteration table should be evaluated only, when all the entries in $X_B$ column comes to be non negative because it may indicate whether n variables can be entered. If any entry is negative then try by entering (n-1) variables instead of n variables.

3. Theoretical Proof of above formulae.

Consider Initial simplex table as following:

**Step 1:** Initial simplex table

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pivot $a_1$</td>
<td>$b_1$</td>
<td>$c_1$</td>
<td>$d_1$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Pivot $b_2$</td>
<td>$c_2$</td>
<td>$d_2$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$b_3$</td>
<td>Pivot $c_3$</td>
<td>$d_3$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$b_4$</td>
<td>$c_4$</td>
<td>$d_4$</td>
</tr>
</tbody>
</table>

**Step 2:** First iteration Simplex table

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$b_1^* = \frac{b_1}{a_1}$</td>
<td>$c_1^* = \frac{c_1}{a_1}$</td>
<td>$d_1^* = \frac{d_1}{a_1}$</td>
</tr>
<tr>
<td>0</td>
<td>$b_2^* = b_2 - \frac{a_2b_1}{a_1}$</td>
<td>$c_2^* = c_2 - \frac{a_2c_1}{a_1}$</td>
<td>$d_2^* = d_2 - \frac{a_2d_1}{a_1}$</td>
</tr>
<tr>
<td>0</td>
<td>$b_3^* = b_3 - \frac{a_3b_1}{a_1}$</td>
<td>$c_3^* = c_3 - \frac{a_3c_1}{a_1}$</td>
<td>$d_3^* = d_3 - \frac{a_3d_1}{a_1}$</td>
</tr>
<tr>
<td>0</td>
<td>$b_4^* = b_4 - \frac{a_4b_1}{a_1}$</td>
<td>$c_4^* = c_4 - \frac{a_4c_1}{a_1}$</td>
<td>$d_4^* = d_4 - \frac{a_4d_1}{a_1}$</td>
</tr>
</tbody>
</table>

**Step 3:** Second iteration Simplex table

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$c_1^{**} = c_1^* - \frac{b_1^<em>c_2^</em>}{b_2^*}$</td>
<td>$d_1^{**} = d_1^* - \frac{b_1^<em>d_2^</em>}{b_2^*}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$c_2^{**} = \frac{c_2}{b_2^*}$</td>
<td>$d_2^{**} = \frac{d_2}{b_2^*}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$c_3^{**} = c_3^* - \frac{b_3^<em>c_2^</em>}{b_2^*}$</td>
<td>$d_3^{**} = d_3^* - \frac{b_3^<em>d_2^</em>}{b_2^*}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$c_4^{**} = c_4^* - \frac{b_4^<em>c_2^</em>}{b_2^*}$</td>
<td>$d_4^{**} = d_4^* - \frac{b_4^<em>d_2^</em>}{b_2^*}$</td>
</tr>
</tbody>
</table>
Step 4: Third iteration Simplex table

<table>
<thead>
<tr>
<th>$\mathbf{y}_1$</th>
<th>$\mathbf{y}_2$</th>
<th>$\mathbf{y}_3$</th>
<th>$\mathbf{y}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$d_{1}^{*<strong>} = d_{1}^{</strong>} - \frac{d_{3}^{<strong>} c_{1}^{</strong>}}{c_{3}^{**}}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$d_{2}^{*<strong>} = d_{2}^{</strong>} - \frac{d_{3}^{<strong>} c_{2}^{</strong>}}{c_{3}^{**}}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$d_{3}^{*<strong>} = \frac{d_{3}^{</strong>}}{c_{3}^{**}}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$d_{4}^{*<strong>} = \frac{d_{3}^{</strong>} c_{4}^{<strong>}}{c_{3}^{</strong>}}$</td>
</tr>
</tbody>
</table>

1. Pivotal elements are occurring in different rows.
2. Result 1:- Expressing third order determinant in terms of second order determinants.

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

perform $R_2 = R_2 - \frac{a_2}{a_1} R_1$

and $R_3 = R_3 - \frac{a_3}{a_1} R_1$

$\therefore A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 - \frac{a_2}{a_1} a_1 & b_2 - \frac{a_2}{a_1} b_1 & c_2 - \frac{a_2}{a_1} c_1 \\ a_3 - \frac{a_3}{a_1} a_1 & b_3 - \frac{a_3}{a_1} b_1 & c_3 - \frac{a_3}{a_1} c_1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 b_1 & a_1 c_1 \\ 0 & a_1 b_2 & a_1 c_2 \end{vmatrix}$

$= a_1 (k_1 k_4 - k_2 k_3) = a_1 \left( \frac{1}{a_1^2} \right) \left[ \begin{vmatrix} a_1 & b_1 & a_1 \\ a_2 & b_2 & a_3 \\ a_3 & b_3 & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & b_1 & a_1 \\ a_2 & b_2 & a_3 \\ a_3 & b_3 & c_3 \end{vmatrix} \right]$

$= a_1 \left( \frac{1}{a_1} \right) \left[ a_1 b_1 a_3 c_1 - a_1 a_3 b_3 c_1 \right]$

3. We have made use of determinants to express the above entries and obtained the results as follows

$b_1^* = \frac{b_1}{a_1} \ldots 1$
$b_2^* = \frac{b_2}{a_2} \ldots 4$
$b_3^* = \frac{b_3}{a_3} \ldots 7$
$b_4^* = \frac{b_4}{a_4} \ldots 10$
$c_1^* = \frac{c_1}{a_1} \ldots 2$
$c_2^* = \frac{c_2}{a_2} \ldots 5$
$c_3^* = \frac{c_3}{a_3} \ldots 8$
$c_4^* = \frac{c_4}{a_4} \ldots 11$

$d_1^* = \frac{d_1}{a_1} \ldots 3$
$d_2^* = \frac{d_2}{a_2} \ldots 6$
$d_3^* = \frac{d_3}{a_3} \ldots 9$
$d_4^* = \frac{d_4}{a_4} \ldots 12$
\[ c_1^{**} = \frac{b_1^* c_2^*}{b_2^*} = \frac{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \cdot \begin{vmatrix} a_3 & c_1 \\ a_4 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} \]

\[ = \frac{1}{a_1} \left( c_1 \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} - b_1 \begin{vmatrix} a_1 & a_2 \\ a_2 & a_3 \end{vmatrix} \right) / \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \]

\[ = \frac{1}{a_1} \left( \begin{vmatrix} 0 & b_1 & c_1 \\ a_1 & a_2 & c_2 \\ a_2 & a_3 & c_3 \end{vmatrix} \right) / \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \]

\[ c_1^{**} = \frac{c_1}{c_2^{**}} \]

\[ c_2^{**} = \frac{c_2}{c_3^{**}} \]

\[ c_3^{**} = \frac{c_3}{c_4^{**}} \]

\[ c_4^{**} = \frac{c_4}{c_5^{**}} \]

\[ \therefore c_5^{**} = \frac{c_5}{c_6^{**}} \] (using result 1)

\[ d_1^{**} = \frac{d_1}{d_2^{**}} \]

\[ d_2^{**} = \frac{d_2}{d_3^{**}} \] (using result 1)
Also using result 1 we can write

\[ d_3^* = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \ldots 19 \]

Now

\[ d_1^{***} = \frac{c_1^{**} \cdot d_1^{**}}{c_3^{**}} = \begin{vmatrix} b_2^* & c_2^* & d_2^* \\ b_3^* & c_3^* & d_3^* \end{vmatrix} \begin{vmatrix} b_2^* & c_2^* \\ b_3^* & c_3^* \end{vmatrix} \begin{vmatrix} b_2^* \\ b_3^* \end{vmatrix} \]

\[ d_1^{***} = \frac{1}{b_2^*} \begin{vmatrix} b_2^* & c_2^* & d_2^* & b_2^* & c_2^* & d_2^* \\ b_3^* & c_3^* & d_3^* & b_3^* & c_3^* & d_3^* \end{vmatrix} \begin{vmatrix} b_2^* \\ b_3^* \\ b_2^* \\ b_3^* \end{vmatrix} \begin{vmatrix} b_2^* \\ b_3^* \end{vmatrix} \]

\[ d_1^{***} = \begin{vmatrix} b_3 & c_1 & d_1 \\ a_1 & a_1 & a_1 \\ a_2 & a_2 & a_2 \\ a_3 & a_3 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \ldots 21 \]

Numerator of (21) is

\[ = b_1 \left( \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} - \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} \right) - \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} \]

\[ = \frac{1}{a_1^2} \left( b_1 \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} - c_1 \begin{vmatrix} a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} + d_1 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \right) \]

\[ = \frac{-1}{a_1^2} \left( b_1 \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} - a_1 \begin{vmatrix} b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} + a_2 \begin{vmatrix} b_1 & c_1 & d_1 \\ b_3 & c_3 & d_3 \end{vmatrix} - a_3 \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \end{vmatrix} \right) \]
\[
\frac{1}{a_1 b_2 c_2 d_2} \cdots 22
\]

Denominator of 21 is
\[
\frac{1}{a_1 b_2 c_2 d_2} \cdots 23 \text{ using 15}
\]

\[
d_1^{***} = \begin{vmatrix}
    b_1 & c_1 & d_1 \\
    b_2 & c_2 & d_2 \\
    b_3 & c_3 & d_3 \\
\end{vmatrix}
/ \begin{vmatrix}
    a_1 & b_1 & c_1 \\
    a_2 & b_2 & c_2 \\
    a_3 & b_3 & c_3 \\
\end{vmatrix}
\cdots 24 \text{ using 22 and 23}
\]

\[
d_2^{***} = \begin{vmatrix}
    c_2^* & d_2^* \\
    c_3^* & d_3^* \\
\end{vmatrix}
/ \begin{vmatrix}
    b_2^* & c_2^* \\
    b_3^* & c_3^* \\
\end{vmatrix}
\cdots 25
\]

Numerator of 25
\[
= \frac{d_1 c_2 c_3 d_3}{b_2^*}
\]

\[
= \frac{1}{a_1 b_2 c_2 d_2} \left( \begin{vmatrix}
    a_1 & d_1 & c_1 \\
    a_2 & d_2 & c_2 \\
    a_3 & d_3 & c_3 \\
\end{vmatrix}
- \begin{vmatrix}
    a_1 & c_1 & d_1 \\
    a_2 & c_2 & d_2 \\
    a_3 & c_3 & d_3 \\
\end{vmatrix} \right)
\]

\[
= \frac{1}{a_1 b_2^*} \begin{vmatrix}
    a_1 & c_1 & d_1 \\
    a_2 & c_2 & d_2 \\
    a_3 & c_3 & d_3 \\
\end{vmatrix} \cdots 26
\]

since \(c_3^* = \frac{1}{a_1 b_2^*} \cdots 27\)

\[
d_2^{***} = \begin{vmatrix}
    a_1 & d_1 & c_1 \\
    a_2 & d_2 & c_2 \\
    a_3 & d_3 & c_3 \\
\end{vmatrix} \cdots 28
\]

since \(\begin{vmatrix}
    a_1 & c_1 & d_1 \\
    a_2 & c_2 & d_2 \\
    a_3 & c_3 & d_3 \\
\end{vmatrix} = (-1)\begin{vmatrix}
    a_1 & d_1 & c_1 \\
    a_2 & d_2 & c_2 \\
    a_3 & d_3 & c_3 \\
\end{vmatrix} = \begin{vmatrix}
    a_1 & d_1 & c_1 \\
    a_2 & d_2 & c_2 \\
    a_3 & d_3 & c_3 \\
\end{vmatrix} \cdots 29 \text{ using 15 and 19}
\]

Now,
\[
d_4^{***} = \frac{c_2^* d_2^*}{c_3^*} \cdots 30
\]

Numerator of 30 is
\[
= \begin{vmatrix}
    b_2^* & c_2^* & b_2^* \\
    b_2^* & c_2^* & b_2^* \\
\end{vmatrix}
\]
\[
\begin{align*}
\text{Int. J Latest Trend Math} & \quad \text{Vol-4 No 2 June, 2014} \\
& = \frac{1}{b_2^2} \left( b_2^* c_2^* d_2^* \right) \\
& = \frac{1}{b_2^2} \left( b_3^* c_2^* d_2^* \right) \\
& \quad \text{...31} \\
\text{now} \\
& = \frac{1}{a_1^2} \left( \begin{array}{cccc}
| a_1 & b_1 | & | a_1 & c_1 | & | a_1 & d_1 |
| a_2 & b_2 | & | a_2 & c_2 | & | a_2 & d_2 |
| a_3 & b_3 | & | a_3 & c_3 | & | a_3 & d_3 |
| a_4 & b_4 | & | a_4 & c_4 | & | a_4 & d_4 |
\end{array} \right)
\left( \begin{array}{cccc}
| a_1 | & | a_1 | & | a_1 | & | a_1 |
| a_2 | & | a_2 | & | a_2 | & | a_2 |
| a_3 | & | a_3 | & | a_3 | & | a_3 |
| a_4 | & | a_4 | & | a_4 | & | a_4 |
\end{array} \right)
\left( \begin{array}{cccc}
| a_1 | & | a_1 | & | a_1 | & | a_1 |
| a_2 | & | a_2 | & | a_2 | & | a_2 |
| a_3 | & | a_3 | & | a_3 | & | a_3 |
| a_4 | & | a_4 | & | a_4 | & | a_4 |
\end{array} \right)
\left( \begin{array}{cccc}
| a_1 | & | a_1 | & | a_1 | & | a_1 |
| a_2 | & | a_2 | & | a_2 | & | a_2 |
| a_3 | & | a_3 | & | a_3 | & | a_3 |
| a_4 | & | a_4 | & | a_4 | & | a_4 |
\end{array} \right)
\end{align*}
\]
Here we can find $d_1^{***}, d_2^{***}, d_3^{***}$ and $d_4^{***}$ using following formula.

\[
\begin{align*}
d_1^{***} &= \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} \\
&= \frac{1}{R} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\
d_2^{***} &= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\
&= \frac{1}{R} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\
d_3^{***} &= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\
&= \frac{1}{R} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
\end{align*}
\]

Where $R = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

4. Statement of the Problem

Solve the LPP: Max $Z = 950x_1 + 1221.5x_2 + 1325x_3$

Subject to the constraints:

\[
\begin{align*}
5x_1 + 5x_2 + 10x_3 &\leq 1,00,000 \\
10x_1 + 10x_2 + 10x_3 &\leq 1,80,000 \\
5x_1 + 10x_2 + 10x_3 &\leq 1,20,000 \\
x_1 &\geq 6,000 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{align*}
\]

5. Solution of the Problem:

Max $Z = 950x_1 + 1221.5x_2 + 1325x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 - Mx_8$

Subject to the constraints:

\[
\begin{align*}
5x_1 + 5x_2 + 10x_3 + x_4 &\leq 1,00,000 \\
10x_1 + 10x_2 + 10x_3 + x_5 &\leq 1,80,000 \\
5x_1 + 10x_2 + 10x_3 + x_6 &\leq 1,20,000 \\
x_1 - x_7 + x_8 &\geq 6,000 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{align*}
\]

(where $x_5, x_6, x_7 \rightarrow$ slack variables)

We now employ the simplex method where we choose the entering vector which is most negative.
Step (1): (Initial table)

<table>
<thead>
<tr>
<th>C_B</th>
<th>X_B</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
<th>P_5</th>
<th>P_6</th>
<th>P_7</th>
<th>P_8</th>
<th>x_B</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>x_4</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>950</td>
<td>-M</td>
</tr>
<tr>
<td>0</td>
<td>x_5</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1212.5</td>
<td>18,000</td>
</tr>
<tr>
<td>0</td>
<td>x_6</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1325</td>
<td>24,000</td>
</tr>
<tr>
<td>-M</td>
<td>x_8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-950</td>
</tr>
</tbody>
</table>

Step (2): Introduce P_1 and drop P_8

<table>
<thead>
<tr>
<th>C_B</th>
<th>X_B</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
<th>P_5</th>
<th>P_6</th>
<th>P_7</th>
<th>P_8</th>
<th>x_B</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>x_4</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>950</td>
<td>7,000</td>
</tr>
<tr>
<td>0</td>
<td>x_5</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>1,200</td>
<td>12,000</td>
</tr>
<tr>
<td>0</td>
<td>x_6</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>90,000</td>
<td>9,000</td>
</tr>
<tr>
<td>950</td>
<td>x_1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>6,000</td>
<td>-</td>
</tr>
</tbody>
</table>

Step (3): Introduce P_2 and drop P_4

<table>
<thead>
<tr>
<th>C_B</th>
<th>X_B</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
<th>P_5</th>
<th>P_6</th>
<th>P_7</th>
<th>P_8</th>
<th>x_B</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1325</td>
<td>x_3</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>1/10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>½</td>
<td>7,000</td>
<td>14,000</td>
</tr>
<tr>
<td>0</td>
<td>x_5</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>50,000</td>
<td>10,000</td>
</tr>
<tr>
<td>0</td>
<td>x_6</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>20,000</td>
<td>4,000</td>
</tr>
<tr>
<td>950</td>
<td>x_1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>6,000</td>
<td>-</td>
</tr>
</tbody>
</table>

| 0   | -550| 0   | 132.5| 0   | 0   | 0   | -287.5 |
Step (4): Introduce $P_1$ and drop $P_4$

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1325</td>
<td>$x_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/5</td>
<td>0</td>
<td>-1/10</td>
<td>1/2</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>0</td>
<td>$x_5$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>5</td>
<td>30,000</td>
<td>6,000</td>
</tr>
<tr>
<td>1212.5</td>
<td>$x_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1/5</td>
<td>0</td>
<td>1/5</td>
<td>0</td>
<td>4,000</td>
<td>-</td>
</tr>
<tr>
<td>950</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>6,000</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 12,000$, $x_2 = 4,000$, $x_3 = 2,000$ and Max $Z = 1,89,00,000$.

6. Here we apply quick simplex method.

Step (1): (Initial table)

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$x_4$</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,00,000</td>
</tr>
<tr>
<td>0</td>
<td>$x_5$</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,80,000</td>
</tr>
<tr>
<td>0</td>
<td>$x_6$</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1,20,000</td>
</tr>
<tr>
<td>-M</td>
<td>$x_8$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>6,000</td>
<td>6,000</td>
</tr>
</tbody>
</table>

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 12,000$, $x_2 = 4,000$, $x_3 = 2,000$ and Max $Z = 1,89,00,000$.

6. Here we apply quick simplex method.
Here we introduce simultaneously $P_1, P_2, P_3$ three vectors and outgoing vectors are $P_8, P_6, P_4$. So we get direct fourth simplex table using above formulae. Introduction is such that Pivotal element should be in different row.

Here first entering variable $P_1$ and corresponding outgoing $P_8$

Then second entering variable $P_2$ and corresponding outgoing $P_6$

And third entering variable $P_3$ and corresponding outgoing $P_4$

i.e. we introduce simultaneously $P_1, P_2, P_3$ all the three vectors and outgoing vectors are $P_8, P_6, P_4$. So we get direct fourth simplex table using above formulae. 

$$R = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 10 & 5 \\ 5 & 10 & 10 \end{vmatrix} = 50$$

(No need to adjust pivotal elements in diagonal form)

These columns correspond to $P_1, P_3, P_2$ respectively contains rows and columns of vectors having pivotal elements or $R$ is obtained by considering the determinant made up from column of entering variables by considering rows where pivotal elements are there.

We shall call $R$ as key determinant and $|R|$ is the denominator for every element in the fourth simplex table.

First column of $R$ corresponds to new variable $P_1$ because outgoing variable is $P_8$ in that column and it will be replaced by $P_1$.

Second column of $R$ corresponds to new variable $P_3$ because outgoing variable is $P_4$ in that column and it will be replaced by $P_3$.

Third column of $R$ corresponds to new variable $P_2$ because outgoing variable is $P_6$ in that column and it will be replaced by $P_2$.

Now to find numerators for each entry in the fourth simplex table, if we replace $i$th column of $R$ (Here $i=1,2,3$) by the original vector $P_j$ then we get the entries corresponding to new variables.

To find other entries in fourth simplex table.

$$Column\; P_4 = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \Rightarrow x_1 = \begin{vmatrix} 0 & 0 & 0 \\ 10 & 5 & 1 \\ 10 & 10 & 0 \end{vmatrix} = 0, x_2 = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 10 & 1 \\ 5 & 10 & 0 \end{vmatrix} = \frac{-1}{5}, x_3 = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 1 & 5 \\ 5 & 0 & 10 \end{vmatrix} = \frac{1}{5}$$

$$x_5 = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = 0$$

New $P_4 = \begin{vmatrix} \frac{1}{5} \\ 0 \\ -\frac{1}{5} \end{vmatrix}$
Column $P_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ \quad \because \quad x_1 = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 10 & 0 \\ 10 & 10 & 0 \end{bmatrix} R = 0, \quad x_2 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 10 & 0 \\ 5 & 10 & 0 \end{bmatrix} R = 0, \quad x_3 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 0 & 5 \\ 5 & 0 & 5 \end{bmatrix} R = 0$

And $x_5 = \begin{bmatrix} 0 \\ 1 \\ 10 \\ 10 \\ 10 \end{bmatrix} R = 1$

New $P_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Column $P_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ \quad \because \quad x_1 = \begin{bmatrix} 0 & 0 & 0 \\ 10 & 5 & 0 \\ 10 & 10 & 1 \end{bmatrix} R = 0, \quad x_2 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 10 & 0 \\ 5 & 10 & 0 \end{bmatrix} R = 1$

New $P_6 = \begin{bmatrix} -1/10 \\ -1 \\ 1/5 \\ 0 \\ 0 \end{bmatrix}$

Column $P_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ \quad \because \quad x_1 = \begin{bmatrix} 0 & 0 & -1 \\ 10 & 5 & 0 \\ 10 & 10 & 1 \end{bmatrix} R = -1, \quad x_2 = \begin{bmatrix} 1 & 0 & -1 \\ 5 & 10 & 0 \\ 5 & 10 & 0 \end{bmatrix} R = 0, \quad x_3 = \begin{bmatrix} 1 & -1 & 0 \\ 5 & 0 & 5 \\ 5 & 0 & 5 \end{bmatrix} R = 1/2$

and $x_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 10 \\ 10 \\ 10 \\ 0 \end{bmatrix} R = 5$

New $P_6 = \begin{bmatrix} 1/2 \\ 5 \\ 0 \\ -1 \end{bmatrix}$

**Step (4):** Introduce $P_1$ and drop $P_4$

<table>
<thead>
<tr>
<th></th>
<th>950</th>
<th>1212.5</th>
<th>1325</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
<td>$X_B$</td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_3$</td>
<td>$P_4$</td>
<td>$P_5$</td>
<td>$P_6$</td>
<td>$P_7$</td>
</tr>
<tr>
<td>1325</td>
<td>$x_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/5</td>
<td>0</td>
<td>-</td>
<td>1/10</td>
</tr>
<tr>
<td>0</td>
<td>$x_5$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>1212.5</td>
<td>$x_2$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1/5</td>
<td>0</td>
<td>1/5</td>
<td>0</td>
</tr>
<tr>
<td>950</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>22.5</td>
<td>0</td>
<td>110</td>
<td>-287.5</td>
<td></td>
</tr>
</tbody>
</table>
Here we need one more iteration that is step 5

**Step (5): Introduce \( P_7 \) and drop \( P_5 \)**

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( X_B )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
<th>( P_8 )</th>
<th>( x_B )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1325</td>
<td>( x_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/5</td>
<td>-1/10</td>
<td>0</td>
<td>0</td>
<td>2,000</td>
<td>1325</td>
</tr>
<tr>
<td>0</td>
<td>( x_7 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td>-1/5</td>
<td>1</td>
<td>6,000</td>
<td>1212.5</td>
</tr>
<tr>
<td>1212.5</td>
<td>( x_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1/5</td>
<td>0</td>
<td>1/5</td>
<td>0</td>
<td>4,000</td>
<td>950</td>
</tr>
<tr>
<td>950</td>
<td>( x_1 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td>-1/5</td>
<td>0</td>
<td>12,000</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>22.5</td>
<td>57.5</td>
<td>52.5</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since all \( Z_j - C_j \geq 0 \) hence an optimum basic feasible solution has been reached.

\[ \Rightarrow \text{optimum solution is } x_1 = 12,000, \ x_2 = 4,000, \ x_3 = 2,000 \text{ and Max } Z = 1,89,00,000. \]

**7. Statement of the Problem-II**

Solve the LPP: Max \( Z = x_1 + 2x_2 + 3x_3 - x_4 \)

Subject to the constraints:

\[ x_1 + 2x_2 + 3x_3 = 15 \]
\[ 2x_1 + x_2 + 5x_3 = 20 \]
\[ x_1 + 2x_2 + x_3 + x_4 = 10 \]
\[ x_1, x_2, x_3, x_4 \geq 0 \]

**8. Solution of the Problem.**

Max \( Z = x_1 + 2x_2 + 3x_3 - x_4 \)

Subject to the constraints:

\[ x_1 + 2x_2 + 3x_3 + x_5 = 15 \]
\[ 2x_1 + x_2 + 5x_3 + x_6 = 20 \]
\[ x_1 + 2x_2 + x_3 + x_4 = 10 \]
\[ x_1, x_2, x_3, x_4 \geq 0 \ (\text{Where } x_5, x_6, x_7 \rightarrow \text{slack variables}) \]

We now employ the simplex method where we choose the entering vector which is most negative.

**Step (1): (Initial table)**

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( X_B )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
<th>( P_8 )</th>
<th>( x_B )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M</td>
<td>( x_5 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>15</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>-M</td>
<td>( x_6 )</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>20</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>( x_4 )</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Step (2): Introduce $P_3$ and drop $P_6$

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$x_B$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M</td>
<td>$x_5$</td>
<td>-1/5</td>
<td>7/5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-3/5</td>
<td>3</td>
<td>15/7</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>2/5</td>
<td>1/5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>-1</td>
<td>$x_4$</td>
<td>3/5</td>
<td>9/5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1/5</td>
<td>6</td>
<td>30/9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step (3): Introduce $P_2$ and drop $P_5$

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$x_B$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>-1/7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5/7</td>
<td>-3/7</td>
<td>15/7</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>3/7</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1/7</td>
<td>2/7</td>
<td>25/7</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>$x_4$</td>
<td>6/7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-9/7</td>
<td>4/7</td>
<td>15/7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15/6</td>
</tr>
</tbody>
</table>

Step (4): Introduce $P_1$ and drop $P_4$

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/6</td>
<td>3/6</td>
<td>-1/3</td>
<td>5/2</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-3/6</td>
<td>1/2</td>
<td>0</td>
<td>5/2</td>
</tr>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>7/6</td>
<td>-9/6</td>
<td>4/6</td>
<td>5/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>M+1</td>
<td>M</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.
optic solution is \( x_1 = 5/2, x_2 = 5/2, x_3 = 5/2 \) and \( x_4 = 0 \) Max. \( Z = 15 \).

9. Here we apply quick simplex method.

**Step (1):** (Initial table)

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( X_B )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
<th>( x_B )</th>
<th>Ratio 1</th>
<th>Ratio 2</th>
<th>Ratio 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M ( x_5 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>18</td>
<td>0</td>
<td>15</td>
<td>15/2</td>
<td>5</td>
</tr>
<tr>
<td>-M ( x_6 )</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>20</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>-1 ( x_4 )</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>-3M-2 ( x_3 )</td>
<td>-3M-4</td>
<td>-8M-4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \begin{array}{c c c c c c}
\uparrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow \\
\end{array} \]

Here first entering variable \( P_3 \) and corresponding outgoing \( P_6 \)

Then second entering variable \( P_2 \) and corresponding outgoing \( P_4 \)

And third entering variable \( P_1 \) and corresponding outgoing \( P_5 \)

i.e. we introduce simultaneously \( P_3, P_2, P_1 \) all the three vectors and outgoing vectors are \( P_6, P_4, P_5 \). So we get direct fourth simplex table using above formulae.

(we are selecting \( y_5 \) as choice of \( P_6 \) and \( P_4 \) is already over, to have outgoing variables in different rows.)

\[ \therefore RR = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \\ 1 & 2 & 1 \end{vmatrix} = 6 \]

(No need to adjust pivotal elements in diagonal form)

These columns correspond to \( P_1, P_2, P_3 \) respectively \( R \) contains rows and columns of vectors having pivotal elements or \( R \) is obtained by considering the determinant made up from column of entering variables by considering rows where pivotal elements are there.

We shall call \( R \) as key determinant and \( |R| \) is the denominator for every element in the third simplex table.

First column of \( R \) corresponds to new variable \( P_1 \) because outgoing variable is \( P_5 \) in that column and it will be replaced by \( P_1 \).

Second column of \( R \) corresponds to new variable \( P_2 \) because outgoing variable is \( P_4 \) in that column and it will be replaced by \( P_2 \).

Third column of \( R \) corresponds to new variable \( P_3 \) because outgoing variable is \( P_6 \) in that column and it will be replaced by \( P_3 \).

Now to find numerators for each entry in the third simplex table, if we replace ith column of \( R \)(Here i = 1, 2, 3) by the original vector \( P_i \) then we get the entries corresponding to new variables.

(Here when we replace first column we get values for \( P_1 \), when we replace second column we get values for \( P_2 \), when we replace third column we get values for \( P_3 \)).

To find new values in \( X_B \) column.
\[ x_1 = \frac{15}{2} \quad \frac{2}{3} \quad \frac{3}{2} \quad \frac{20}{3} \quad \frac{1}{5} \quad \frac{10}{2} \quad \frac{1}{1} \quad \frac{5}{2} \quad \frac{x_2}{\frac{1}{2}} \quad \frac{2}{3} \quad \frac{20}{3} \quad \frac{1}{5} \quad \frac{10}{2} \quad \frac{1}{1} \quad \frac{5}{2} \quad \frac{x_3}{\frac{1}{2}} \quad \frac{2}{3} \quad \frac{20}{3} \quad \frac{1}{5} \quad \frac{10}{2} \quad \frac{1}{1} \quad \frac{5}{2} \]

Since all the values are positive, non-negativity constraint is satisfied and it indicates that such transformation is allowed.

To find other entries in third simplex table.

Column \( P_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \):

\[ x_1 = \frac{7}{6} \quad \frac{x_2}{\frac{1}{6}} \quad \frac{x_3}{\frac{3}{6}} \]

New \( P_4 = \begin{bmatrix} 7/6 \\ 1/6 \\ -3/6 \end{bmatrix} \)

Column \( P_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \):

\[ x_1 = \frac{-9}{6} \quad \frac{x_2}{\frac{3}{6}} \quad \frac{x_3}{\frac{3}{6}} \]

New \( P_5 = \begin{bmatrix} -9/6 \\ 3/6 \\ 3/6 \end{bmatrix} \)

Column \( P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \):

\[ x_1 = \frac{4}{6} \quad \frac{x_2}{\frac{-2/6}{0}} \quad \frac{x_3}{\frac{0/6}{0}} \]

New \( P_6 = \begin{bmatrix} 4/6 \\ -2/6 \\ 0 \end{bmatrix} \)

New \( P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \), New \( P_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \), New \( P_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \)

Therefore new simplex table in third iteration

**Step (4):** New simplex table in third iteration

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>-1</th>
<th>-M</th>
<th>-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( P_1 )</td>
<td>( P_2 )</td>
<td>( P_3 )</td>
<td>( P_4 )</td>
<td>( P_5 )</td>
</tr>
<tr>
<td>1</td>
<td>( x_1 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>7/6</td>
<td>-9/6</td>
</tr>
<tr>
<td>2</td>
<td>( x_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/6</td>
<td>3/6</td>
</tr>
<tr>
<td>3</td>
<td>( x_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-3/6</td>
<td>1/2</td>
</tr>
<tr>
<td>( Z_f - C_f )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>M+1</td>
<td>M</td>
</tr>
</tbody>
</table>
When we rearranged rows according to original third simplex table as $[x_2 \ x_3 \ x_1]$ we get usual simplex table.

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/6</td>
<td>3/6</td>
<td>-1/3</td>
<td>5/2</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-3/6</td>
<td>1/2</td>
<td>0</td>
<td>5/2</td>
</tr>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>7/6</td>
<td>-9/6</td>
<td>4/6</td>
<td>5/2</td>
</tr>
<tr>
<td></td>
<td>$Z_j - C_j$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>M+1</td>
<td>M</td>
<td></td>
</tr>
</tbody>
</table>

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 5/2$, $x_2 = 5/2$, $x_3 = 5/2$ and $x_4 = 0$ Max. $Z = 15$.

**10. Conclusion**

The method gives simplex table after $n$ replacements directly.

**References**