Joint Decision on Integrated Supplier Selection and Stock Control of Inventory System Considering Purchase Discount

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Abstract—Stock control and supplier selection are two vital parts on supply chain and management. Integrating these parts by solving them simultaneously is a good idea for cost reducing. Furthermore, if some suppliers give a purchase discount for their product, how to determine the optimal strategy is interesting. In this paper, the authors propose a mathematical model in a mixed integer quadratic programming with piecewise objective function that can be used to determine the optimal strategy for integrated supplier selection problem and stock control problem of multiproduct inventory system considering purchase discount. Stock control refers to bring the stock level of each product to a reference level given by the decision maker. The authors formulate two mathematical models which are a model in deterministic environment where all parameters are known and a model in probabilistic environment where the demand is random. The authors perform two numerical experiments to evaluate the proposed model. From the results, the optimal strategy was obtained i.e. the optimal product volume purchased from each supplier and the stock level of each product follows the reference level with minimal total cost.

Keywords—probabilistic multi-stage programming, purchase discount, supplier selection, stock/inventory control

1. Introduction

Recently, manufacturers or retailers are facing global competition so they must reduce the spend cost. In this part, an optimal management on their supply chain is needed. Procurement or purchasing cost and storage or inventory cost are two important cost components in supply chain management that have to be reduced [1]. The procurement cost can be reduced by selecting the optimal supplier which occurring a supplier selection problem. The storage cost can be reduced by optimizing the amount of a product in the inventory so that the demand is satisfied but not wasting the storage cost. In some case, the decision maker decides to control the inventory level so that it will be located at some desired level which occurring an inventory control problem. Then, a supplier selection problem and inventory control problem are occurred which the are two important parts in supply chain management that have to be optimized in order to reduce the total spend cost. Some researchers were developed a method to solve a supplier selection. The most method was formulating a mathematical model, for example, mixed-integer program [2], [3], integrating a mixed-integer with other methods like fuzzy, analytic network process and Knowledge-Based Networks [4]–[10], fuzzy-Delphi method [11], performance-evaluation approach [12], [13] and interval-valued intuitionistic fuzzy numbers [14]. Oher works were developed to solve a supplier selection problem under some assumption or condition such as facility disruption [15] and piecewise holding cost [16]. In particular, some researchers were developed a method for inventory control problem solving such as queuing approach [17] and mixed-integer program [18].

Supplier selection and inventory control methods were applied by many researchers in many sectors like automotive industry [19], banking [20], electric industry [21], thermal power plants [22], bioenergy power plants [23], survey data screening [24] and many more. The developing of a method for supplier selection problem solving has been intense likes green supplier selection for emission reducing [25]. As another developing, when many researchers solve a supplier selection problem and inventory control problem in particular way, solving these problems simultaneously is a new approach. Reference [26] was used a predictive control method to solve an integrated supplier selection problem and inventory control for multi-product inventory system whereas [27] was used
probabilistic dynamic programing but there is no any discount on the problem. Furthermore, if there is a purchase discount from the suppliers, how to determine the optimal supplier is a new problem.

Optimizing a problem with uncertain parameter can be approached by using some methods like fuzzy approach or probabilistic optimization approaches. A probabilistic optimization is approaching the uncertainty of the parameter with some probability distribution. The method for probabilistic optimization problem solving can be called as probabilistic programming and one of several methods to solve this problem is probabilistic multi-stage programming that can be illustrated as follows. Let \( t \) denotes the stage of the problem, \( x_t \) denotes the decision variable at stage \( t \) and \( \Omega_t \) denotes the event space at stage \( t \). The problem is solved by generating the scenario tree of the problem. A scenario is one possible outcomes in the future based on the realization of the random parameters and scenario tree is the enumeration of all possible combinations of outcomes illustrated by Fig. 1. The decision is taken based on the past decisions and the realization of the random parameters.

The authors solve the problem by defining the problem first with some assumptions/conditions are hold. Symbols of the parameters and variables of the problem are used for mathematical model formulation. The authors identified the problem if the problem is in deterministic situation or probabilistic situation. Then, a mathematical model is formulated as a mathematical optimization problem and the optimal strategy is calculated by using mathematical optimization method i.e. integer quadratic programming. Finally, the corresponding solution is implemented.
2.1 Problem Definition

A problem with the value of all of parameters are known with certainty is simpler than a problem with uncertain parameter(s). But in the fact, there are so many uncertain parameters. The value of demand from the customer can be certain, uncertain or both of them. In this section, the authors formulate the mathematical model for each of these environments.

Given a multi-product, multi-supplier and multi-period supplier selection problem. Let the symbols of parameters and variables for this problem are given in Table 1.

Table 1. Parameters and variables of the problem

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Index of products</td>
</tr>
<tr>
<td>( s )</td>
<td>Index of suppliers</td>
</tr>
<tr>
<td>( t )</td>
<td>Index of time periods</td>
</tr>
<tr>
<td>( X_{t,s,p} )</td>
<td>Amount of product ( p ) purchased from supplier ( s ) at time period ( t ) (unit)</td>
</tr>
<tr>
<td>( I_{t,p} )</td>
<td>Inventory level of product ( p ) at time period ( t ) (unit)</td>
</tr>
<tr>
<td>( U_{t,s,p} )</td>
<td>Purchasing cost per unit of product ( p ) from supplier ( s ) at time period ( t )</td>
</tr>
<tr>
<td>( H_{t,p} )</td>
<td>Holding cost per unit of product ( p ) at time period ( t )</td>
</tr>
<tr>
<td>( D_{t,p} )</td>
<td>Demand of product ( p ) at time period ( t ) (unit)</td>
</tr>
<tr>
<td>( r_{t,p} )</td>
<td>Reference level of product ( p ) at time period ( t ) for stock control purposes</td>
</tr>
<tr>
<td>( C_{s,p} )</td>
<td>Maximum capacity of supplier ( s ) to supply product ( p ) (unit)</td>
</tr>
<tr>
<td>( M_{p} )</td>
<td>Maximum storage capacity of product ( p ) per unit time period (unit)</td>
</tr>
</tbody>
</table>

2.2 Mathematical Model in Deterministic Environment

The first mathematical model is formulated for deterministic environment. This case is occurred when all of parameters are known with certainty. Let the number of the supplier is \( S \) and the number of the time period that the problem will be solved is \( T \). The decision maker decides that the inventory level will be controlled so that it will be located at some point as close as possible to a reference point given by the decision maker. Let \( r_{t} \) denotes the reference point at time period \( t \) and \((I_{t} - r_{t})^{2}\) be the reference tracking objectives. The authors minimize the total cost and the reference tracking objectives as follows

\[
\min J \tag{1}
\]

where

\[
J = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U_{t,s,p}^{(l)} X_{t,s,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{t,p} I_{t,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{t,p} - r_{t,p} \right)^{2}, \quad \text{if} \; d_{t,s,p}^{(0)} \leq X_{t,s,p} \leq d_{t,s,p}^{(1)};
\]

\[
J = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U_{t,s,p}^{(2)} X_{t,s,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{t,p} I_{t,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{t,p} - r_{t,p} \right)^{2}, \quad \text{if} \; d_{t,s,p}^{(1)} \leq X_{t,s,p} \leq d_{t,s,p}^{(2)};
\]

\[
J = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U_{t,s,p}^{(j)} X_{t,s,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{t,p} I_{t,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{t,p} - r_{t,p} \right)^{2}, \quad \text{if} \; d_{t,s,p}^{(j-1)} \leq X_{t,s,p} \leq d_{t,s,p}^{(j)};
\]

or it can be rewritten as

\[
J = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U_{t,s,p}^{(j)} X_{t,s,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{t,p} I_{t,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{t,p} - r_{t,p} \right)^{2}, \quad \text{if} \; d_{s,p}^{(j-1)} \leq X_{t,s,p} \leq d_{s,p}^{(j)};
\]

where the discount for purchasing cost is using the following scheme

\[
U_{t,s,p} \begin{cases} 
U_{t,s,p}^{(1)}, & \text{if} \; d_{s,p}^{(0)} \leq X_{t,s,p} \leq d_{s,p}^{(1)} \\
U_{t,s,p}^{(2)}, & \text{if} \; d_{s,p}^{(1)} \leq X_{t,s,p} \leq d_{s,p}^{(2)} \\
M, & \text{if} \; d_{s,p}^{(j-1)} \leq X_{t,s,p} \leq d_{s,p}^{(j)}
\end{cases}
\tag{2}
\]

for \( \forall t \in T, \forall s \in S \), or it can be rewritten as follows

\[
U_{t,s,p} = U_{t,s,p}^{(j)}, \quad \text{if} \; d_{s,p}^{(j-1)} \leq X_{t,s,p} \leq d_{s,p}^{(j)}, \tag{3}
\]

for \( \forall t \in T, \forall s \in S \), where \( d_{s,p}^{(j)}, j = 0,1,2,...,J \) is the price level for this discount scheme.

The constraints of the model can be explained as follows. Constraint

\[
I_{t-1,p} + \sum_{s=1}^{S} X_{t,s,p} - I_{t,p} \geq D_{t,p}, \quad \forall t \in T, \forall p \in P \tag{4}
\]

is used to ensure that the product in the storage and the purchased product will satisfy the demand.
\[ X_{t,s,p} \leq C_{s,p}, \forall t \in T, \forall s \in S, \forall p \in P \quad (5) \]
is used to ensure that the purchased product form supplier \( s \) is no more than the supplier capacity \( C_s \).

Let the maximum capacity of the storage is \( M \), then constraint

\[ I_{t,p} \leq M_p, \forall t \in T, \forall p \in P \quad (6) \]
is used to ensure that the inventory level does not exceed the storage capacity. Finally, the last constraint which is integer constraint for the purchased product volume is formulated as follows

\[ X_{t,s,p} \in \{0,1,2,\ldots\}, \forall t \in T, \forall s \in S, \forall p \in P. \quad (7) \]

2.3 Mathematical Model in Probabilistic Environment

The second model is formulated for probabilistic environment. This case is occurred when at least one parameter becomes uncertain. This uncertainty is approached by using probability distribution. The authors will solve this problem by using probabilistic multi-stage programming by generating scenario tree and determine the optimal strategy based on this scenario tree that minimized the expected total cost. Let \( P_i \) denotes the probability of scenario \( i \in \Omega \) and \( \Omega \) denotes the event space of the problem for any time period. For probabilistic environment, the authors minimizes the expected total cost which is

\[ \min \mathcal{J} \]

where

\[ \mathcal{J} = \sum_{t=1}^{T} \sum_{p=1}^{P} P_i \left[ \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U^{(j)}_{t,s,p} X_{t,s,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{t,p} I_{t,p} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{t,p} - r_{i,p} \right)^2 \right] \]

if \( d^{(j-1)}_{s,p} < X_{t,s,p} \leq d^{(j)}_{s,p} \),

and the constraints are the same with the first model i.e. model for deterministic case.

3. Numerical Experiment

In this section, the authors give three numerical experiments which are a numerical experiment in deterministic environment where all of parameters are known with certainty, a numerical experiment in probabilistic environment where the demand parameter is random and a numerical experiment for sensitivity analysis purposes. The problem is described as follows. Suppose that a manufacturer has three suppliers which are \( s_1, s_2 \) and \( s_3 \) to supply products \( p_1, p_2 \) and \( p_3 \). The purchasing cost for each product from each supplier is considering discount as shown in Table 2. Suppose that the initial inventory level for each product is 0 item and the holding cost is $1/unit/period for \( p_1 \), $2/unit/period for \( p_2 \) and $4/unit/period for \( p_3 \). Finally, let the warehouse’s maximum capacity is 300 units/period for \( p_1 \), 250 units/period for \( p_2 \) and 250 units/period for \( p_3 \). The decision maker desires that the inventory/stock level must be located at some point as close as possible to a reference level given in the result of each simulation. The authors solve all numerical experiments in LINGO® 16.0 with Windows 8 of OS, AMD A6 2.7GHz of processor and 4 GB of Memory.

3.1. Demand is certain, a deterministic case

For the first simulation, let the demand value of all products are known with certainty. Let the demand value of each product is given in Fig. 2.

![Figure 2](image-url)

**Figure 2.** Demand of the products (P1, P2, P3 refers to p1, p2, p3)

The authors evaluate the model for 10 time periods i.e. \( T = 10 \). The solution of optimization (1) i.e. the optimal product volume purchased from all suppliers for time periods 1 to 10, is summarized in Figure 3. From Figure 3, it can be seen that at time period 1, the optimal purchased product volume is 381 units of \( p_3 \) from \( s_3 \), 201 units of \( p_1 \) from \( s_2 \) 358 units of \( p_2 \) from \( s_2 \) and 179 unit of \( p_1 \) from \( s_2 \). The inventory level of \((p_1, p_2, p_3)\) at time period 1 is \((100, 110, 47)\) units where the reference level is \((100, 110, 50)\). The evolution of the inventory level of all products and their reference levels can be seen in Figure 4. It can be seen that the actual inventory level follows the reference level well.

**Table 2.** Purchasing cost & supplier capacity

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Product</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>P1</td>
<td>$1</td>
</tr>
<tr>
<td>S1</td>
<td>P2</td>
<td>$2</td>
</tr>
<tr>
<td>S1</td>
<td>P3</td>
<td>$4</td>
</tr>
<tr>
<td>S2</td>
<td>P1</td>
<td>$1</td>
</tr>
<tr>
<td>S2</td>
<td>P2</td>
<td>$2</td>
</tr>
<tr>
<td>S2</td>
<td>P3</td>
<td>$4</td>
</tr>
<tr>
<td>S3</td>
<td>P1</td>
<td>$1</td>
</tr>
<tr>
<td>S3</td>
<td>P2</td>
<td>$2</td>
</tr>
<tr>
<td>S3</td>
<td>P3</td>
<td>$4</td>
</tr>
</tbody>
</table>
Figure 3. The optimal product volume purchased from the suppliers (S1, S2, S3 refers to s1, s2, s3; DL1, DL2 refers to discount level 1, discount level 2)

Table 3. Scenario tree of the problem in probabilistic environment

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Time period (t)</th>
<th>Demand p1 (units)</th>
<th>Solution</th>
<th>Inventory (unit)</th>
<th>Probability</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>200</td>
<td>Fig. 5.a</td>
<td>99 110 48</td>
<td>0.064</td>
<td>48838</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>200</td>
<td></td>
<td>100 119 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>200</td>
<td></td>
<td>90 120 37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>200</td>
<td>Fig. 5.b</td>
<td>99 110 48</td>
<td>0.096</td>
<td>50837</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>200</td>
<td></td>
<td>100 119 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>300</td>
<td></td>
<td>90 119 37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>200</td>
<td>Fig. 5.c</td>
<td>99 110 48</td>
<td>0.096</td>
<td>50838</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td></td>
<td>100 120 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>200</td>
<td></td>
<td>89 119 39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>300</td>
<td>Fig. 5.d</td>
<td>100 110 47</td>
<td>0.216</td>
<td>54838</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td></td>
<td>100 120 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>300</td>
<td></td>
<td>90 119 38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. The evolution of the inventory level of the products and their reference level

(a)  

(b)  

(c)  

(d)  

Figure 5. Solution of problem in Section 3.2
3.2. Demand is uncertain, a probabilistic case

Suppose that the demand of some product is uncertain, the authors approach this uncertainty with a probability distribution. Due to computers capacity limit, the authors evaluate the model with 3-by-3 time periods, the random variable is only demand for product \( p_1 \) with two samples where the probability distribution for \( D_{1,t}, \forall t \in T \) is given by Table 4.

<table>
<thead>
<tr>
<th>( D_{1,t}, \forall t \in T )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.4</td>
</tr>
<tr>
<td>300</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Let the demand value for \( p_2 \) is 200 units for each time period and the demand value for \( p_3 \) is 300 units for each time period. The remaining parameters are following the first problem i.e. problem when demand is certain. The authors solve the optimization problem (8) in LINGO 16.0 by using probabilistic multi-stage programming where the model class is PIQP (pure integer quadratic programming).

Firstly, the authors evaluate this problem for only 3 time periods and (8) generates 8 scenarios as shown by Table 3. The value of the objective function (8) is the expected total cost which is the sum of the multiplication of the probability of the scenario and the total cost of the scenario which is \((0.064)(\$48838) + (0.096)(\$50837) + \ldots + (0.216)(\$54838) = \$52438\). Based on the scenarios given in Table 3, the optimal decision at a time period can be determined after the random parameter for the corresponding time period is revealed. For example, let the demand of \( p_1 \) at time period 1 is 200 units, then the optimal decision is purchasing 310 units of \( p_2 \) from \( s_1 \), 200 units of \( p_1 \) from \( s_2 \), 300 units of \( p_1 \) from \( s_3 \), 99 units of \( p_2 \) from \( s_2 \) and 48 units of \( p_3 \) from \( s_3 \) (see Fig. 5a or 5b or 5c). The optimal decision for time period 2 and 3 can be determined after the demand of \( p_1 \) at time period 2 and 3 are revealed. Let the demand of \( p_1 \) at time period 2 is 300 and 200 units at time period 3 (see scenario 3). Then, the optimal decision at time period 2 is purchasing 210 units of \( p_2 \) from \( s_1 \), 50 units of \( p_3 \) from \( s_2 \), 301 units of \( p_1 \) from \( s_1 \) and 250 units of \( p_3 \) from \( s_3 \) whereas the optimal decision at time period 3 is purchasing 199 units of \( p_2 \) from \( s_1 \), 189 units of \( p_1 \) from \( s_2 \), and 291 units of \( p_3 \) from \( s_3 \) (see Fig. 5c). These decisions give (99,110,48) units of \((p_1,p_2,p_3)\) in the storage at time period 1 where the reference level is \((10,110,50)\), \((100,120,48)\) units of \((p_1,p_2,p_3)\) in the storage at time period 2 where the reference level is \((100,120,50)\), and (89,119,39) units of \((p_1,p_2,p_3)\) in the storage at time period 3 where the reference inventory level is \((100,130,50)\) units. It can be observe that for time periods 1 and 2, the actual inventory level for each product is closed to the reference level. For time period 3, the actual inventory level is following the reference level although there is a sufficiently large gap between them. It was caused by the time period 3 is the last time period for optimization. Hence, the model was decided to not store the product in the inventory for future periods.

To observe how the comparison between the inventory level of each product and its reference level, the authors evaluate the model for 10 time periods and the result i.e. the inventory level and the reference level is given by Fig. 6. From Fig 6, it can be seen that the actual inventory level of each product at time periods 1, 2, 4, 5, 7, 8 and 10 is very close to the reference level, but for time periods 3, 6 and 9, the actual inventory level of each product is sufficiently far from the reference. The authors observe that this was caused by the number of time period evaluation which is 3 time periods for each model evaluation which means that it gives an assumption to the model that the problem is only optimized for 3 time periods and there is no optimization after that.

3.3 Impact of Demand’s Uncertainty

The last numerical experiment is used to analyze the impact of the demand’s uncertainty to the expected total cost. Let the demand’s uncertainty for each product is normally distributed with mean 400 and standard deviation \( \sigma > 0 \). The authors evaluate model (8) with \( \sigma = 10, 20, 50, 100, 200 \) where the sample size of each random variable is 2 samples.
From Figure 7, it can be seen that the expected total cost becomes larger if the demands standard deviation becomes larger. It was caused by the range of the demands uncertainty is become wider.

4. Concluding Remarks and Future Research

In this paper, a mathematical model in an integer quadratic model and a probabilistic integer quadratic model were formulated to determine the joint decision of an integrated supplier selection problem and trajectory tracking control problem of a multi-product inventory system with purchase discount in deterministic and probabilistic environments. Numerical experiments were considered in deterministic environment where the corresponding optimization was solved by using integer quadratic programming and probabilistic environment where the demand value of some products are random and the corresponding optimization was solved by using probabilistic multi-stage programming. From the results, it can be conclude that the joint decision was determined i.e. the optimal product volume purchased from each supplier selection and the inventory/stock level followed the reference level.

In the future research, the authors will develop the mathematical model considering several uncommon conditions like imperfect service from a supplier, backlog of a product, late on delivery, etc. Furthermore, the authors will using a metaheuristic method to solve the corresponding optimization problem in order to reduce the computational time of the problem.

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