Distribution Center Location Selection using an Extension of Fuzzy TOPSIS Approach

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Abstract-Distribution center location is a very important problem in practice. This paper proposes a generalized fuzzy TOPSIS approach to support for distribution center location selection process. In the proposed approach, the ratings of alternatives and importance weights of criteria for distribution center location selection are represented by generalized triangular fuzzy numbers. Then, the membership functions of the final fuzzy evaluation value in the proposed approach are developed based on the linguistic expressions. Finally, this study applies the proposed generalized fuzzy TOPSIS to a real case of distribution center location selection in a company demonstrating its advantages and applicability.

Keywords-Distribution center, location selection, TOPSIS, generalized fuzzy numbers

1. Introduction

Selecting distribution centers location is one of the most important activities for a company to reduce transportation costs and enhance operation efficiency. However, it is not an easy task to optimize distribution center locations. To select a suitable location for distribution centers, many quantitative and qualitative criteria must be considered in decision process such as investment cost, climate condition, resource availability, possibility of expansion, transportation availability, human resources, proximity to suppliers, and closeness to demand market, etc [1-3]. Therefore, distribution center location selection can be seen as a multi-criteria decision making (MCDM) problem.

Although several fuzzy MCDM and non-fuzzy methods have been developed for distribution center location selection [1-2, 4-16], all of them used normal fuzzy numbers in their calculation. However, in many cases it is not possible to restrict the membership function to the normal form.

In recent years, TOPSIS (technique for order performance by similarity to ideal solution) [17] has been a popular technique for solving MCDM problems. The fundamental idea of TOPSIS is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. Some recent applications can be found in [18-24]. In this paper, a generalized fuzzy TOPSIS approach is developed to support for distribution center location selection process. In the proposed approach, the ratings of alternatives and importance weights of criteria for distribution center location selection are represented by generalized triangular fuzzy numbers. Then, the membership functions of the final fuzzy evaluation value in the proposed approach are developed based on the linguistic expressions. Finally, this study applies the proposed generalized fuzzy TOPSIS to a real case of distribution center location selection in a company demonstrating its advantages and applicability.

The rest of the paper is organized as follows. Section 2 reviews the basic concepts of generalized fuzzy numbers. Section 3 proposes a generalized fuzzy TOPSIS approach. The proposed generalized fuzzy TOPSIS approach is applied to solve the real case of company in Section 4. Finally, conclusions are drawn in Section 5.

2. Literature review

In literature, many methods have been developed for distribution location selection based on the concept of accurate measure and crisp evaluation [4-11]. Nevertheless, the values for the qualitative criteria are often imprecisely defined for the decision-makers. Additionally, the rating values of
alternatives for subjective criteria and importance weights of criteria are usually assessed with linguistic terms. Obviously, the precision-based methods are not adequate to resolve the distribution center location selection problem.

Although several fuzzy MCDM methods have been developed for distribution center location selection [1-2, 12-16], all of them used normal fuzzy numbers in their calculation. Ref. [1] applied the fuzzy TOPSIS to evaluate and select the best location for implementing an urban distribution center. Sensitivity analysis was performed to determine the influence of criteria weights on location planning decisions for urban distribution centers. Ref. [2] proposed a MCDM approach for selecting distribution centre location using an improved fuzzy preference relation. Ref. [3] presented a new fuzzy MCDM method to solve the distribution center location selection problem. By calculating the difference of final evaluation value between each pair of DC locations, a fuzzy preference relation matrix was constructed to represent the intensity of the preferences of one plant location over another. And then, a stepwise ranking procedure was proposed to determine the ranking order of all candidate locations. Ref. [15] developed a hybrid method, which incorporates the axiomatic fuzzy set and TOPSIS techniques to select competitive regions in logistics. Ref. [16] applied a fuzzy integrated hierarchical decision-making approach to solve the distribution center location selection problem. A case study adopted was discussed to show the effectiveness and robustness of the proposed methodology over the existing conventional hierarchical approaches.

However, in many cases it is not to possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers and their arithmetic operations [25]. It seems that no one has developed and applied the generalized fuzzy numbers for solving the distribution center location selection problem.

3. Generalized fuzzy numbers

This section briefly reviews some basic concepts of generalized fuzzy numbers as the following:

A. Basic concepts of generalized fuzzy numbers

Based on Ref. [25] and Ref. [26], a generalized trapezoidal fuzzy number can be represented by

\[ A = (a, b, c, d; w) \]

where \( 0 < w \leq 1 \) and \( a, b, c \) and \( d \) are real numbers. Fig. 1 is an illustration of the generalized trapezoidal fuzzy number. The membership function \( f_A \) of the generalized trapezoidal fuzzy numbers satisfies the following conditions:

(a) \( f_A \) is a continuous mapping from \( \mathbb{R} \) to the closed interval \( [0, w] \). \( 0 \leq w \leq 1 \);

(b) \( f_A(x) = 0 \), for all \( x \in (-\infty, a] \);

(c) \( f_A \) is strictly increasing on \( [a, b] \);

(d) \( f_A(x) = w \), for all \( x \in [b, c] \);

(e) \( f_A \) is strictly decreasing on \( [c, d] \);

(f) \( f_A(x) = 0 \), for all \( x \in (d, \infty] \).

In Fig. 1, if \( w = 1 \), then the generalized trapezoidal fuzzy number \( A \) is called a normal trapezoidal fuzzy number and denoted as \( A = (a, b, c, d; 1) \). If \( a = b \) and \( c = d \), then \( A \) is called a crisp interval. If \( a < b = c < d \), then \( A \) becomes a generalized triangular fuzzy number, and can be denoted by \( A = (a, b, d; w) \) or \( A = (a, b, d; 1) \) if \( w = 1 \). If \( a = b = c = d \) and \( w = 1 \), then \( A \) is called a crisp value.

B. Arithmetic operations on generalized fuzzy numbers

Ref. [25] presented arithmetical operations between generalized trapezoidal fuzzy numbers based on the extension principle.

Let \( A \) and \( B \) be two generalized trapezoidal fuzzy numbers, i.e., \( A = (a_1, a_2, a_3, a_4; w_A) \) and \( B = (b_1, b_2, b_3, b_4; w_B) \), where \( a_1, a_2, a_3, a_4, b_1, b_2, b_3 \) and \( b_4 \) are real numbers.
and \( b_i \) are real values, \( 0 \leq w_A \leq 1 \) and \( 0 \leq w_B \leq 1 \).

Some arithmetic operators between the generalized fuzzy numbers \( A \) and \( B \) are defined as follows:

(i). Generalized trapezoidal fuzzy numbers addition (+):

\[
A(+B) = (a_i,a_i,a_i,a_i;w_A)+(b_i,b_i,b_i,b_i;w_B)
\]

\[
= (a_i + b_i, a_i + b_i, a_i + b_i, a_i + b_i; \min(w_A,w_B))
\]

where \( a_i, a_i, a_i, a_i, b_i, b_i, b_i, b_i \) are real values.

(ii). Generalized trapezoidal fuzzy numbers subtraction (−):

\[
A(-B) = (a_i,a_i,a_i,a_i;w_A)-(b_i,b_i,b_i,b_i;w_B)
\]

\[
= (a_i - b_i, a_i - b_i, a_i - b_i, a_i - b_i; \min(w_A,w_B))
\]

where \( a_i, a_i, a_i, a_i, b_i, b_i, b_i, b_i \) are real values.

(iii). Generalized trapezoidal fuzzy numbers multiplication (\( x \)):

\[
A(x)B = (a_i,b_i,c_i,d_i;\min(w_A,w_B))
\]

where \( a_i = \min(a_i \times b_i, a_i \times b_i, a_i \times b_i, a_i \times b_i) \),

\( b_i = \min(a_i \times b_i, a_i \times b_i, a_i \times b_i, a_i \times b_i) \),

\( c_i = \max(a_i \times b_i, a_i \times b_i, a_i \times b_i, a_i \times b_i) \),

\( d_i = \max(a_i \times b_i, a_i \times b_i, a_i \times b_i, a_i \times b_i) \).

It is obvious that if \( a_i, a_i, a_i, a_i, b_i, b_i, b_i, b_i \) are all positive real numbers, then

\[
A(x)B = (a_i \times b_i, a_i \times b_i, a_i \times b_i, a_i \times b_i; \min(w_A,w_B))
\]

(iv). Generalized trapezoidal fuzzy numbers division (\( l \)):

The inverse of the fuzzy number \( B \) is

\[
1/B = (1/b_1,1/b_2,1/b_2,1/b_2;w_B)
\]

where \( b_1, b_2, b_2 \) and \( b_2 \) are non-zero positive numbers or all non-zero negative real numbers. Let \( a_i, a_i, a_i, a_i, b_i, b_i, b_i, b_i \) be non-zero positive real numbers. Then, the division of \( A \) and \( B \) is as follows:

\[
A(l)B = (a_i,a_i,a_i,a_i;w_A)/(b_i,b_i,b_i,b_i;w_B)
\]

\[
= (a_i/b_i, a_i/b_i, a_i/b_i, a_i/b_i; \min(w_A,w_B))
\]

4. Proposed a generalized fuzzy TOPSIS approach

This section develops a generalized fuzzy TOPSIS approach for supporting the distribution center location selection process by the following procedure:

A. Aggregate ratings of alternative versus criteria

Assume that a committee of \( l \) decision makers \((M_t,t=1,\ldots,l)\) is responsible for evaluating \( m \) alternatives \((A_i,i=1,\ldots,m)\) under \( n \) selection criteria \((C_j,j=1,\ldots,n)\). A fuzzy MCDM problem can be concisely expressed in matrix format as:

\[
C_1 \quad C_2 \quad \cdots \quad C_n
\]

\[
A_i = \begin{bmatrix}
\hat{x}_{i1} & \hat{x}_{i2} & \cdots & \hat{x}_{in}
\end{bmatrix}
\]

\[
M_t = \begin{bmatrix}
A_{1t} & A_{2t} & \cdots & A_{lt}
\end{bmatrix}
\]

Let \( x_{ij} = (a_{ij},b_{ij},c_{ij};\sigma_{ij}) \), \( i = 1,\ldots,m \), \( j = 1,\ldots,h \), \( t = 1,\ldots,l \), be the suitability rating assigned to alternative \( A_i \), by decision maker \( M_t \), for subjective \( C_j \). The averaged suitability rating, \( x_{ij} = (a_{ij},b_{ij},c_{ij};\sigma_{ij}) \), can be evaluated as:

\[
x_{ij} = \frac{1}{l} \oplus (x_{ij1} \oplus x_{ij2} \oplus \cdots \oplus x_{ijn} \oplus \cdots \oplus x_{ijn})
\]

where \( a_{ij} = \frac{1}{l} \sum_{t=1}^{l} a_{ijt} \), \( b_{ij} = \frac{1}{l} \sum_{t=1}^{l} b_{ijt} \), \( c_{ij} = \frac{1}{l} \sum_{t=1}^{l} c_{ijt} \), and \( \sigma_{ij} = \min \sigma_{ijt} \).

B. Aggregate the importance weights

Let \( w_j = (a_j,p_j,q_j;\sigma_j) \), \( w_j \in \mathbb{R}^* \), \( j = 1,\ldots,n \), \( t = 1,\ldots,l \) be the weight assigned by decision maker \( M_t \), to criterion \( C_j \). The averaged weight, \( w_j = (a_j,p_j,q_j) \), of criterion \( C_j \), assessed by the committee of \( l \) decision makers can be evaluated as:

\[
w_j = (1/l) \oplus (w_{j1} \oplus w_{j2} \oplus \cdots \oplus w_{jl})
\]
where
\[
\omega_i = (1/1)^{\sum_{j=1}^{k} \omega_i}, \quad p_j = (1/1)^{\sum_{j=1}^{k} p_j}, \quad q_j = (1/1)^{\sum_{j=1}^{k} q_j},
\]
and \( \sigma_j = \min \sigma_j \).

C. Construct the weighted fuzzy decision matrix

Considering the different weight of each criterion, the weighted decision matrix can be computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted decision matrixes \( T_i \) are defined as:

\[
G_i = \left( \frac{1}{n} \sum_{j=1}^{n} r_{ij} \otimes w_j, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n. \right)
\]

D. Calculation of \( A^+, A^-, d^+_i \) and \( d^-_i \)

The fuzzy positive-ideal solution (FPIS, \( A^+ \)) and fuzzy negative ideal solution (FNIS, \( A^- \)) are obtained as:

\[
A^+ = (1, 1, 1; \max \sigma_i)
\]

\[
A^- = (0, 0, 0; \min \sigma_i)
\]

The distance of each alternative \( A_i, i = 1, \ldots, m \) from \( A^+ \) and \( A^- \) is calculated as:

\[
d^+_i = \sqrt{\sum_{j=1}^{n} (G_{ij} - A^+)^2}
\]

\[
d^-_i = \sqrt{\sum_{j=1}^{n} (G_{ij} - A^-)^2}
\]

where \( d^+_i \) represents the shortest distance of alternative \( A_i \), and \( d^-_i \) represents the farthest distance of alternative \( A_i \).

E. Obtain the closeness coefficient

The closeness coefficient of each alternative, which is usually defined to determine the ranking order of all alternatives, is calculated as:

\[
CC_i = \frac{d^-_i}{d^+_i + d^-_i}
\]

A higher value of the closeness coefficient indicates that an alternative is closer to PIS and farther from NIS simultaneously. The closeness coefficient of each alternative is used to determine the ranking order of all alternatives and identify the best one among a set of given feasible alternatives.

5. Application for distribution center location evaluation problem

In this section, the proposed generalized fuzzy TOPSIS approach is applied on the case of Viglacera Company to solve the distribution center location selection and evaluation. Due to increase in customer demand, the company’s managers intend to establish a new distribution center to expand their business. However, the managers of this company have confused the issue concerning how to select the location for distribution center to maximize their profit. In order to help the company select the most suitable distribution center location and test the efficacy of the proposed method, the proposed approach was applied to the process of evaluating and selecting distribution center location for this company. The data used as input to implement the proposed method were collected by means of semi-structured interviews with the top managers and head of departments. Three company managers were required to make their evaluation separately, according to their preferences for the importance weights of criteria and the ratings of alternative based on each criterion.

A. Determining the distribution center location criteria and aggregating importance weights of criteria

Following a survey of the literature and discussions with company’s top managers and head of departments, six criteria were chosen to select the distribution center location including expansion possibility (\( C_1 \)), closeness to demand market (\( C_2 \)), human resources (\( C_3 \)), availability of acquirement material (\( C_4 \)), investment cost (\( C_5 \)), transportation availability (\( C_6 \)).

After the determination of the distribution center location selection criteria, each of the three managers established the level of each criteria by means of a linguistic variable. An important weight set of \( Q \) was used to express opinions on the criteria: \( Q = \{ \text{UI}, \text{OI}, \text{I}, \text{VI}, \text{AI} \} \), where UI = Unimportant = (0.0, 0.0, 0.3; 0.7), OI = Ordinary
Important = (0.2, 0.3, 0.4; 0.8), I = Important = (0.3, 0.5, 0.7; 0.8), VI = Very Important = (0.6, 0.8, 0.9; 0.9), and AI = Absolutely Important = (0.8, 0.9, 1.0; 1.0). Table 1 displays the importance weights of six criteria from the three managers. Using Chen’s arithmetic operations, the aggregated weights of criteria from the decision making committee can be obtained as presented in Table 1.

Table 1: The importance weights of the criteria evaluated by managers

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Managers</th>
<th>( w_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>I VI I</td>
<td>(0.400, 0.600, 0.767; 0.8)</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>VI AI AI</td>
<td>(0.733, 0.867, 0.967; 0.9)</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>I I I</td>
<td>(0.500, 0.700, 0.833; 0.8)</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>VI AI AI</td>
<td>(0.667, 0.833, 0.933; 0.9)</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>VI AI VI</td>
<td>(0.667, 0.833, 0.933; 0.9)</td>
</tr>
</tbody>
</table>

B. Aggregate ratings of alternatives versus criteria

Three managers use the linguistic rating set \( S = \{ VP, P, F, G, VG \} \), where VP = Very Poor = (0.0, 0.0, 0.2; 0.6), P = Poor = (0.1, 0.3, 0.5; 0.7), F = Fair = (0.3, 0.5, 0.7; 0.8), G = Good = (0.6, 0.8, 0.9; 0.9), and VG = Very Good = (0.8, 0.9, 1.0; 1.0), to evaluate the suitability of the distribution center locations under each criteria.

Using Chen’s arithmetic operations, the aggregated suitability ratings of five distribution center locations, i.e. \( A_1, \ldots, A_5 \) versus six criteria, i.e. \( C_1, \ldots, C_6 \), from three managers can be obtained as shown in Table 2.

Table 2: The linguistic ratings evaluated by decision makers

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Locations</th>
<th>Managers</th>
<th>( R_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>F G F</td>
<td>(0.400, 0.600, 0.700; 0.8)</td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td>G F G</td>
<td>(0.500, 0.700, 0.800; 0.8)</td>
<td></td>
</tr>
<tr>
<td>( C_1 )</td>
<td>VG V G G</td>
<td>(0.733, 0.867, 0.933; 0.9)</td>
<td></td>
</tr>
<tr>
<td>( A_4 )</td>
<td>G G G</td>
<td>(0.600, 0.800, 0.867; 0.9)</td>
<td></td>
</tr>
<tr>
<td>( A_5 )</td>
<td>F G G</td>
<td>(0.500, 0.700, 0.800; 0.8)</td>
<td></td>
</tr>
</tbody>
</table>

C. Determine the weighted fuzzy decision matrix

This matrix can be obtained by multiplying each aggregated rating by its associated fuzzy weight using Chen’s arithmetic operation of generalized fuzzy numbers. Table 3 shows the weighted ratings of each distribution center location.
Table 3: Weighted ratings of each distribution center location

<table>
<thead>
<tr>
<th>Distribution center location</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.193, 0.344, 0.461; 0.8)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.217, 0.368, 0.486; 0.8)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.195, 0.347, 0.461; 0.8)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.169, 0.323, 0.442; 0.8)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.230, 0.385, 0.499; 0.8)</td>
</tr>
</tbody>
</table>

D. Calculation of $A^+, A^-, d^+$ and $d^-$

As shown in Table 4, the distance of each distribution center location from $A^+$ and $A^-$ can be calculated by Equations 8-11.

Table 4: The distance of each distribution center location from $A^+$ and $A^-$

<table>
<thead>
<tr>
<th>Distribution center locations</th>
<th>$d^+$</th>
<th>$d^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1.422</td>
<td>0.418</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1.440</td>
<td>0.396</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1.402</td>
<td>0.442</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1.416</td>
<td>0.424</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1.422</td>
<td>0.418</td>
</tr>
</tbody>
</table>

E. Obtain the closeness coefficient

The closeness coefficients of distribution center locations can be calculated by Equation (12), as shown in Table 5. Therefore, the ranking order of the five distribution center locations is $A_5 > A_2 > A_1 > A_3$. Consequently, the best distribution center locations is $A_5$.

Table 5: Closeness coefficients of distribution center locations

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Closeness coefficient</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.341</td>
<td>4</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.364</td>
<td>2</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.343</td>
<td>3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.322</td>
<td>5</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.378</td>
<td>1</td>
</tr>
</tbody>
</table>

6. Conclusions

Distribution center location selection is the MCDM problem that is affected by several criteria. This paper proposed the generalized fuzzy TOPSIS model to solve the distribution center location selection problem. In the proposed approach, the ratings of alternatives and relative importance weights of criteria for are expressed in linguistic values which are represented by the generalized triangular fuzzy numbers. An application was given to illustrate the applicability of the proposed approach. The results indicate that the proposed generalized fuzzy TOPSIS approach is practical and useful. The proposed approach can also be applied to other management problems under similar settings.

References


