Selection of Priority Sequence of Investor's Portfolio with the Use of the Supply Chain Management in the Criteria of "Against Nature" Game

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Abstract - As part of the research related to the improvement of the portfolio theory of H. Markowitz, J. Tobin, and W. Sharpe for use in established as well as developing modern stock markets, the authors proposed a concept and a numerical algorithm to choose the priority sequence of financial portfolios of non-institutional investors, taking into account their preferences and projected changes in the parameters included in financial portfolios assets. In the absence of an optimization model suited to the market, an investor's ability to choose a portfolio from several alternatives that satisfy the preferences in terms of profitability, risk, and liquidity significantly improves the quality of an investment decision by employing supply chain strategy. The authors' concept provides for the formation of a set of alternative financial portfolios with different characteristics in a priori given quantity: the selection of an integral quality indicator of the investment portfolio, the formation of the priority sequence of portfolios using the game theory method "against nature", and the synthetic "game" Wald-Savage criterion, which allows the consideration of the investor's predisposition for risk–return pair. A comparative analysis of investment decisions based on the "classical" portfolio theory and the author’s concept allowed the conclusion that the proposed approach and the numerical method are correct and are better in comparison to traditional methods and efficiency algorithms when applied to the portfolio investment tasks.

Keywords - portfolio theory, portfolio models of H. Markowitz - J. Tobin - W. Sharpe, stock market in transition, non-institutional investor, Sharpe ratio, supply chain strategy.

1. Introduction

We recall that the classic task of optimizing the investment portfolio set of G. Markowitz is to select a set of financial assets traded on the stock market with a total value not exceeding the investor’s budget, which provides maximum profitability and limits risk or, conversely, having minimum risk while limiting the expected profitability. The profitability of a financial asset is the average return over the observed time period, while the risk is defined as the standard deviation (SD) of the average value of the planned return. This risk is based on the law of large numbers and Chebyshev’s inequality, which states that the smaller the SD of a financial asset's return is, the less likely it is to deviate from the average value [1, 2].

The optimal portfolio investment problem, implemented in the form of a well-known Markowitz model [1,3,4], is widely used in Russian (e.g., FINAM, Troika Dialog, Uralisib, Alfa Capital, Renaissance Investment Management) and foreign (e.g., UFG Asset Management, Raiffeisen Capital Management) management companies (MC) when placing financial assets of investors, such as individuals and legal entities [5,6].

To illustrate the current involvement of countries in stock market operations, we present the World Bank data concerning the ratio of capitalization of the stock market of countries to GDP values in 2017 (data for countries where this value exceeds 40%) (Figure 1). This indicator became significant in the financial analysis following Buffet’s1 statement that it is the best indicator for drawing conclusions about overestimation or underestimation of the market as a whole [2]. A value above 100% indicates the first phenomenon, while a value of about 50% indicates the second one.

Based on these values, not all countries meet the market efficiency condition. This circumstance indicates that the direct use of the results of the classical portfolio theory is fraught in the developing countries, and particularly in the Russian stock market, with a number of problems related to their institutional features, among which we highlight the following2:

1 The American entrepreneur Warren Buffet, one of the most famous investors, is one of the richest people in the world and the second wealthiest U.S. citizen [2].
2 The authors have not considered the current situation in the Russian stock market, which does not give a comprehensive view of its dynamics in the context of macroeconomic instability.
- high market volatility, reflecting fluctuations in demand, supply and prices of financial assets, which initiates the appropriateness of the following portfolio formation scheme: “Assets characterized by profitability\(^3\) and liquidity\(^4\) — sold and newly acquired assets — asset holding time taking into account the listed quality indicators” [3,7];
- the factor of discreteness - the investment portfolio includes financial assets that are mainly traded in whole lots [8].

3By risk, further in the work, we mean the specific (non-systematic) risk inherent in a particular security and determined by the level of yield volatility. In this paper, the authors did not set the task of developing a theoretical approach to the analysis and risk assessment of portfolio investments. In the future, we will proceed from the premise of stock market growth, when financial instruments selected at the stage of express analysis are deprived of general market risk and their profitability fluctuates in tune with the market.

4Portfolio liquidity on a time horizon \([0; T]\) is the share (in value terms) of a portfolio excluding discounted profit and loss streams that provides a reverse conversion to cash or cash surrogates (financial leasing) on this horizon.

5Institutional investor is a legal entity acting as a holder of funds (contributions, shares) and investing them in securities and real estate (including the right to real estate) for the purpose of making a profit. Institutional investors include investment and pension funds, insurance, and credit unions (including banks). Non-institutional investor is a natural or legal person who uses the services of professional market participants (brokers) [2,7].
synthetic Wald-Savage criterion, we created a priority sequence of investment strategies and developed the corresponding portfolios from the specified set.

3. Research Findings and Discussion

In view of the newly-emerging needs of investors and market “movements,” the classic Markowitz model can be supplemented. However, such an approach cannot be recognized as effective because a change of the tools requires careful study, the theoretical justification of modifications, and the further testing of the model in the market, all of which are different in terms of both their functioning conditions and the parameters of traded assets. We choose another approach, namely: the formation of a financial portfolio and taking into account the preferences of investors, which, in this case, can be called “additional.” We demonstrate the possibilities of this approach in the appendix, which details the task of forming a medium-term investment portfolio of a moderately aggressive investor with a low investment budget. Such investors include non-institutional investors, whose aim is to keep money under the conditions of inflation. The expectations of return on investment in the portfolios under consideration are not higher than the market average; however, requirements regarding the reliability of financial instruments in the portfolio are more stringent: investors are tolerant of risk, but the value of risk may not exceed a priori set level. Also, a timely withdrawal from a transaction requires an appropriate level of liquidity regarding the instruments. This group of investors focuses on the medium-term investment horizon. It is proposed to consider the prospects of using the Wald-Savage synthetic criterion suggested by L. Labsker in the task of selecting the optimal portfolio for investors of the group under consideration. The choice of tools of “games against nature” models is subject to inherent the stock market uncertainty caused by unstable macroeconomic situation, volatile market conditions, expectations of market participants and other factors that have a direct impact on securities quotations. The Wald-Savage synthetic criterion allows us to evaluate the optimality of behavior strategies considered by the subject – an agent of market interaction (in this case – the non-institutional investor) from the perspective of winning and taking risks. It is a linear combination of the Wald and Savage criteria with coefficients, which determine quantitative assessment of the subject’s winning and risk preference [11]. It is proposed to build a variant of the traditional Markowitz model for non-institutional investor, according to the results of which it is planned to get at least m alternative portfolios of financial instruments homogeneous in terms of liquidity, investment conditions, size of the investment budget and difference in profitability and risk. The Markowitz model for forming an optimal portfolio for a group of non-institutional investors, which would create the basis to solve the task, is briefly described below with introduction of a formal definition of the Wald-Savage synthetic criterion. The portfolio model consisting of n securities is as follows:

\[
\begin{align*}
\sum_{k=1}^{n} r_k w_k & \rightarrow \max; \\
\sqrt{\sum_{k=1}^{n} \sum_{l=1}^{n} w_k w_l \sigma_{kl}} & \leq \sigma_p; \\
\sum_{k=1}^{n} w_k &= 1; \\
w_k & \geq 0,
\end{align*}
\]

(1)

where: k, l are asset indices; \( r_k \) – an average expected return of the \( k \)-th asset of the investment portfolio; \( \sigma_{kl} \) – the covariance of returns of the \( k \)-th and \( l \)-th assets in the securities portfolio; \( \sigma_p \) – the risk level acceptable for the investor; \( w_k \) – a share of the \( k \)-th financial asset in the investment portfolio [1].

The Wald-Savage synthetic criterion includes:

- The Wald criterion, which enables to determine the strategy optimality from the perspective of winning;

- The Savage criterion, which allows selecting a strategy from the perspective of gaming risk.

The strategy, which provides the W-maximum winning among minimum winnings in the pure strategies, is optimal in a set of pure strategies according to the Wald criterion, or the W-optimal strategy. The optimal solution selected in this manner eliminates the risk, and regardless of the state of “nature”, the obtained result cannot be lower than W. This criterion is called “the principle of guaranteed result” in the literature and defined as the criterion of “extreme pessimism about the winnings” [11]. It is applied in cases when the subject is aimed at unwillingness to lose rather than win, which corresponds exactly to formalization of preferences of the non-institutional investors’ group under consideration. The strategy, which provides minimum risk among maximum risks in the S pure strategies, is optimal in a set of pure strategies according to the Savage criterion, or the S-optimal strategy. This criterion is also defined as “the criterion of extreme pessimism” in the literature, since when choosing such strategy; the subject is focused on the highest risk, namely, that the “nature” would be in the worst condition for the player [11]. Their linear combination, as mentioned above, will allow approaching the optimal investor strategy selection from the perspective of winning and risk.
introduce coefficients characterizing the degree of the investor’s winning and risk preference: \( r \in [0,1] \) and \((1 - r)\) – for formal description of synthetic criterion. The choice of a numerical value of the \( r \) indicator is subjective, depending on the required expected return and risk tolerance [11]. The Wald-Savage criterion with a winning indicator \( r \in [0,1] \) will be defined as follows:

\[
Q_{WS}(r) = rW_i - (1 - r)S_i.
\]

(2)

where: \( W_i \) is the efficiency of \( A_i \) strategy according to the Wald criterion; \( S_i \) – the efficiency of \( A_i \) strategy according to the Savage criterion, \( i \in I \).

\[
Q_{WS}(r) = \max\{Q_{WS}(r): i \in I\}.
\]

(3)

where \( Q_{WS}(r) \) is the value of game in pure strategies.

Let us call strategy \( A_i \) on the set of the S pure strategies optimal provided that:

\[
Q_{WS}(r) = Q_{WS}(r).
\]

(4)

An optimal set \( Q_{WS}(r) \) in the set of the S pure strategies is defined as \( S_{opt}^{Q_{WS}(r)} \).

It is proved in the cited paper that each strategy, being optimal on the set of pure strategies by the Wald-Savage criterion, is optimal on the set of S by both the Wald and Savage criteria. Also, when \( r \in (0,1) \), the structure of the set of \( S_{opt}^{Q_{WS}(r)} \) strategies being optimal on the set of pure strategies by the Wald-Savage criterion with the winning coefficient \( r \), does not depend on the values \( r \in (0,1) \) [11].

For practical use of the model described above we suggest to use the following algorithm, originally proposed by L. Labsker and improved for the purposes of this paper. We believe that it is necessary to introduce the following assumptions: (1) a non-institutional investor selects a strategy of investment from the ranked order of at least \( m \) security portfolios obtained by calculations; (2) risk limits are set externally; (3) no restrictions on the liquidity of financial instruments are imposed, as it is assumed that the portfolios are formed in the stock markets from the assets with high liquidity. The proposed algorithm includes:

1. To form investment portfolios according to the “classical” Markowitz model (1) using financial instruments, which meet the investor’s requirements for risk, profitability and liquidity, in calculations, to define characteristics of portfolios.

The use of the Wald-Savage synthetic criterion requires identification of an indicator for comparative evaluation of portfolios. It is proposed to use the Sharpe ratio\(^7\) for this purpose.

2. To form a matrix of A winnings, the elements of which will be the Sharpe’s ratios of the formed portfolios in the periods under consideration;

3. Using the formula

\[
W = \min\{q_{ij} : i = 1, \ldots, n\}, j = 1, \ldots, m.
\]

(5)

to find efficiency indicators \( W_i \), \( i \in I \), of strategies \( A_i \), \( i \in I \), according to the Wald criterion, value of game \( W_s \) in the pure strategies according to the Wald criterion.

4. To determine a set of strategies, which are optimal in the set of pure strategies according to the Wald criterion: \( S_{opt}^{Q_{WS}(r)} \).

5. To create \( R \) risk matrix on the basis of matrixA.

6. Based on the \( R \) matrix data to calculate indicators \( S_i \), determine game value according to the Savage criterion in pure strategies, \( S_s \) by formula:

\[
S_s = \min\{r_{ij} : i = 1, \ldots, n\}, j = 1, \ldots, m.
\]

(6)

7. To determine a set of strategies, which are optimal in the set of pure strategies according to the Savage criterion: \( S_{opt}^{Q_{WS}(r)} \).

8. Based on the data from steps 4 and 7 to verify the feasibility of the condition:

\[
S_{opt}^{Q_{WS}(r)} \cap S_{opt}^{Q_{WS}(r)} = \emptyset;
\]

If this condition is not met, the set of strategies, which are \( Q_{WS}(r) \)-optimal on \( S \) set, has the following structure:

\( \text{Vol. 8, No. 3, June 2019} \)
\[
S_{\text{opt}}^{Q_W(r)} = \begin{cases} 
S_{\text{opt}}^{Q_W(r)} \cap S_{\text{opt}}^{Q_S(r)}, & r \in (0, 1) \\
S_{\text{opt}}^{Q_W(r)}, & r = 1 
\end{cases}
\]

(7)

Otherwise, we move on to the next step.

9. Based on the data from steps 4 and 6 to determine the value of game \(S_{\text{opt}}^{Q_W(r)}\) in strategies of set \(S_{\text{opt}}^{Q_S(r)}\) according to the Savage criterion.

10. Based on the data from steps 3 and 7 to calculate the value of game \(W_{S_{\text{opt}}^{Q_W(r)}}\) in strategies of set \(S_{\text{opt}}^{Q_S(r)}\) according to the Wald criterion.

11. Based on the data from steps 4 and 7 to determine the set of strategies, which are not optimal on a set of pure strategies by both the Wald and Savage criteria.

12. For each strategy determined in step 11, to verify the correctness of inequality using steps 3, 6, 9, 10:

\[
\left( S_{\text{opt}}^{Q_W(r)} - S_s \right) W_i - \left( W_s - W_{S_{\text{opt}}^{Q_S(r)}} \right) S_i < W_{S_{\text{opt}}^{Q_S(r)}} S_{\text{opt}}^{Q_W(r)} - W_s S_s.
\]

(8)

If this inequality is not correct for at least one strategy, the calculations are completed, and the structure \(S_{\text{opt}}^{Q_W(r)}\) is not clear. If the inequality is correct, we move on to the next step.

13. Based on the data from steps 3 and 7 to determine the set \((S_{\text{opt}}^{Q_S(r)})_W\), optimal on \(S_{\text{opt}}^{Q_W(r)}\) according to the Wald criterion.

14. Based on the data from steps 4 and 6 to determine the set \((S_{\text{opt}}^{Q_W(r)})_S\), optimal on \(S_{\text{opt}}^{Q_S(r)}\) according to the Savage criterion.

15. Based on the data from steps 3, 6, 9, 10 to calculate \(r_{Q_W}\) using formula:

\[
r_{Q_W} = \frac{S_{\text{opt}}^{Q_W(r)} - S_s}{S_{\text{opt}}^{Q_S(r)} - S_s + (W_s - W_{S_{\text{opt}}^{Q_S(r)}})}.
\]

(9)

16. Based on the data from steps 4, 7, 13, 14, 15 to determine the structure of the set of optimal pure strategies \(S_{\text{opt}}^{Q_W(r)}\) using formula:

\[
S_{\text{opt}}^{Q_W(r)} = \begin{cases} 
S_{\text{opt}}^{Q_W(r)} \cap S_{\text{opt}}^{Q_S(r)}, & \text{with } r = 0 \\
S_{\text{opt}}^{Q_W(r)} \cap S_{\text{opt}}^{Q_S(r)}, & \text{with } 0 < r < r_{Q_W} \\
S_{\text{opt}}^{Q_W(r)} \cap S_{\text{opt}}^{Q_S(r)}, & \text{with } r_{Q_W} < r < 1 \\
S_{\text{opt}}^{Q_W(r)} \cap S_{\text{opt}}^{Q_S(r)}, & \text{with } r = 1.
\end{cases}
\]

(10)

Let us consider the following example, in which the data on Sharpe ratios at successive time intervals (six observable periods) are used to select the priority sequence from six pre-compiled investment portfolios. Initial data are represented by a matrix of winnings.

\[
\begin{array}{cccccccc}
\Pi_i & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 & \Pi_6 & W_i \\
A_i & \hline 
A1 & 0.0953 & 0.2681 & 0.1750 & -0.2681 & 0.0729 & -0.0807 & -0.2681 \\
A2 & 0.0221 & 0.2213 & 0.1871 & -0.2631 & 0.0612 & -0.1127 & -0.2631 \\
A3 & 0.0217 & 0.2205 & 0.1866 & -0.2630 & 0.0611 & -0.1135 & -0.2630 \\
A4 & 0.4458 & 0.0611 & -0.1195 & -0.2393 & 0.0832 & -0.2587 & -0.2587 \\
A5 & 0.0451 & 0.2366 & 0.1886 & -0.2658 & 0.0643 & -0.1057 & -0.2658 \\
A6 & 0.1583 & 0.1243 & 0.2144 & -0.2184 & 0.0742 & -0.0702 & -0.2184 \\
\beta_j & 0.4458 & 0.2681 & 0.2144 & -0.2184 & 0.0832 & -0.0702 & W_s = -0.2184 \\
\end{array}
\]

\[(11.1)\]

Efficiency indicators \(W_i, i=1,2,...,6\) of strategies \(A_i, i=1,2,...,6\) are calculated in the last column of the matrix according to the Wald criterion. The last line contains the indices of favorability \(\beta_j, i=1,2,...,6\) of the states of nature \(\Pi_i, i=1,2,...,6\). Let us determine the structure of the set \(S_{\text{opt}}^{Q_W(r)}\) of strategies, which are optimal in a set of pure strategies according to the Wald-Savage synthetic criterion, in accordance with the above algorithm. Recall that the winnings matrix used in the interpretation consists of Sharpe ratios for six portfolios and for various "states of nature" (the periods for which these coefficients are calculated). Efficiency indicators of strategies according to the Wald criterion are found and shown in the last column of matrix (11.1). The value of the game in pure strategies according to the Wald criterion is as follows: \(W_s = -0.2184\). It follows from the last column that \(W_6 = W_s = -0.2184\), which means that strategy A6 is optimal according to the Wald criterion.
Consequently, $S_{\text{opt}}^{Q_\text{w}(r)} = \{A6\}$. Let us form a risk matrix generated by the winnings matrix (11.1):

\[
\begin{array}{cccccccc}
\text{Ai} & \text{Pi} & \text{Pi} & \text{Pi} & \text{Pi} & \text{Pi} & \text{Pi} & \text{Si} \\
A1 & 0.3505 & 0.0000 & 0.0394 & 0.0497 & 0.0103 & 0.0105 & 0.3505 \\
A2 & 0.4237 & 0.0468 & 0.0273 & 0.0447 & 0.0220 & 0.0426 & 0.4237 \\
A3 & 0.4241 & 0.0476 & 0.0278 & 0.0446 & 0.0221 & 0.0434 & 0.4241 \\
A4 & 0.0000 & 0.2070 & 0.3339 & 0.0209 & 0.0000 & 0.1886 & 0.3339 \\
A5 & 0.4007 & 0.0315 & 0.0258 & 0.0474 & 0.0189 & 0.0355 & 0.4007 \\
A6 & 0.2875 & 0.1439 & 0.0000 & 0.0000 & 0.0090 & 0.0000 & 0.2875 \\
\end{array}
(11.2)
\]

The indicators are calculated and presented in the last column of matrix (11.2). The value of the game according to the Savage criterion is as follows $S_r = 0.2875$. A set of strategies $S_{\text{opt}}^{Q_\text{w}(r)}$, which are optimal in a set of pure strategies according to the Savage criterion, consists of a single strategy $A6$. Consequently, $S_{\text{opt}}^{Q_\text{w}(r)} = \{A6\}$. Using matrices (11.1) and (11.2), we find the value of the criterion for each strategy at the ends of segment [0, 1] by formula (2) and present the obtained values in Table 1.

![Table 1](image)

![Table 2](image)

The results of the calculations show the following: the left end $Q_{WS_4}(0)$ of section $Q_{WS_4}(r)$ of strategy $A4$ is less than the indicator at the left end of strategy $A6$, and the right end $Q_{WS_4}(1)$ of strategy $A4$ is more than the right ends of strategies $A1$, $A2$, $A3$, and $A5$. Therefore, it is possible to determine the mutual intersections of the segments $Q_{WS_i}(r), i = 1, \ldots, 6$, which appears shown in Table 2. In the cells, “x” indicates the intersection of Table 2 in the cells.

![Table 3](image)

Next, we find $r$ values at the intersection of each segment, solving the equation $Q_{WS_4}(r) = Q_{WS_5}(r)$. Let us obtain the following $r$ values for each intersection:

- $r_{12} = 0.9350$;
- $r_{23} = 0.9738$;
- $r_{15} = 0.9552$;
- $r_{25} = 0.8939$; and $r_{65} = 0.8950$.

Table 3 presents the values of efficiency indicators $Q_{WS_i}(r), i = 1, \ldots, 6$ at $r = 0, r_{12}, r_{23}, r_{15}, r_{25}, r_{35}, 1$ and strategy numbers are presented in Table 3 in order of priority.

<table>
<thead>
<tr>
<th>Value of $r$ indicator</th>
<th>Values of efficiency of $Q_{WS_i}(r) = rW_i - (1 - r)S_i$ pure strategies $Ai$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.35048  -0.42373  -0.42408  -0.33390  -0.40072  -0.28753</td>
</tr>
<tr>
<td>0 &lt; $r &lt; 0.8939$</td>
<td>0.3  0.5  6  2  4  1</td>
</tr>
<tr>
<td>0.8939</td>
<td>-0.27688 -0.28010 -0.28013 -0.26671 -0.28010 -0.22574</td>
</tr>
<tr>
<td>0.8939 &lt; $r &lt; 0.8950$</td>
<td>0.3  0.4  6  2  4  1</td>
</tr>
<tr>
<td>0.8950</td>
<td>-0.27679 -0.27993 -0.27996 -0.26663 -0.27996 -0.22566</td>
</tr>
</tbody>
</table>
Therefore, efficiency indicators have been calculated for each strategy. The strategies are ranked in a non-growing order (the numbers are specified in the table under efficiency indicators). If a pure strategy number in the priority sequence for \( r \) in the interval is found, a priority sequence position number will be assigned for the strategy, which would be general for the ends of this interval. For example, for strategy A1, the general priority sequence position number is 6 if the value of the winning-indicator at the end of the interval is (0.9738; 1). Therefore, at any value of \( r \) from this interval, strategy A1 will take the sixth place. The obtained sequences allow recommendations to be offered to a non-institutional investor. By choosing the least risky option, the following priority sequence of growing order (the numbers are specified in the table) is obtained:

\[
\begin{array}{cccccc}
\text{Interval} & 3 & 4 & 5 & 2 & 5 & 1 \\
\hline
0.8950 < r < 0.9350 & 3 & 4 & 5 & 2 & 5 & 1 \\
0.9350 & 0.9550 & -0.27350 & -0.27350 & -0.27351 & -0.26363 & -0.27455 & -0.22290 \\
0.9350 < r < 0.9552 & 3 & 3 & 5 & 2 & 6 & 1 \\
0.9552 & 0.9738 & -0.27184 & -0.27026 & -0.27027 & -0.26211 & -0.27184 & -0.22151 \\
0.9552 < r < 0.9738 & 5 & 3 & 4 & 2 & 5 & 1 \\
0.9738 & 1 & -0.26815 & -0.26306 & -0.26305 & -0.25874 & -0.26579 & -0.21841 \\
\end{array}
\]

\[
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\]

Therefore, we consider a simpler solution to this problem using another classical criterion of “games against nature” - the pessimism-optimism criterion of Hurwitz, which is as follows: Hurwitz criterion \( Hurw(\alpha) \) with a coefficient \( \alpha \in [0,1] \) of optimality of pure strategies on the set \( S \) with respect to the winnings. This coefficient expresses a quantitative “measure of optimism” of a player—a non-institutional investor—when the player is choosing a strategy. The coefficient is determined by this measure of optimism from subjective considerations on the basis of statistical studies of the results of decision-making by the stock market agents and personal decision-making experiences in similar situations. The pure strategy optimality according to the pessimism-optimism criterion of Hurwitz \( Hurw(\alpha) \) is defined by an indicator:

\[
Hurw(\alpha) = \alpha i^+ + (1-\alpha) i^- 
\]

(12)

where \( i^+ \) is the efficiency indicator of strategy \( A_i \) according to the maxi max criterion, \( i^- \) is the efficiency indicator of the strategy according to the Wald criterion. Pure strategy \( A_5 \) with the highest efficiency indicator \( Hurw(\alpha) \) is called optimal for the set of pure strategies according to the Hurwitz criterion with the coefficient of optimism \( \alpha \) with respect to winnings:

\[
Hurw(\alpha) = \max\{Hurw(\alpha): i \in I\}
\]

(13)

It is obvious that when \( \alpha = 0 \), the Hurwitz criterion is transformed into the Wald criterion for the optimality of pure strategies; when \( \alpha = 1 \), the Hurwitz criterion is transformed into the maxi max criterion for the optimality of pure strategies. Let us calculate the performance indicators by the pessimism-optimism criterion of Hurwitz for the strategies under consideration. The extended payoff matrix (13.1) has the following form:

\[
\begin{array}{cccccccc}
\text{Ai} & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 & a_i^+ & a_i^- & Hurw(\alpha) \\
\hline
A1 & 0.095 & 0.268 & 0.175 & -0.268 & 0.073 & -0.081 & 0.268 & -0.268 & 0.536 \alpha-0.268 \\
A2 & 0.022 & 0.221 & 0.187 & -0.263 & 0.061 & -0.113 & 0.221 & -0.263 & 0.484 \alpha-0.263 \\
A3 & 0.022 & 0.221 & 0.187 & -0.263 & 0.061 & -0.114 & 0.221 & -0.263 & 0.484 \alpha-0.263 \\
A4 & 0.446 & 0.061 & -0.119 & -0.239 & 0.083 & -0.259 & 0.446 & -0.259 & 0.705 \alpha-0.259 \\
A6 & 0.045 & 0.237 & 0.189 & -0.266 & 0.064 & -0.106 & 0.237 & -0.266 & 0.502 \alpha-0.266 \\
\end{array}
\]

(13.1)

From the column “\( a_i^+ \)”, we obtain: \( a_i^+ = \max a_i^+ = -0.218 \), and, therefore, the set of optimal strategies according to the Wald criterion is: \( S_{opt}^W = \{A_5\} \). From the column “\( a_i^- \)”, we obtain: \( a_i^- = \max a_i^- = 0.446 \), and, therefore, the set of optimal strategies according to the maxi max criterion is: \( S_{opt}^M = \{A_5\} \).

Using the found sets \( S_{opt}^W \) and \( S_{opt}^M \), we calculate:

\[
a^+_{\alpha(+)\alpha} = \max\{A4\} = \max(-0.259) = -0.259, \\
a^+_{\alpha(-)\alpha} = \max\{A6\} = \max(0.214) = 
\]
method of forming a priority sequence of financial assets portfolios proposed in this paper allow expanding the possibilities of portfolio theory taking into account the prospects of changing not only the parameters of securities selected as investment instruments, but also such an important integral characteristic of the portfolio as the Sharpe ratio.

References