A Modified Reverse Supply Chain with Remanufacturing for Sustainable Product Cycle

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Abstract—This paper deals with a new model, in which the stationary demand for a product can be fulfilled by remanufactured products along with newly procured product leading to minimum consumption of virgin raw materials. The remanufactured products are assumed to be as good as new ones and the returned items from the customers can be remanufactured at a fixed rate. The model helps in maintaining the goodwill of the customers by not allowing frequent stock outs by providing safety stock. A model is proposed and analyzed depending on the relationship between different parameters. An interpretive modelling based approach has been employed to model the reverse logistics variables typically found in reverse supply chains. A methodology is used for the calculation of optimum level for the newly manufactured items and the optimum level of the returned items for remanufacturing simultaneously. The major objective is to minimize the waste and gain the competitive advantage of cost of conversion. Moreover the company can sustain in the same line of business for a longer period of time.

Keywords—Product recovery, remanufacturing, reverse supply chain, sustainable products

1. Introduction

A stiff increase in the demand for goods and commodities has resulted in the reduction of non-renewable resources with a high percentage of land fill of waste. This has shifted the focus of traditional supply chain towards recovery options for the end of life products and products returned from various stages. Reverse logistics is the process of moving goods from their typical final destination for the purpose of capturing value (through reuse, repair, recycle, refurbish, remanufacture and cannibalize) or proper disposal. It is a process whereby supply chains can become more environmental friendly by reducing the amount of virgin materials used. It is observed that all the sales transactions carried in many product-based supply chains are not final with the payment recovery at the point of sales as they need to cope up with returns of the product due to recalls, warranty claims, service returns, recovery at the end-of-use, disposal at the end-of-life etc. Thus, the reverse distribution, which is from consumer to producer, has gained tremendous importance in the recent years. Reverse logistics stands for all the operations related to reuse of products coming back from customers, excess inventory of products and materials including collection, disassembly and processing of used products, product parts, and/or materials. This concept is complicated by the following typical figures on critical thinking.

• 50% of customers with bad return experience will not buy from brand again.
• In 60% of the cases cost to process a returned product can be higher than value of product.
• Lack of insight from returns data not used for improving revenue, growth and profitability
• Liability from non-compliance on waste regulations

These figures make most successful companies focus on their core competencies – delighting customers through stellar forward supply chain but experience a persistent lack of control over their reverse logistic processes leading to high cost, poor customer service, reduced asset recovery, low profitability, loss of shareholder value and decreased competitiveness. Organizations and communities are forced to consider recovery alternatives such as reuse, repair, recycle,
refurbish, remanufacture and cannibalize, rather than discarding of the products after end of life. The different product recovery options are shown in fig.1. The Product recovery aims to minimize the amount of waste sent to landfill sites by recovering materials and parts from old or outdated products.

Product recovery and reuse of products and materials is not a new experience. Waste paper recycling, metal scrap brokers, and deposit systems for soft drink bottles are all examples that are in practice since long. These cases find recovery of the used products is economically more attractive than disposal. Out of the different recovery options, this paper emphasizes on the remanufacturing part.

A model is developed which operates both on direct manufacturing as well as remanufacturing. A mathematical model is developed and care has been taken to optimize the inventory level of direct manufacturing as well as the remanufacturing with safety stock to avoid the stock out due to demand fluctuation. The aim is the reutilization of resources and therefore focuses on integration of the upstream and downstream chains. A generalized model for remanufacturing is shown in fig-2. This model is a combination of direct manufacturing and remanufacturing.

2. Related work

A supply chain is a network of facilities and distribution options that executes the functions of procurement of materials, conversion of these materials into intermediate and finished products, and the distribution of these finished products to customers. Supply chains exist in both service and manufacturing organizations, although the complication of the chain may vary greatly from industry to industry and organization to organization. The literature review, for the sake of clear understanding of this paper is divided into two distinct divisions, Forward supply chain and Reverse supply chain with specific orientation towards inventory control.

2.1 The Forward Supply Chain

Much has been said in the past and research is still going on the effectiveness of the forward supply chain. According to Frankel [1], much emphasis should be given on the key components (supplier, manufacturing and customer) and factors responsible for successful collaboration of supply chain to make the forward supply chain effective. Product recovery includes collection, disassembly, cleaning, sorting, repairing, reconditioning, reassembling and testing. Brennan et al [2]. The ability of supply chain members to successfully design and execute solutions that facilitate inventory arriving on time is critical to developing stronger supply chains. Arshinder et al [3] has stressed upon the coordination between various members of the supply chain. Supply chains are generally complex and are characterized by numerous activities spread over multiple functions which differ from organizations to organizations. Therefore it becomes a challenge for the effective coordination of the supply chains. The authors elaborately discussed with the gaps in the coordination process. Edward [4] investigated and presented a tutorial overview of inventory management. The author tried to categorize the inventory problems & associated models based on dimensions. He concluded that there exists a continuing gap between theory and practice of SC pertinent to inventory management and suggested a number of research topics that will bridge the gap. Hesham, K. Alfares [5] has presented a model of an inventory system with stock-dependent demand, in which the holding cost is a step function of storage time. Two types of holding cost variation in terms of storage time have been
considered: retroactive increase, and incremental increase. Simple optimization algorithms have been developed, and numerical examples have been solved. From the analysis he concluded that both the optimal order quantity and the cycle time decrease when the holding cost increases.

2.2 The Reverse Supply Chain

Reverse Supply chain is the coordination and control, physical pickup and delivery of the material, parts, and products from the field to reuse, repair, recycle, refurbish, remanufacture and cannibalize, and subsequent returns back to the field where appropriate.

2.3 Repair and Recycle

Imre Dobos[6] in his work has analyzed a production–recycling system. His analysis consists of two types of models. First model is related to minimization of the EOQ related costs and the second one is the generalization of the first model with linear waste disposal, recycling, production and buyback costs. According to the author, the pure strategy (either production or recycling) is optimal when compared with the mixed strategy. Oh, Y. H, Hwang, H[7] considered a recycling system where the supplier receives a fixed portion of recyclable material from customers. Like used cans and crashed bottles, recyclable materials become raw materials of new ones. In order to meet the demand, he also purchases additional raw material from outside. He did find out the optimal solution by considering/comparing the total cost of (P*, 1) and (1, O*) where (p*, O*) is the “P” production set-up and “O” orders. Two cases like (P, 1) and (1, O) are discussed. The order quantity, production setup and the production lot is optimized based on minimization of the total cost. Koh S.G., et al.[8] obtained the economic order quantity for newly produced products and the optimal inventory level of recoverable items to start the recovery process simultaneously in a joint EOQ and EPQ model. According to the relationship with the parameters, a numerical model is proposed with one setup for recovery and many orders for new products. The system was modeled under four situations and a solution procedure is established to find out the optimal control parameters. Knut Richter [9] modeled a situation where some share of the used products is collected and later repaired; the other products are disposed outside according to some waste disposal rate. This model is extended to the case of variable setup numbers n and m for production and repair within some collection time interval.

2.4 Remanufacturing

A Study on a unified general inventory model for integrated new production and remanufacturing of returned items in an infinite planning horizon is presents by Alamri [10]. This work involves the joint production and remanufacturing options. A closed form for the total relevant cost is considered with production, remanufacturing, demand, return, and deterioration rates as arbitrary functions of time. Finally a mathematical model showing the global optimality to the underlying inventory system is introduced.

Teuner, R. H., and Vlachos, D[11] studied a single item hybrid production system with manufacturing and remanufacturing. It is assumed that remanufacturing is profitable and that, on average, there are more demands than returns. Teunter, R. Laan, E.V.D.[12] provided a DCF(discounted cash flow) inventory model with disposal and remanufacturing as while analyzing average cost inventory models. It is of common use to add the discount rate times the capital tied up in a product, to the out-of-pocket holding cost rate. The author suggest that, one should be very careful when applying the average cost approach for more complex models with remanufacturing and disposal as no set of holding cost rate will lead to DCF optimal. Richter, K and Sombrutzki, M [13] in his paper have discussed the reverse Wagner/Whitin’s dynamic production planning and inventory control model and some of its extensions. Their model can efficiently deal with several combinations of reverse and original models. The restriction of the proposed model is that if the quantity of used products does not match the demand of remanufactured goods, the methods proposed here, fails. Therefore the design of appropriate algorithms seems to be another important research direction. Chung, S.L, et al [14] analyzed an inventory system with traditional forward-oriented material flow as well as a reverse material flow supply chain. In the reverse material flow, the used products are returned, remanufactured and shipped to the retailer for resale. They proposed a multi-echelon inventory system with remanufacturing capability. The
analytical results of this study show a substantial profit increase using the integrated approach.

The objective of the present work is to build a sustainable model for a manufacturing facility using remanufacturing concept in order to meet the average demand economically in the light of the model explained by Koh, S. G. et al [8]. Further the work attempts to determine the various model variables by developing a mathematical model. The focus of study in this work is aimed at the following.

- To determine the optimal order quantity of newly manufactured products to be mixed with the line for meeting the average market demand.
- To determine the optimal quantity of returned items that needs to be there for initiating the remanufacturing process.
- To determining the safety stock considering the fluctuation of the collection rate of returned items for remanufacturing stabilizing the diversified opinions in the field of returned item inventory.

The primary concern here is environmental benefits where use of return items will decrease the depletion rate of natural resources. Successful implementation of the method can reduce the inventory cost of finished goods. This model helps in reduction of the raw material supplies thereby minimizing the risk associated with the environment.

3 The proposed Model

The significance of the proposed model is that, the disadvantages associated in the model explained by Koh, S. G. et al [8] (where stock outs are not considered) are taken care by the introduction of the safety stock. As the model uses remanufacturing of recovered products, there is a decrease in the total cost and there will be an increase in the productivity. Additionally, the possibility of loss of goodwill from the customers resulting from stock outs is minimized in the proposed model. The primary concern is environmental benefits where the use of return items will decrease the depletion rate of resources. Successfully implementing the method can reduce the cost of finished goods. This helps in reduction of the raw material supplies.

3.1 Model Description

As shown in the figure -3, there are two cycles acting simultaneously. The first one is the remanufacturing cycle and the second one is serviceable cycle comprising of products both from direct manufacturing and remanufacturing. The remanufacturing starts after a certain amount of collection (R). The remanufactured goods are assumed to be as good as new and can be a perfect substitute for the direct manufactured products. As there are certain uncertainties in the collection rate of the returned items for remanufacturing therefore we propose a safety stock of finished goods/serviceable that provides a cushion during the fluctuation of the return collection rate. The mathematical calculation deals with the optimal value of the quantity ‘Q’ for direct manufacturing, the optimal value ‘R’, to start remanufacturing and the buffer stock.

![Figure 3. Model setup for remanufacturing and newly procured items to satisfy the market demand](image)

3.2 Model Parameters

The following are the various process parameters used in the present model.

- r: the number of returned items collected from the customers in unit time (units/time)
- p: the capacity of the remanufacturing process (units/time)
- d: the demand of finished products/serviceable(units/time)
- C_s: the set up cost for the remanufacturing process (Rs/setup)
- C_0 : the ordering cost for new items (Rs/order)
- C_h1 : the inventory holding cost for the returned items for remanufacturing(Rs/unit/time)
C\(_{h2}\) : the inventory holding cost for the newly manufactured items (Rs/unit/time)

T : Cycle time.

t : Time after which remanufacturing initiates.

The model is intended to analyze the behavior and find an optimal value for the following process variables.

- R : the inventory level for remanufactured items,
- Q : the order quantity for the newly procured items, and
- S: the amount of safety stock to take care of the fluctuation of the collection rate for returned items for remanufacturing.

While formulating the model the following assumptions are made:

- The average demand is ‘d’.
- Returned products are collected for remanufacturing with an average rate ‘r’.
- The remanufacturing capacity is ‘p’.
- The cost parameters are known and remain constant.
- Purchase and remanufacturing lead times are constant.
- Remanufacturing rate is greater than the collection rate.
- Demand rate is greater than the collection rate.

### 3.3 Mathematical Formulation

The cost of remanufactured items comprises of the setup cost and the inventory holding cost. So the total cost for remanufacturing cycle

\[ C_s + \frac{C_{h1}}{2}RT \]  \hspace{1cm} (1)

The cost of serviceable items comprises of order cost, inventory holding cost considering the triangles \(\alpha\) and \(\beta\).

Order cost = \(C_0\)  \hspace{1cm} (2)

Holding cost for triangle \(\alpha\) in Fig-3 is

\[ \frac{p(p - d)C_{h2}}{2d} \left( T - t \right)^2 \]  \hspace{1cm} (3)

Holding cost for the triangle \(\beta\) in Fig-3 is

\[ \frac{p(p - d)C_{h2}}{2d} \left( T - t \right)^2 \]  \hspace{1cm} (4)

It can also be shown that \(T = \frac{pR}{r(p - r)} \) \hspace{1cm} (5)

And \(t = \frac{R}{r} \) \hspace{1cm} (6)

Using equation 1-5,

We can have the total cost, \(TC = \frac{r(p - r)(C_s + C_0)}{pR} + \frac{C_{h1}}{2}R + \frac{C_{h2}}{2d(p - r)} \left\{ \frac{p(d - r)^2}{r} + r(p - d) \right\}R \)  \hspace{1cm} (7)

Since equation-7 is a convex function of R we can have the optimal value of R by differentiating the total cost (TC) with respect to R and then equating it to zero.

Hence, the optimal value of R can be written as \(R = \sqrt{\frac{r(p - r)(C_s + C_0)}{pA_1}} \)  \hspace{1cm} (8)

Where

\[ A_1 = \frac{C_{h1}}{2} + \frac{C_{h2}}{2d(p - r)} \left\{ \frac{p(d - r)^2}{r} + r(p - d) \right\} \]  \hspace{1cm} (9)

The value of Q can be calculated by using the equation,

\[ Q = \frac{p(d - r)R}{r(p - r)} \]  \hspace{1cm} (10)

### 3.4 The Safety Stock

Since uncertainty lies in the remanufacturing cycle due to fluctuation of the collection rate, it is preferred to consider the additional stock of newly produced items in stock which will be adequate to satisfy the demand of the serviceable items when there is a decrease in the value of ‘r’. Further it is also assumed that the collection of the returned items continues throughout the cycle, hence the
safety stock calculation should be made for the entire cycle time and it can be taken as;

Safety Stock = \( Z\sigma_T \). Here, \( Z \) = number of standard deviations for a specified service level, which can be directly used from the normal distribution table (For example service level 95% means that, \( Z = 1.65 \)).

And \( \sigma_T \) = the standard deviation of the collection rate of items for remanufacturing for the total time period. The value of \( \sigma_T \) can be determined by the square root of the sum of the variances for each unit time during the total cycle time period.

\[
\sigma_T = \sqrt{\sum_{i=1}^{T} \sigma_{d_i}^2},
\]

Where \( \sigma_{d_i} \) is the standard deviation for each unit time during the total time period.

It is to be remembered here that, while calculating the total cost there will be two components. The first one is the fixed component for the safety stock which has to be maintained constant throughout the cycle and the second one is as calculated by using equation (7).

4 An Example Problem

In order to validate the model we consider a sample problem with the following data,

\( r = 100 \) units /month,
\( p = 300 \) units /month,
\( d = 200 \) units/month,
\( C_0 = \)Rs 10/order for new items
\( C_s = \)Rs 20/setup for remanufacturing process
\( C_{h1} = \) Rs 1/remanufacturable unit /month, and
\( C_{h2} = \) Rs 2/serviceable unit/month.

The standard deviation of collection rate for returned items for remanufacturing for each day is 5. The service level to be maintained is 95%. The purchase cost for new items = Rs8 per unit and

\[ \text{The purchase cost for the collection of returned items for remanufacturing} = \text{Rs 4 per unit.} \]

We can have the value of \( R \equiv 37 \) units; \( Q = 55 \) units and \( T = 0.555 \) months \( = 16.65 \) days \( = 17 \) days (say) using equation (7), equation (9) and equation (5) respectively.

Safety stock
\[
= Z\sigma_T = 1.65 \times \sqrt{\sum_{i=1}^{T} \sigma_{d_i}^2} = 1.65 \times \sqrt{\sum_{i=1}^{17} 5^2} = 1.65 \times \sqrt{17 \times 25} = 1.65 \times 2061 \equiv 34 \text{Units}
\]

Hence, the total cost per cycle;

\[ TC = \text{Total cost of inventory without safety stock as calculated by equation (7) + the cost of safety stock + the cost of purchase of new items + the cost of purchase of returned items for remanufacturing.} \]

\[ \text{Now } TC_{\text{inventory without safety stock}} = \text{Rs 109.55, } \]
\[ \text{Cost of safety stock = } 34 \times 8 = \text{Rs 272, } \]
\[ \text{Cost of purchase of new items without safety stock = } 55 \times 8 = \text{Rs 440, and } \]
\[ \text{Cost of purchase of returned items } = rT \times 20 = \text{Rs 222} \]

The total cost = 109.55 + 272 + 440 + 222 = Rs1043.55 say Rs1044

5 Results and Analysis

In order to make an analysis of the model and the associated inventory system, the developed mathematical model was tried to find out the relationship of the various parameters at different situations. Accordingly the parameters were chosen and their relationships are determined. Table.1 presents a concise state of conditions for analyzing the behaviour of these identified parameters. The values used in the analysis are the standard values as used in typical production units. A brief interpretation of the behaviour is presented to examine the validity of the model.

Table 1. Process Parameters for Model Analysis
As shown from the graphs, Fig.4 $C_0$ vs lot size $R$ and $Q$, it is clear that the value of $R$ is more than $Q$ at small value of $C_0$. As the value of $C_0$ increases, $R$ and $Q$ agree with each other at around $C_0=5.5$.the difference between $R$ and $Q$ shows an increase with higher values of $C_0$, where the relative increase in $Q$ value is more.

![Figure 4. Effect of order cost ($C_0$) on the system](image)

The effect of variation of $C_s$ (Fig. 5) has appreciable effect on $Q$ which decreases rapidly in the beginning but increases at a rather slow rate after $C_s$ reaches a value of around 20. After which it has a little effect on the difference between $R$ and $Q$. Both these quantities vary almost linearly with the variation of $C_s$ maintaining a constant difference.

![Figure 5. Effect of setup cost for remanufacturing ($C_s$) on the system](image)

The effect of variation of $p$ in Fig. 6 shows at small values of $p$, the quantities $R$ and $Q$ are close to each other. With increase of $p$, the $Q$ value increases more as compared to the value of $R$.

![Figure 6. Effect of capacity of recovery process (p) on the system](image)

The effect of variation of $d$ as shown in Fig. 7 depicts that at small values of $d$, the quantities $R$ and $Q$ are close to each other. With increase of $d$, the $Q$ value increases more as compared to the value of $R$.

![Figure 7. Effect of demand rate (d) on the system](image)

The effect of variation of $C_{h1}$ (Fig. 8) shows that at small values of $C_{h1}$, the quantities $R$ and $Q$ are close to each other. With increase of $C_{h1}$, the $Q$ value increases more as compared to the value of $R$. 

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<tr>
<th>Relationship</th>
<th>Constants</th>
<th>Variables</th>
</tr>
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<tbody>
<tr>
<td>$C_0$ vs. $R$, $Q$</td>
<td>$r$, $p$, $C_s$, $C_{h1}$</td>
<td>$d$, $C_0$, $C_{h2}$</td>
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Relationship Constants Variables

$C_0$ vs. $R$, $Q$ $r$, $p$, $C_s$, $C_{h1}$ $d$, $C_0$, $C_{h2}$

$C_s$ vs. $R$, $Q$ $r$, $d$, $C_o$, $C_{h2}$ $C_s$, $p$, $C_{h1}$

$d$ vs. $R$, $Q$ $r$ $d$, $p$, $C_s$, $C_o$, $C_{h1}$, $C_{h2}$

$p$ vs. $R$, $Q$ $r$, $C_o$, $C_{h2}$ $p$, $d$, $C_s$, $C_{h1}$

$C_{h1}$ vs. $R$, $Q$ $r$, $C_o$, $C_{h2}$ $C_{h1}$, $p$, $C_o$, $d$

$C_{h2}$ vs. $R$, $Q$ $r$, $p$, $C_s$, $C_{h1}$ $C_{h2}$, $C_o$, $d$
The effect of variation of $C_{h2}$ (Fig. 9) shows that at small values of $C_{h2}$, the quantities $R$ and $Q$ are close to each other. Even below the value of 1, the value of $R$ is higher than $Q$. With increase of $C_{h2}$, the $Q$ value increases more as compared to the value of $R$.

**Figure 8.** Effect of holding cost for returned items for remanufacturing ($C_{h1}$) on the system

From the above analysis it is clear that we have to keep the parameters $C_0$, $p$, $d$, $C_{h1}$ and $C_{h2}$ as small as possible. This will help in maintaining a balance between the $R$ and $Q$ with both the values remaining close to each other. The aim is to use the returned items for remanufacturing equivalently with the directly procured products.

**6 Conclusion**

The proposed model tries to satisfy the average demand of the product both from newly procured items and remanufactured items. For simplicity the model assumes one complete procurement cycle and one remanufacturing cycle in the total cycle time. Additionally, the inclusion of safety stock helps in not allowing frequent stock outs in the remanufacturing cycle due to variation of collection rate of returned items for remanufacturing.

This model can be modified with the consideration of waste disposal cost which is not considered in this model. The researchers can extend this model with the collection rate as a random variable.

**References**


