Lead-Time Estimation Approach using the Process Capability Index

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Abstract — This research proposes a methodology to estimate the customer order lead-time in supply chain based on the process capability index in manufacturing and service applications. The cases when the process output is normally distributed and when it is not are considered. The proposed method is used to examine the current process capability to deliver the orders before the promised lead-time. If the process was found to be incapable, the method can be used to revise the current lead-time to a proper value according to a desired service reliability level selected by the management. A Case study is presented to evaluate the capability index for the delivery processes in a multinational company in Egypt. The case presented estimates for the capability indices when delivering products belonging to \( G_1, G_2, \) and \( G_3 \) classes. Computation results reveal that the indices are 0.22, -0.257, and 1.01 respectively. Therefore, the process is incapable of delivering the \( G_1 \) and \( G_2 \) products before their promised due-date. The delivery process, however, does better job when delivering products belonging to class \( G_3 \). The proposed estimation methodology was employed to revise the lead-time for the incapable cases. The relationships between the system capability indices in both service and manufacturing applications, delivery system reliabilities and the percentages of orders delivered after their promised due-dates are presented.

Keywords — Lead-time Estimation, Process Capability Index, Delivery System Reliability, Statistical Analysis, Service Achievement Index, Service Quality

1. Introduction

Lead-time is one of the critical measures in supply chain, production and quality management. Decreased lead-time is the main criterion of having a responsive supply chain. Accurate lead-time estimation is essential in the process of improving customer satisfaction.

Several research papers were presented regarding lead-time estimations. Authors have utilized computer simulation to estimate the average time to complete a customer order or a job. Other authors utilized queuing theory as well as statistical models. Many assumptions were tackled in conducting the simulation experiment; such as: assuming stable power supply with no power outage; equal production output for all employees, no maintenance performed during production process and so on. If the lead-time is estimated based on the process mean; plenty of orders will be delivered after the promised due date which would lead to dissatisfied customers.

This paper presents a new lead-time estimation methodology based on the production or the service processes capability index. The index compares the process specification width, or the company standard lead-time to the process variability spread that can be estimated from company’s actual lead-time database. The estimated lead-time following this proposed methodology guarantees lower probability of late customer order deliveries; which maximizes the customer satisfaction.

2. Literature Review

In this section, a review of the literature on lead-time estimation methodologies, process capability indices in both service and manufacturing systems are presented.

Several authors have followed various approaches for lead time estimation including: (a) Simulation approach; (b) queuing theory; (c) logistic operating curves; (d) stochastic analysis; (e) statistics; (f) artificial intelligent; (g) and Hybrid
Process capability index provides a quantitative assessment to the process performance to determine the degree its output conforms to specifications [3]. It is measured as the ratio between the tolerance spread and natural spread of a process as shown in the following equation that was first presented in [4] when the process output is normally distributed and the process mean is centered between specification limits:

\[ C_p = \frac{USL - LSL}{6\sigma} \]

Where \( C_p \) is the capability index, \( USL \) is the upper specification limit, \( LSL \) is the lower specification limit, and \( \sigma \) is the process standard deviation of the in-control process. A \( C_p \) value less than 1.0 indicates the tolerance spread is narrower than the natural spread which indicates that the process is incapable. However if it is more than 1, it indicates the process tolerance is wider than the natural spread, and the process is capable. Industry recommended that the \( C_p \) value is more than 1.33 [3]. Ref. [5] has reviewed the process capability literature for a manufacturing process. In their research paper, they cited [6] proposing a process capability index estimator for a non-centered process that follows a normal probability distribution, when the process output falls between the specification limits, \( C_{pk} \):

\[ C_{pk} = \min \left( \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right) \]

Where \( \mu \) and \( \sigma \) are the mean and standard deviation of the process, or the parameters of the in-control process. The authors of ref. [7] cited some researchers developing the relationship between the process capability indices \( C_p \) and \( C_{pk} \) as:

\[ C_{pk} = (1 - k)C_p \]

Where \( k \) can be determined as:

\[ k = \frac{M - \mu}{(USL - LSL)/2} \]

Where variable \( M \) represents the speciation interval midpoint that can be stated as follows:

\[ M = (USL + LSL)/2 \]

Ref. [8] has listed some of the assumptions for the process capability formulae: (a) process must be in-control state; and (b) the upper and lower specifications limits are able to represent the customer specifications. The authors also stated that in production practice, when the process capability \( C_p \geq 1.67 \), it is rated high; when it is 1.33 \( \leq C_p < 1.67 \), the process capability is rated moderate high; when the value is 1.0 \( \leq C_p < 1.33 \), the process capability is rated as ordinary; and when it is 0.67 \( \leq C_p < 1.0 \), it is rated moderately poor; and when it is less than 0.67 it is rated poor.

One problem with those indices is that they do not take into considerations the difference between the process mean and the target value of the process mean. This change will indicate how good the process is. Ref. [5] also cited ref. [9] and ref. [10] presenting two measures of capability indices taken into considerations the process mean target value for both the centered \( C_{pm} \) and non-centered case \( C_{pmk} \) as shown:

\[ C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \]

\[ C_{pmk} = \min \left\{ \frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\} \]

Ref. [11] considered the case when the process variability follows the lognormal probability distribution. He developed an \( C_{pk} \) index for this case as well as proposed a formula to determine the proportion of nonconforming items in the population. The proposed \( C_{pk} \) index is:

\[ C_{pk} = \min \left\{ \frac{Ue^{-\xi} - 1}{e^{3\sigma} - 1}, \frac{1 - Le^{-\xi}}{1 - e^{-3\sigma}} \right\} \]
Ref. [12] presented a generalization to the method presented in ref. [13] to estimate the capability index for a data set with asymmetric tolerance in the case when the process output does not follow the normal probability distribution. They proposed the following estimators:

$$C_p = \frac{USL - LSL}{X_{99.866} - X_{0.135}}$$

$$C_{pk} = \text{Min} \left\{ \frac{USL - X_{50}}{X_{99.866} - X_{50}}, \frac{X_{50} - LSL}{X_{50} - X_{0.135}} \right\}$$

The upper specification limit shown in the equations above will be replaced by the company promised lead-time, and the lowest possible time, which is zero, will replace the lower specification limit. The estimator is:

$$\hat{C}_{pk} = \text{Min} \left\{ \frac{LT - \bar{x}}{3\hat{\sigma}}, \frac{\bar{x} - LSL}{3\hat{\sigma}} \right\}$$

The other version of the previous equation is:

$$\hat{C}_{pk} = \hat{C}_{p}(1 - \hat{k})$$

3. Lead-Time Estimation Methods

In this section, the proposed lead-time estimation methods for both manufacturing and service systems are introduced. Those methods evaluate the capability of the existing process to deliver the orders before the promised due date and revise the current lead-time to a more reasonable one in case the process is found to be incapable.

3.1 Lead time Estimation in Manufacturing

Section 3.1.1 presents a method to evaluate the existing process capability of the delivery before the promised due date and if the process is found to be incapable, Section 3.1.2 presents a method to predict the proper lead-time based on the required service level.

3.1.1 Evaluation of the Current Process Capability

In this section, existing process capability index evaluation is introduced. If the process is normally distributed, the following process capability index $C_{pk}$ is used:

$$C_{pk} = \text{Min} \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\}$$

The upper specification limit shown in the equations above will be replaced by the company promised lead-time, and the lowest possible time, which is zero, will replace the lower specification limit. The estimator is:

$$\hat{C}_{pk} = \text{Min} \left\{ \frac{LT - \bar{x}}{3\hat{\sigma}}, \frac{\bar{x} - LSL}{3\hat{\sigma}} \right\}$$

The other version of the previous equation is:

$$\hat{C}_{pk} = \hat{C}_{p}(1 - \hat{k})$$
Where: 

$$\hat{C}_p = \frac{USL - LSL}{6\sigma} = \frac{LT}{6\bar{x}}$$

The value of $k$ is determined as:

$$k = \frac{|M - \bar{x}|}{(USL - LSL)/2} \quad 0 \leq k \leq 1$$

After substituting the upper and lower specification limits in the equation:

$$k = \frac{|LT - 2\bar{x}|}{LT} \quad 0 \leq k \leq 1$$

If the process output does not follow the normal probability distribution, the estimator proposed by [12] is used as shown in example 2.

**Example 1:** A manufacturing company promises to deliver the orders in 30 days. The manufacturing process has a mean of 20.6 days and the process standard deviation is 4.5 days. In the following steps, a test is conducted to examine the process capability of delivery in the stated lead-time assuming that it is normally distributed. This test is conducted using the two $\hat{C}_{pk}$ versions presented in the previous section. The following is the $\hat{C}_{pk}$ estimator using the first version:

$$\hat{C}_{pk} = \frac{lead\ time - \bar{x}}{3\bar{x}} = \frac{30 - 20.6}{3 \times 4.5} = 0.6963$$

The following are the steps to perform the test using the second version of the $\hat{C}_{pk}$ index:

$$\hat{C}_{pk} = \frac{LT}{6\bar{x}} = \frac{30}{6(4.5)} = 1.11111$$

The value of factor $k$ is:

$$k = \frac{|LT - 2\bar{x}|}{LT} = \frac{|30 - 2 \times 20.6|}{30} = \frac{11.2}{30} = 0.3733$$

Therefore $\hat{C}_{pk}$ is:

$$\hat{C}_{pk} = \hat{C}_p (1 - \hat{k}) = 1.111(1 - 0.3733) = 0.6963$$

As shown, the capability index following the two formulas produces the same answer; which is 0.6963. Since the capability index has low value, the process is incapable of delivering the products to the customers in the promised lead-time.

**Example 2:** The company advertises a shorter lead-time, 15 days, for the other product classification. Data analysis shows that the data are not normally distributed, the median lead-time, $X_{50} = 11$ days, and the value of $X_{99.866} = 14.5$ days and $X_{0.134} = 0$. The following steps show the estimation of the capability index in the case when the process output is not normal.

$$C_{pk} = \min \left\{ \frac{LT - X_{50}}{X_{99.866} - X_{50}}, \frac{X_{50} - LSL}{X_{0.134}} \right\}$$

$$C_{pk} = \min \left\{ \frac{15 - 11}{14 - 11}, \frac{11 - 0}{11 - 0} \right\} = 1.333$$

Since the process capability index is high, the process is capable to deliver the products to the customers before the promised due date.

3.1.2 Lead-Time Estimation

This section presents the lead-time estimation methodology when the output follows either the normal or non-normal distribution. Starting with the normally distribution case. The management has to select the required system capability as stated by Chai and Zhu (2010) in Table 1.

<table>
<thead>
<tr>
<th>Service Rank</th>
<th>Rating</th>
<th>$C_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>High</td>
<td>$C_p \geq 1.67$</td>
</tr>
<tr>
<td>II</td>
<td>Moderate High</td>
<td>$1.33 \leq C_p &lt; 1.67$</td>
</tr>
<tr>
<td>III</td>
<td>Ordinary</td>
<td>$1.0 \leq C_p &lt; 1.33$</td>
</tr>
<tr>
<td>IV</td>
<td>Moderate</td>
<td>$0.67 \leq C_p &lt; 1.0$</td>
</tr>
<tr>
<td>V</td>
<td>Poor</td>
<td>$C_p &lt; 0.67$</td>
</tr>
</tbody>
</table>

Table 1: Capability index and Service Rating in Manufacturing Applications

The process capability index $C_{pk}$ is:

$$\hat{C}_{pk} = \min \left\{ \frac{LT - \bar{x}}{3\bar{x}}, \frac{\bar{x}}{3\bar{x}} \right\}$$

This approach replaces the upper specification limit by the company promised lead-time, maximum possible time (USL = Lead Time), and the lowest possible time, which is zero, will replace the lower specification limit (LSL = 0), as shown in Figure 1. The figure presents the upper and lower control limits of a process that is not centered between the specification limits. The process center is $\mu$ and the midpoint between the specification limits is $M$. The process output is assumed to follow the normal
probability distribution with process mean \( \mu \) and standard deviation \( \sigma \). The area under the right tail of the normal distribution represents the probability of late delivered orders.

\[
\text{System will be able to meet is estimated as:} \\
\overline{LT} = \bar{x} + 3\hat{C}_{pk}
\]

If a \( \hat{C}_{pk} \) value of 1.69 is selected

\[
\overline{LT} = 20.6 + 3(4.5)(1.69) = 43.4 \text{ days}
\]

Therefore, a more appropriate lead-time estimate is 44 days. If the lead-time estimate is found to be too long, a process improvement program should be carried out to reduce both the process variability and mean. To determine the process reliability as well as the percentage of late delivery orders at the current and proposed lead-times,

\[
R = P[T \leq LT] = P(Z \leq \frac{30 - 20.6}{4.5}) = 0.9816
\]

\[
R = P[T \leq LT] = P(Z \leq \frac{44 - 20.6}{4.5}) = 0.9999
\]

The percentages of late orders in both cases which are estimated in parts per million (PPM) is determined. For the case when lead-time was set at 30 days, 18,359 orders will be late per million. But when the lead-time is set to 44 days, about 0.1 orders for each million will be late.

Table 2 presents the relationship between process capability index, reliability of the order delivery system, and proportion of late orders in parts per million orders.

### 3.2 Lead time Estimation in Service Applications

The concept of process capability index fits very much in manufacturing applications but needs to be modified if wants to be considered in service oriented applications. The upper and lower specification limits identify the differences between the non-defective and the defective products in manufacturing but in service the customer may specify a range of values that he/she will be equally satisfied with service quality [14].

The author proposed the following formula to be used to compute the service capability index assuming uniformly distributed random variable and the customer prefers lower values of service performance to higher values as in the case of lead-time.
Table 2: Relationship between process capability and System Reliability

<table>
<thead>
<tr>
<th>LT − μ</th>
<th>Cpk</th>
<th>R</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1σ</td>
<td>0.333</td>
<td>0.8413</td>
<td>158,655</td>
</tr>
<tr>
<td>1.5σ</td>
<td>0.500</td>
<td>0.933</td>
<td>66,807</td>
</tr>
<tr>
<td>2σ</td>
<td>0.667</td>
<td>0.977</td>
<td>22,750</td>
</tr>
<tr>
<td>2.5σ</td>
<td>0.833</td>
<td>0.994</td>
<td>6,209</td>
</tr>
<tr>
<td>3σ</td>
<td>1.000</td>
<td>0.998</td>
<td>1,349</td>
</tr>
<tr>
<td>3.5σ</td>
<td>1.167</td>
<td>0.9997</td>
<td>233</td>
</tr>
<tr>
<td>4σ</td>
<td>1.333</td>
<td>0.9999</td>
<td>32</td>
</tr>
<tr>
<td>4.5σ</td>
<td>1.500</td>
<td>0.9999</td>
<td>3</td>
</tr>
<tr>
<td>5σ</td>
<td>1.667</td>
<td>0.9999</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3: Capability index and Service Rating in Service Applications

<table>
<thead>
<tr>
<th>Service Rank</th>
<th>Rating</th>
<th>S_Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>High</td>
<td>S_Q ≥ 1.67</td>
</tr>
<tr>
<td>B</td>
<td>Moderate High</td>
<td>1.33 ≤ S_Q &lt; 1.67</td>
</tr>
<tr>
<td>C</td>
<td>Ordinary</td>
<td>1.0 ≤ S_Q &lt; 1.33</td>
</tr>
<tr>
<td>D</td>
<td>Moderate</td>
<td>0.67 ≤ S_Q &lt; 1.0</td>
</tr>
<tr>
<td>E</td>
<td>Poor</td>
<td>S_Q &lt; 0.67</td>
</tr>
</tbody>
</table>

Accordingly, the S_Qi values are the standard Z values of a normal distribution. Using the table of the standard normal distribution, the range of −3 < S_Qi < 3 includes 99.73% of the values when the service process is in control. To determine the lead time in this case:

\[ S_Q \leq U_l - \mu_i \]

The lead time = \[ U_l = \mu_i + S_Q \sigma_i \]

4. Case Study

This case study utilizes a sample of lead-time data of 2013 customer orders from the data file of a multinational company operating in Egypt. The company classifies their products into 3 classes: \[ G_1, G_2 \text{ and } G_3[17]. \] The data file includes 1143 lead-time observations for products belonging to the \[ G_1 \] class, 682 observations for products belonging to the \[ G_2 \] class and 188 observations for the \[ G_3 \] class. Below is the summary statistics of the average, standard deviation, sample size, and promised lead-time for the products belonging to the \[ G_1 \] class:

<table>
<thead>
<tr>
<th>Lead time Statistics for products [ G_1 ] (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Count</td>
</tr>
<tr>
<td>Promised Lead-time</td>
</tr>
</tbody>
</table>

When the process output is normal, the formula proposed in reference [15] is used to compute the service capability index or service achievement index:

\[ S_{Qi} = \frac{U_l - \mu_i}{\sigma_i} \]
As a result, the service quality is poor. The percentage of late delivery orders is 41% and the service reliability is 59%. The service process has to be improved to reduce process average and variability or the promised lead-time has to be revised to a reasonable value the system will be able to provide. The current process can provide a lead time is $25.69 + 1.67 \times 19.579 = 58.39$ days with percentage of late orders of 4.7% and service reliability 95.3%. The following table presents the summary statistics for $G_2$ products:

<table>
<thead>
<tr>
<th>Lead time Statistics for products $G_2$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Count</td>
</tr>
<tr>
<td>Promised</td>
</tr>
</tbody>
</table>

$$P_{Q,G_2} = \frac{\text{Lead time} - \bar{x}}{s} = \frac{15 - 19.34}{16.75} = -0.257$$

As a result, the service quality is poor. The percentage of late delivery orders is 39.7% and the service reliability is 60.3%. The service process has to be improved to reduce process average and variability or the promised lead-time has to be revised to a reasonable value the system will be able to provide. The current process can provide a lead time is $19.3 + 1.67 \times 16.75 = 47.3$ days.

The following table presents the summary statistics for $G_3$ products:

<table>
<thead>
<tr>
<th>Lead time Statistics for products $G_3$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Count</td>
</tr>
<tr>
<td>Promised</td>
</tr>
</tbody>
</table>

For products G3 category,

$$P_{Q,G_3} = \frac{\text{Lead time} - \bar{x}}{s} = \frac{60 - 37.82}{21.91} = 1.01$$

As a result, the service quality is poor. The percentage of late delivery orders is 5.6% and the service reliability is 84.4%. The service process has to be improved to reduce process average and variability or the promised lead-time has to be revised to a reasonable value the system will be able to provide. The current process lead time corresponding to the excellent service is $37.82 + 1.67 \times 21.91 = 74.4$ days.

The table below presents the relationship between service capability index $C_{pk}$, service reliability $R$, and the proportion of late orders $LD$ (late orders per thousand orders):

<table>
<thead>
<tr>
<th>$C_{pk}$</th>
<th>R</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.333</td>
<td>0.631</td>
<td>369</td>
</tr>
<tr>
<td>0.500</td>
<td>0.691</td>
<td>309</td>
</tr>
<tr>
<td>0.667</td>
<td>0.748</td>
<td>252</td>
</tr>
<tr>
<td>0.833</td>
<td>0.798</td>
<td>202</td>
</tr>
<tr>
<td>1.000</td>
<td>0.841</td>
<td>159</td>
</tr>
<tr>
<td>1.167</td>
<td>0.878</td>
<td>122</td>
</tr>
<tr>
<td>1.333</td>
<td>0.908</td>
<td>91</td>
</tr>
<tr>
<td>1.500</td>
<td>0.933</td>
<td>67</td>
</tr>
<tr>
<td>1.667</td>
<td>0.952</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 4: Relationship between process capability and System Reliability

5. Conclusion

This research paper presents an approach to estimate the customer order lead-time in service and manufacturing applications. The method used the process capability index to examine the ability of the current process to deliver the products before the promised due date and develop a lead-time estimator to predict the lead time if the process is found to be incapable. Mathematical models for the cases when the process output is normal and the one when it is not normal. The proposed model is valid when the process is in-control state. A Case study is presented to evaluate the capability index for the delivery processes in a multinational company in Egypt. The case presented estimates for the capability indices when delivering products belonging to $G_1, G_2$, and $G_3$ classes. Computation results reveal that the indices are 0.22, -0.257, and 1.01 respectively. Therefore, the process is incapable of delivering the $G_1$ and $G_2$ products in their promised due-date. The process, however,
does better job when delivering products belonging to class $G_3$. The proposed estimation methodology was employed to revise the lead-time for the incapable cases. Relationships between process capability, process reliability, and proportion of late order delivery are tabulated for service and manufacturing applications.

References