Study of Influence of Geometrical Structure Parameters of INGAN Single Quantum Well Laser on Time Response and Power-Current Characteristics by Solving Rate Equations

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Abstract—A new visual and analytical model has been presented to study effects of structural parameters on laser performance of InGaN single quantum well (SQW) lasers based on solving rate equations by using user friendly programming software, Delphi. This model has provided a graphical user interface (GUI) for researchers by which one can work directly with all laser parameters throughout the simulation work and analysis without the need to access source code [1]. We have studied effects of different parameters such as quantum well thickness, separate confinement heterostructure (SCH) thickness and cavity length on the fundamental characteristics of the laser which include laser time response (P-t), and output power-current (P-I) characteristics. Meanwhile related features such as turn-on delay time of lasing, threshold current and slope efficiency have been investigated.

Keywords—quantum well laser, rate equations, InGaN/GaN, Delphi, 4th Runge-kutta method.

1. Introduction

III-nitride semiconductor laser diodes have found much attention in the last few years due to their electrical and optical properties such as broad band gap energies and wide coverage of electrostatics spectrum from infrared (IR) to visible and ultraviolet (UV). They have spacious range of applications in various fields such as data storages, full color displays, illumination and medical applications [1,2].

There are several equation sets such as Poisson equation, Schrödinger equation, current continuity equations, scalar wave equation and photon rate equation that are used to characterize performance parameters of quantum well (QW) lasers. These equation sets are coupled in the laser simulation and are solved consequently by getting feedback from each other. One simple and important equations set which is used to study laser characteristics in almost all powerful and expensive simulation tools such as Crosslight, ISE TCAD and SILVACO is rate equations. A numerical analysis of rate equations is a powerful tool to study of carriers and photons behavior in semiconductor lasers. The differential rate equations are used to estimate simulated and spontaneous emissions of laser in almost all semiconductor simulation softwares [1, 3-9].

In our last work, a primary model, an analytical, visual and open source model based on solving rate equations for InGaN SQW lasers by using Delphi programming software was presented. The most prominent feature of the model was being an open source model. Since Delphi programming software is an object oriented programming software, researchers could work directly with the laser parameters throughout the simulation. It is a feature that could not be seen in the most of mentioned expensive softwares [1]. In this work, the last model has been developed by adding the solutions of the Schrödinger equation to the model. The solutions of Schrödinger equation provide the possibility of exact analysis of structure parameters such as well thickness, well mole fraction, barrier or SCH thickness and mole fractions. In this effort, by considering the improved model by adding the solution of Schrödinger equation, effects of different parameters such as quantum well thickness, separate confinement heterostructure (SCH) thickness and cavity length on the principle performance characteristics of the SQW laser which include laser time response (P-t), and output power-current (P-I) characteristics have been studied. Meanwhile related features such as turn-on delay time of lasing, threshold current and slope efficiency have been investigated.

2. Theoretical Model and Parameters used in the Modeling

The schematic diagram of the SQW laser and the
carrier transport processes which is considered in our model are shown in Fig.1. Carrier transport processes include two independent carrier rate equations for the SCH or barrier layers, $N_{\text{sch}}$ for the region to the left and $N_{\text{sch}2}$ for the region to the right of the well, and one equation for $N_1$ that represents the distribution of carriers in the well [1, 3, 8].

Figure 1. Schematic view of carrier transport processes in a SQW laser structure.

By assuming the current injection from the left cladding to the SCH, the three rate equations for the carriers in the left SCH, quantum well and right SCH, respectively are [1]:

$$
\frac{dN_{\text{sch}}}{dt} = \eta I \frac{N_{\text{sch}}}{\tau_s} + \frac{N_{\text{sch}}}{\tau_{\text{sch}}(N_{\text{sch}})} + \frac{N_{\text{well}}}{\tau_c} + \frac{N_{\text{sch}2}}{\tau_s} \xi \tau_c \tau_s \tau_\text{sc} \tau_\text{sch} (N_{\text{sch}}) \eta
$$

(1)

$$
\frac{dN}{dt} = (1 - \frac{N_{\text{sch}}}{\tau_c} \frac{N_{\text{sch}}}{\tau_{\text{sch}}(N_{\text{sch}})} + (1 - \frac{N_{\text{sch}2}}{\tau_c} \frac{N_{\text{sch}2}}{\tau_{\text{sch}2}(N_{\text{sch}2})}) - \tau_\text{sc} \tau_\text{sch} (N_{\text{sch}2}) - \tau_\text{sch} (N_{\text{sch}}) \eta)
$$

(2)

$$
\frac{dN_{\text{sch}2}}{dt} = \frac{N_{\text{sch}2}}{\tau_s} + \frac{N_{\text{sch}2}}{\tau_{\text{sch}2}(N_{\text{sch}2})} + \frac{N_{\text{well}}}{\tau_c} + \frac{N_{\text{sch}1}}{\tau_s} \xi \tau_c \tau_s \tau_\text{sch} \tau_{\text{sch}2} (N_{\text{sch}2}) \eta
$$

(3)

The above mentioned three rate equations are coupled with a photon rate equation:

$$
\frac{dS}{dt} = \Gamma_{\text{sc}} \nu_s g(N_1)(1 - \alpha \gamma) S - \frac{S}{\tau_p} + \frac{N_1}{\alpha \gamma(N_1)} \eta
$$

(4)

Where $\eta$ is the internal quantum efficiency, $I$ is the injected current, $q$ is the electronic charge, $V_{\text{sch}}$ is the volume of a SCH layer, $V_{\text{well}}$ is the volume of well, $\tau_s$ is the carrier transport time in the SCH region, $\tau_{\text{sch}}$ is the carrier recombination lifetime in the SCH regions, $\tau_{\text{well}}$ is the carrier thermionic emission/escape time from the well to SCH layers, $\xi$ is the leakage factor from each SCH to another SCH, $\tau_p$ is the carrier recombination lifetime in the quantum well, $\gamma$ is the optical confinement factor of the SQW, $v_s$ is the group velocity, $g(N)$ is the carrier-density dependent gain, $S$ is the photon density that has been normalized with respect to the volume of the well, $\alpha$ is the gain compression factor, $\tau_p$ is the photon lifetime and $\beta$ is the spontaneous emission factor. The carrier recombination lifetimes, $\tau_{\text{sch}}$ and $\tau_{\text{well}}$ are dependent on carrier density which are given by $\eta = (A + BN + CN^2)^{-1}$ where $A$, $B$, and $C$ are the monomolecular, bimolecular and Auger recombination coefficients, respectively. There are various descriptions for the carrier-density dependent gain $g(N)$. Here $g(N)$ is taken as a logarithmic equation:

$$
g(N) = G_0 \ln \left( \frac{AN + BN^2 + CN^3}{AN_0 + BN_0^2 + CN_0^3} \right)
$$

(5)

Where $N$ is the carrier density in the well, $G_0$ is the gain coefficient, and $N_0$ is the carrier density at transparency.

The Schrödinger equation is another important equation that has a significant role in calculation of the parameters such as gain, effective barrier (SCH) height, energy levels of electron and hole in the well and consequently output wavelength which are directly or indirectly used in the rate equations [7, 9, 10]. Figure 2 shows the quantum well structure used in the simulation.

Figure 2: QW band energy Diagrams with considering first energy level of electron and hole

For a quantum well structure shown in the Fig.2, the Schrödinger equation is written as:
\[
\left[ -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V(z) \right] \phi(z) = E \phi(z)
\]

(6)

\[
V(z) = \begin{cases} 
V_0 & |z| = L_{QW} / 2 \\
0 & |z| < L_{QW} / 2 
\end{cases}
\]

(7)

Where \( \phi(z) \) is wave function of each particle (electron, light hole and heavy hole) in the well, \( E \) is energy level of particle, \( L_{QW} \) is well thickness and \( V_0 \) is barrier height that is corresponding to \( \Delta E_c \) for conduction band and \( \Delta E_v \) for valence band. In addition \( m \) is particle mass that equals \( m_e \) in the well and \( m_b \) in the barrier regions.

By assuming even solutions as the results, the wave functions are as following:

\[
\phi(z) = \begin{cases} 
C_1 e^{-\alpha |z| L_{QW} / 2} & |z| = L_{QW} / 2 \\
C_2 \cos k z & |z| < L_{QW} / 2 
\end{cases}
\]

(8)

Where

\[
k = \sqrt{2m_e E / \hbar} 
\]

(9)

\[
\alpha = \sqrt{2m_b (V_0 - E) / \hbar} 
\]

(10)

The energy levels of each particles in the well (electron, light hole and heavy hole) can be found by substituting equation 11 in the equation 9 and 10.

\[
E_{e-hh} = \frac{\hbar^2}{2m_e} \left( \frac{\pi}{L_{QW}} \right)^2, \quad E_{e-h} = \frac{\hbar^2}{2m_e} \left( \frac{\pi}{L_{QW}} \right)^2, \quad E_{h-h} = \frac{\hbar^2}{2m_b} \left( \frac{\pi}{L_{QW}} \right)^2
\]

(12)

So the first transition energies are:

\[
E_{e-hh} = E_g (QW) + E_e + E_{h-h} 
\]

(13)

\[
E_{e-h} = E_g (QW) + E_e + E_{h-h} 
\]

(14)

All of the parameters which are used in the modeling are based on our last work [1] (table 1 and 2). Parameters related to the well and SCHs materials are based on the interpolation rule of characteristics of the binary nitride material, GaN, InN and AlN as below [1]:

\[
P(Ga \rightarrow In \rightarrow Al) = P_{InN} + yP_{GaN} + xP_{AlN} + xyP_{InGaN} + yxP_{AlGaN} + xyP_{InAlGaN}
\]

(15)

where \( P_{InN}, P_{GaN}, \) and \( P_{AlN} \) are binary parameters of InN, AlN and GaN respectively (table 1). The \( b_{InN}, b_{GaN}, \) and \( b_{AlN} \) are ternary bowing parameters which are -3.4, -1.43 and -0.7 respectively, and \( b_{InAlN} = 0 \) is the quaternary bowing parameter [1].

### Table 1. Room temperature properties of binary III-N materials [1]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Ga</th>
<th>In</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandgap energy ( E_g ) (eV)</td>
<td>3.42</td>
<td>0.77</td>
<td>6.28</td>
</tr>
<tr>
<td>Electron mobility (cm²/Vs)</td>
<td>180</td>
<td>32</td>
<td>30</td>
</tr>
<tr>
<td>Electron diffusion constant ( \left( \frac{cm^2}{s} \right) )</td>
<td>39</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>Refractive index near ( E_g )</td>
<td>2.50</td>
<td>2.</td>
<td>2</td>
</tr>
<tr>
<td>Electron effective mass ( m_e )</td>
<td>0.22</td>
<td>0.11</td>
<td>0.4</td>
</tr>
<tr>
<td>Electron affinity (eV)</td>
<td>4.1</td>
<td>5.</td>
<td>1.</td>
</tr>
</tbody>
</table>

The rate equations (eq.1-5) have been solved to get the stationary conditions by a well-known numerical technique based on fourth-order Runge–Kutta (RK4) method. This is one of the best methods to solve ordinary differential equations (ODEs) based on finite difference method. In numerical analysis, the Runge–Kutta methods are an important family of implicit and explicit iterative methods for approximating of solutions of ODEs.
Table 2. All computed parameters which used in the simulation for the InGaN SQW laser with wavelength 419.6 nm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>In fraction in well</td>
<td>0.13</td>
</tr>
<tr>
<td>Energy band gap of well (eV)</td>
<td>2.95</td>
</tr>
<tr>
<td>Thickness of the well (nm)</td>
<td>8</td>
</tr>
<tr>
<td>Width of the well or laser (nm)</td>
<td>300</td>
</tr>
<tr>
<td>Volume of well (nm³)</td>
<td>60e8</td>
</tr>
<tr>
<td>Electron diffusion constant of well (cm²/s)</td>
<td>43.9</td>
</tr>
<tr>
<td>Carrier escape time (ps)</td>
<td>227.34</td>
</tr>
<tr>
<td>Monomolecular Recombination Coefficient of well (Aₔ) (s⁻¹)</td>
<td>0.54e8</td>
</tr>
<tr>
<td>Bimolecular Recombination Coefficient of well (Bₔ) (cm³s⁻¹)</td>
<td>2e-11</td>
</tr>
<tr>
<td>Auger Recombination Coefficient of well (Cₔ) (cm³s⁻¹)</td>
<td>2e-30</td>
</tr>
<tr>
<td>Output wavelength (nm)</td>
<td>419.89</td>
</tr>
<tr>
<td>Refractive Index</td>
<td>2.557</td>
</tr>
<tr>
<td>Internal loss (1/cm)</td>
<td>12.1</td>
</tr>
<tr>
<td>Transparency Carrier Density (cm⁻³)</td>
<td>1.4e19</td>
</tr>
<tr>
<td>Material Gain Coefficient (cm⁻¹)</td>
<td>2000</td>
</tr>
<tr>
<td>Gain Compression Factor (cm⁻¹)</td>
<td>1.9e-17</td>
</tr>
</tbody>
</table>

3. Results and discussion

Figure 3 shows the graphical user interface (GUI) of the main program, which is used to input parameters such as QW and SCH thickness and etc. Simulation results of P-I characteristics of InGaN SQW laser with our software has shown in figure 4. Furthermore figure 5 shows the P-t characteristics of the simulated InGaN SQW laser.
Figure 4: P-I characteristics of InGaN single quantum well (well thickness= 8nm, and emission wavelength= 419.89nm)

Figure 5: P-t characteristics of InGaN single quantum well laser (I= 30mA)

Figure 6 demonstrates P-I characteristics curves of InGaN SQW lasers with different quantum well thicknesses (3.1, 4, 6 and 8 nm). As shown in this figure, the threshold current decreases by decreasing the well thickness. It also shows that the output power increases by decreasing the well thickness. Decreasing well thickness also causes blue shifting the output emission wavelength. Regarding to the equation 12, a thinner well has higher energy levels for electrons and holes in the conduction and valence bands and consequently higher transition energy between electron in the conduction band and hole in the valence band. The higher transition energy results the lower output emission wavelength. Figure 7 illustrates the output emission wavelength of laser as a function of well thickness for different In mole fraction in the well. As shown in this figure, two most important parameters which cause to change output emission wavelength are In mole fraction and well thickness. Broken line in the Fig. 7 shows the emission wavelength at 419.89nm. It shows that for a specific wavelength, by increasing well thickness, In mole fraction in the well has to increase.
Figure 6: P-I characteristics for different well thickness (Lqw = 3.1, 4, 6, 8 nm (average power is considered))

Figure 7: wavelength vs. well thickness for different In fractions in the well, broken line is wavelength 419.89 nm

Figure 8 exhibits the threshold current and slope efficiency of InGaN SQW laser as a function of well thickness. As shown in this figure by decreasing well thickness threshold current decreases and slope efficiency increases. This can be related to that when the well thickness decreases the rate of electrons capture time to its escape time in the well decreases, therefore radiative recombination increases [1].

Figure 8: Threshold current and slope efficiency for different well thickness

Figure 9: Turn on Delay in different currents for different well thicknesses Lqw = 3.10, 4, 6, 8 nm

Figure 9 shows the turn on delay of the SQW laser in different currents for different well thicknesses from 3.1 nm to 8 nm. As shown in this figure, by increasing well thickness, turn on delay of the SQW laser increases. It is related to the rate of electrons capture time to its escape time in the well and also radiative recombination in the well [1].

Another structural parameter that affect laser efficiency is the In mole fraction of the barrier or barrier height. Parameters such as carrier thermionic
emission/escape time from the well, transport time in the SCHs, the energy levels in the well, output emission wavelength and others are related to the In mole fraction of barrier. Based on the Schrödinger equation, well thickness is related to barrier height and consequently In mole fraction in the barrier. It means that the minimum well thickness in which electrons are in the 2D well region is depending on In mole fraction of barriers. Carrier thermionic emission time from the well and carrier transport time in the SCHs as a function of In mole fraction in the SCHs are shown in the Fig. 10.

**Figure 10:** Carrier thermionic emission time from the well and carrier transport time in the SCHs vs. In mole fraction in the SCHs

Figure 11 displays the P-t characteristic of the InGaN SQW laser for different In mole fraction of SCHs. As shown in this figure by increasing of In mole fraction, turn on delay of the laser increases and output power slightly decreases.

**Figure 11:** The P-t characteristic of the InGaN SQW laser for different In mole fraction of SCHs.

Carrier transport time in the SCHs is one of the most important parameter in the rate equations that is directly related to the SCH thickness and also material parameters of the SCHs through ambipolar diffusion coefficient and electron and hole mobility of SCH material [1]. Figure 12 shows the P-I characteristics of the InGaN SQW laser for different SCH thickness. As shown in this figure, by decreasing the SCH thickness, threshold current decreases and output power increases. Regarding the carrier transport time equation, by decreasing the SCH thickness, carrier transport time in the SCHs decreases [1], it causes to decrease turn on delay of the laser system and also increase current flowed and electron density in the well. Therefore threshold current decreases and radiative recombination increases. Figure 13 shows the turn on delay and the output power of InGaN SQW laser as function of SCH thickness, respectively [1].
Figure 12: Output power-current for different SCH width-8nm-419.9nm

Figure 13: Turn on delay and output power for different SCHs at 419.9nm emission wavelength and different injection current

As shown in these figures, longer cavity length lasers have higher threshold current and turn on delay and lower output power and slope efficiency. It can be explained in a manner that the laser with higher cavity length has lower electrical resistance. So the laser needs higher injection current to start lasing. Since the rate equations normalized with respect to the system volume, by increasing the cavity length, current density and electron density are decreased; therefore the radiative recombination, output power and consequently slope efficiency are decreased.

Figure 14: Turn on delay and output power for different cavity length at 419.89 nm emission wavelength and 40 and 50 mA injection current

Figure 15: Threshold current and slope efficiency for different cavity length at 419.89 nm emission wavelength.

4. Conclusions

The presented work demonstrates a new visual, open source and analytical model based on solving
rate equations by user friendly Delphi programming software. Our model in comparison with other analytical software and expensive simulators provides directly access to the parameters of lasers throughout the simulation and analysis. In this research, effects of structural parameters on laser performance of InGaN single quantum well (SQW) lasers based on solving rate equations have been studied. We have investigated different effects of structure parameters such as quantum well thickness, separate confinement heterostructure (SCH) thickness and cavity length on the principle characteristics of the laser which include laser time response (P-t), and output power-current (P-I) characteristics. Meanwhile related features such as turn-on delay time of lasing, threshold current and slope efficiency have been studied. The results indicated that the laser with thinner well thickness has higher threshold current and turn on delay and lower output power and slope efficiency. They also illustrated that increasing In mole fraction and thickness of SCH layer cause to increase threshold current and turn on delay and decrease output power of the InGaN SQW laser. The results also indicated that the laser with longer cavity length has higher threshold current and turn on delay and lower output power and slope efficiency. The results also showed that our model has good agreement with experimental works done by Bai et al [11].

Acknowledgments

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References


