Performance Analysis of SSK and Trellis Coded Spatial Modulation in MIMO

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Abstract - Multiple antennas in wireless system offer a practical way to extend next generation communication capabilities. Spatial modulation (SM) in which multiple antennas are used to convey information besides the conventional M-array signal constellations, In a new MIMO(Multiple Input Multiple Output) transmission technique, which has recently been proposed as an alternative to V-BLAST(Vertical Bell Labs Layered Space Time). In this scheme, a trellis encoder and an SM mapper are jointly designed to take advantage of the benefits of both spatial modulation with trellis coding (SM-TC) and space shift keying (SSK). A pair wise error probability (PEP) upper bound is determined for the SM-TC scheme in uncorrelated quasi-static Rayleigh fading channels and it has been compared with Trellis code spatial modulation. Also we illustrate SSK's strength by studying its interaction with the fading channel. We obtain tight upper bounds on bit error probability performance gains over APM systems (3 dB at a bit error rate of 10^-5), making SSK an interesting candidate for future wireless applications. The SM-TC schemes achieve significantly better error performance than the classical space-time trellis codes and coded V-BLAST systems at the same spectral efficiency, yet with reduced decoding complexity.

Index Terms — Multiple antenna array, Spatial Modulation, Trellis coding.

1. Introduction

Multiple-input multiple output (MIMO) transmission is an emerging technologies, which offer significant improvements in channel capacity and reliability compared to single antenna transmission systems [1]. Their unprecedented improvements over single antenna systems have spawned a wealth of research in multiple-input Multiple-output (MIMO) communications, which fall under three general themes.

The first is spatial multiplexing: Spatial multiplexing requires synchronizing all antennas to transmit at the same time, and introduces interference from all antennas during reception, making for complex detection schemes. Secondly, diversity transmission diversity systems exploit the spatial domain as a coding mechanism to increase reliability (i.e., diversity). These types of systems also require synchronizing all antennas to transmit at the same time. Finally, the third category is hybrid transmission: both spatial multiplexing and diversity concepts are integrated.

Some common pitfalls amongst MIMO systems are:

- Inter-channel interference (ICI): Introduced by coupling multiple symbols in time and space.
- Inter-antenna synchronization (IAS): In the BLAST and OSTBC architectures, the detection algorithms assume that all symbols are transmitted at the same time. Hence, IAS is necessary to avoid performance degradation [2], consequently increasing transmitter overhead.
- Radio frequency (RF) chains: Although multiple antenna elements are relatively inexpensive to deploy, and the digital signal processing requirements are feasible due to increased industry growth, the necessary RF elements are not as simple to implement [3]. These RF chains are bulky, expensive, and necessary for each antenna that is used. SM provides some advantages compared to classical MIMO transmission systems in which all antennas transmit simultaneously. Since only one transmitter antenna is active during each symbol transmission, ICI is completely eliminated in SM and this results in much lower (linear) decoding complexity. In recent studies, inspired by SM, a space-shift keying (SSK) modulation scheme in which only antenna indices are used to transmit information has been proposed [4]. In the TCM technique is used in conjunction with SM to partition the transmit antennas into subsets by maximizing the spacing between antennas of the same subset and only the information bits that
determine the transmit antenna number are convolutionally encoded.

2. SSK Modulation Technique

SSK is proposed, in which distinct multipath characteristics from different antennas are used to discriminate between transmitted symbols. The receiver determines which mode of transmission is used (either one antenna or both antennas are activated) in order to detect the message. In SSK, antenna indices are used as the only means to relay information the SSK modulation technique, in which the spatial domain is solely exploited to convey information.

The general system model consists of a MIMO wireless link with \( N_t \) transmit and \( N_r \) receive antennas, which is illustrated in Fig. 1. A random sequence of independent bits \( b = [b_1, b_2, \cdots, b_k] \) enter a channel encoder with output \( c = [c_1, c_2, \cdots, c_n] \), where \( k \) and \( n \) represent the number of encoder inputs and outputs, respectively. The pseudo randomly interleaved sequence \( c \pi \) then enters an SSK mapper, where groups of \( m = \log_2 (N_t) \) bits are mapped to a constellation vector \( x = [x_1, x_2, \cdots, x_{N_t}] \), with a power constraint of unity (i.e. \( E[x'Hx] = 1 \)). In SSK, one antenna remains active during transmission and hence (ideally), only one RF chain is required. However, due to pulse shaping, the transmitted pulse will extend a few symbol periods, and restrict the RF chain from being switched to another antenna. In the UWB framework however, the cost of RF chains is fixed regardless of the number of antennas since pulse shaping is not required. The discussion in this paper may be extended to UWB indoor communications using the appropriate statistical channel models given in [5]. The modulated signal is then transmitted over an \( N_r \times N_t \) wireless channel \( H \), and experiences an \( N_r - \dim \) additive white Gaussian (AWGN) noise \( \eta = [\eta_1, \eta_2, \cdots, \eta N_r] \). The received signal is given by \( y = \sqrt{\rho} Hx + \eta \), where \( \rho \) is the average signal to noise ratio (SNR) at each receive antenna, and \( H \) and \( \eta \) have independent and identically distributed (iid) entries according to \( CN(0, 1) \).

At the receiver side, the SSK detector estimates the antenna index that is used during transmission, and demaps the symbol to its component bits \( \hat{w} \). As an enhancement, turbo CM (TuCM) [6]–[7] provides performance improvements over TCM by incorporating the turbo principle (turbo codes with iterative decoding).

3. Trellis Coded Space Modulation

The considered SM-TC system model is shown in Fig. 2. The independent and identically distributed (i.i.d.) binary information sequence \( u \) is encoded by a rate \( R = k/n \) trellis (convolutional) encoder whose output sequence \( v \) enters the SM mapper. The spatial modulator is designed in conjunction with the trellis encoder to transmit \( n \) coded bits in a transmission interval by means of the symbols selected from an \( M \)-level signal constellation such as \( M \)-ary phase-shift keying (M-PSK),\( M \)-ary quadrature amplitude
modulation (M-QAM), etc., and of the antenna selected from a set of \( n_T \) transmit antennas such that \( n = \log_2 (MnT) \).

The SM mapper first specifies the identity of the transmit antenna determined by the first \( \log_2 n_T \) bits of the coded sequence \( v \). It then maps the remaining \( \log_2 M \) bits of the coded sequence onto the signal constellation employed for transmission of the data symbols. Due to trellis coding, the overall spectral efficiency of the SM-TC would be \( M \) bits/s/Hz.

### 4. Performance

SSK’s performance is derived using the well known union bounding technique [8]. The average BER for SSK is union bounded as

\[
P_{e,\text{bit}}^{SSK} \leq \sum_{j=1}^{N_T} \sum_{j' \neq j} \frac{2N(j, j')}{N_T} P(\mathbf{x}_j \rightarrow \mathbf{x}_{j'})
\]

where \( N(j, \hat{j}) \) is the number of bits in error between \( \mathbf{x}_j \) and \( \mathbf{x}_{\hat{j}} \), \( P(\mathbf{x}_j \rightarrow \mathbf{x}_{\hat{j}}) \) denotes the pairwise error probability (PEP) of deciding on \( \mathbf{x}_j \) given that \( \mathbf{x}_j \) is transmitted, and where the index in the summation is simplified since \( N(j, \hat{j}) \) is symmetric. Using (1), we obtain the PEP conditioned on \( H \) as

\[
P(\mathbf{x}_j \rightarrow \mathbf{x}_{\hat{j}} \mid H) = P(\mathbf{d}_j > \mathbf{d}_j \mid H)
\]

and

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{t^2}{2}} dt
\]

We define \( \kappa \) as

\[
k \Delta = \frac{\rho}{2} \| \mathbf{h}_j - \hat{\mathbf{h}}_j \|^2 = \sum_{n=1}^{2N_r} \alpha_n^2
\]

where \( \alpha_n \sim N(0, \sigma_n^2) \) with \( \sigma_n^2 = \frac{\rho}{2} \)

SSK’s effective constellation space \( \mathbf{X}_{\text{eff}} \), and is given by

\[
P_{e,\text{bit}}^{NR-1} \leq \eta_{\text{neighbor}} N_{\text{avg}} \gamma_{\alpha}^{NR-1} \sum_{k=0}^{N_r-1} \left( N_r - 1 - k \right)
\]

Where \( N_{\text{avg}} = \frac{N_{\Sigma}}{N_T (N_T - 1)} \) represents the average number of neighbors.

The demodulator first computes the a posteriors \( \hat{\mathbf{h}}_j \) and \( \hat{\mathbf{h}}_{\hat{j}} \) using the well known union bounding technique [8]. The average BER for SSK is union bounded as

\[
P_{e,\text{bit}}^{SSK} \leq \sum_{j=1}^{N_T} \sum_{j' \neq j} \frac{2N(j, j')}{N_T} P(\mathbf{x}_j \rightarrow \mathbf{x}_{j'})
\]

where \( \hat{\mathbf{h}}_j = \text{Re}\{ (\mathbf{y} - \sqrt{\frac{\rho}{2}} \mathbf{h}_j)^H \mathbf{h}_j \} \),


![Figure 2 - SM TC](image-url)
5. PEP of ST TC Scheme

In this scheme one receive antenna is assumed; however, all results can be easily extended to any number of receive antennas. A pairwise error event of length \( N \) occurs when the Viterbi decoder decides in favor of the spatially modulated symbol sequence \( \hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N) \) when \( x=(x_1, x_2, \ldots, x_N) \) is transmitted, where \( x_n=(i_n, s_n), S_n \in \mathcal{X} \) is the transmitted symbol over the \( i_n \) th antenna \( (1 \leq i_n \leq n_T) \) at the \( n \) th transmission interval. Let the received signal is given by

\[
Y_n = \alpha_n S_n + w_n
\]

Let \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_N) \) and \( \beta = (\beta_1, \beta_2, \ldots, \beta_N) \) denote the sequences of fading coefficients corresponding to transmitted and erroneously detected SM symbol sequences, \( x \) and \( \hat{x} \), respectively. The CPEP for this case is given by

\[
\Pr(x \to \hat{x} | \alpha, \beta) = P_e(x \to \hat{x} | \alpha_n, S_n)
\]

Where \( m(y, x; \alpha) = \sum_{n=1}^{N} m(y_n, s_n, \alpha_n) = - \sum_{n=1}^{N} \left| y_n - \alpha_n s_n \right|^2 \) is the decision metric for \( x \), since \( y_n \) is Gaussian when conditioned on \( \alpha_n \) and \( S_n \), then the above equation can be expressed as

\[
\Pr(x \to \hat{x} | \alpha, \beta) = P_e(x \to \hat{x} | \alpha_n, S_n) \geq \sum_{n=1}^{N} \left| y_n - \alpha_n s_n \right|^2 \cdot x)\]

The CPEP of the SM-TC scheme is calculated from above equation as

\[
\Pr(x \to \hat{x} | \alpha, \beta) = Q\left( \frac{-m_d}{\sigma_d} \right) = Q\left( \frac{\sum_{n=1}^{N} A_n}{2N_0} \right)
\]

Where \( A_n = | \alpha_n s_n - \beta_n s_n |^2 \)

Using the bound \( Q(x) \leq e^{-\frac{x^2}{2}} \), the CPEP of SM-TC Scheme can be upper bounded by

\[
\Pr(x \to \hat{x} | \alpha, \beta) \leq \frac{1}{2} \exp\left( -\frac{N}{4} \sum_{n=1}^{N} | \alpha_n s_n - \beta_n s_n |^2 \right)
\]

Where \( \frac{E_s}{N_0} = \frac{1}{N_0} \) is the average signal to noise ratio. The UPEP of the SM-TC is obtained as follows

\[
p_e(x \to \hat{x}) \leq \frac{1}{2 \det\left( \frac{\gamma}{4} KS + I \right)}
\]

Note that, \( \text{rank}(KS) = \text{rank}(S) \). Since the rank of \( S \) is not changed upon multiplication by a non-singular matrix.

6. BIT Error Probability Performance

Due to dependence between the terms of CPEP the use of transfer function technique which consider error events with all lengths, we are restricted to an approximation of the average BEP, which considers the error events with lengths up to a finite value given as \( \text{[9]} \)

\[
P_b \approx \frac{1}{c} \sum_{\hat{x}} \left[ \frac{1}{K} \sum_{x \in \mathcal{X}} e(x, \hat{x}) P_e(x \to \hat{x}) \right]
\]

Where \( k \) is the number of input bits per trellis transition, \( e(x, \hat{x}) \) is the number of bit errors associated with each error event, and \( c \) is the total number of different realization of \( x \). Due to the design symmetry, we consider only the path pairs originating from the first state. If we consider \( N=2 \), there are 16 error events of type 3 and 6, and 8 error events of type 4 and 5, respectively. Each path-pair corresponding to the error events of Types 3, 4 and 5 contributes one bit error, while this value is equal to two for Type 6 from above equation we obtain

\[
P_b = \frac{1}{32} \left[ 16P_e(x \to \hat{x})_1 + 8P_e(x \to \hat{x})_6 + 8P_e(x \to \hat{x})_5 + 32P_e(x \to \hat{x})_6 \right]
\]

Code Error Probability

The binary input output symmetric (BIOS) property of BICM systems, the performance analysis is tractable. 6 We assume a convolution coded system, concatenated with an SSK modulator, but can be generalized to other concatenated schemes as well all codeword pairs must be considered in order to obtain an analytical expression for the error probability.
Most often, however, simple union bound on the BER is used. From [10], [11], the bit error probability for BICM under ML decoding is closely upper bounded by

$$P_{\text{e, bit}} \leq B(X)_{\text{upper}} = \sum_{d=0}^{i} B_{d} X^{d}$$

Where \( B(X) \)

\((Ai,d \) represents the number of codeword’s in \( C \)

$$\exp\left(\frac{d \tau(\hat{s})}{2\pi d \tau''(\hat{s})}\right)$$

PEP\(_{\text{BICM}}(d,\mu,\chi,\rho) \approx \frac{1}{2d \sqrt{\pi \rho d}} \left(1 + \frac{\rho}{N_{\tau}}\right) \frac{1}{\sqrt{2\pi \rho d}}$$

where \( \tau(\hat{s}) \) is the cumulate generating function of the random variable \( \Lambda \) defined in (2), \( \tau''(\hat{s}) \)

represents its second derivative, and with \( \hat{s} = \frac{1}{2} \) for BIOS channels. An error-free feedback assumption implies that each bit is transmitted using an equivalent BPSK type system. Consequently, we are able to directly use the results of [44], and obtain closed form PEP bounds on error-free feedback.

Noting that for SSK, \( \Lambda \) defined in (2), \( \Lambda \)

\(\Lambda \) represents the effective SNR, we obtain the saddle point approximation for PEP as

$$\text{PEP}_{\text{BICM-ID}}(d,\mu,\chi,\rho) = \frac{1}{N_{\tau}} \frac{1}{\sqrt{2\pi \rho d}}$$

The error-free feedback bounds for SSK are independent of the constellation size.

7. SSK Vs STTC, SMTC

The figure shows the bit error performance of space shift keying modulation with \( M=8 \), spatial modulation trellis code with \( r=0 \), space time trellis code with 8 stages spatial modulation trellis code is shown to perform both spatial modulation trellis code and space shift keying.

8. Conclusion

We have studied the performance of SSK and SM-TC for MIMO wireless links. We laid out SSK fundamentals as the building ground for hybrid modulation schemes. In this the bit error probability of the coded system, and showed performance gains over APM (for convolutional and turbo codes). All of SM’s merits are also inherent in SSK, but with lower computational overhead and with relaxed APM hardware requirements. These advantages make SSK a promising candidate for low complexity transceivers in next generation communication systems. IN SM-TC. Although one transmit antenna is active during transmission, for quasi-static fading channels, we benefit from the time diversity provided by the SM-TC mechanism. SM-TC codes which offer significant error performance improvements over alternatives while having a lower decoding complexity.
References