Momentum and Reversals: An Alternative Explanation by Non-Conserved Quantities

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Abstract—The momentum effect in stock trading means that stocks performing well in the past will do so in the future, too. A recent (seemingly) proof of it would be a big discovery: Stock prices would obey laws similar to the Newtonian equation of motion. However, using the recent result that stock prices are distinct from stock values, the whole mystery disappears without a trace. Stock prices fluctuate chaotically (in a mathematical sense). Therefore the momentum within stock prices is easily explained by a self-fulfilling prophecy as long as enough people believe in it. In the recent experimental "proof" of the momentum effect, stocks had been traded thousands of times. In generalizing the well-known average cost effect, we give a second quantitative explanation for the observed results.

Keywords—Momentum, chaos; intrinsic value; conserved quantity; average cost method.

1. Introduction

The efficient market hypothesis is one of the tenets of finance theory. In its strongest form, it postulates that past price movements should give no useful information about the future ones. Therefore investors should have no logical reason to prefer the winners of any period to the losers, because both should be priced fairly already (Dimson et al. (2008)). The efficient market hypothesis applies the classical theories of competition to finance by stating that competition among rational investors determines prices, so that they reflect the consensus estimate of fair value in the light of all available information. Despite the growing evidence on price distortions in forms of e.g. systematic mispricing, periodic price bubbles and collapses and levels of volatility vastly greater than the underlying dividend streams, the efficient market hypothesis has remained the dominant paradigm in finance (Vayanos and Woolley (2009)). However, the latest capital market booms and crashes, culminating in socially costly crisis like the one starting in summer 2007, have discredited the idea that markets are efficient. In consequence, the conception that prices reflect fair values has to be questioned (Vayanos and Woolley (2010)).

One of the conundrums in this area is the so called momentum effect (cf. Fama and French (1993), The Economist (2011)). In short is says that a stock having behaved well in the past will do so in the future, too. It has a "moment of inertia", just like a massive body. If that is true, just picking last year's best performing stocks should be a good advice. It is of course in contrast to the standard advice of choosing undervalued stocks, which most likely performed lousy recently.

A very thorough analysis of the momentum effect has been performed lately by Dimson et al. (2008). In many different stock markets, partly dating back over more than a hundred years, the authors simulated the following: Based on the stocks' last 12 month-performance, each month the stock market was separated into three classes

1. **Winners**, i.e. the 20% of best performing stocks.
2. **In-betweens**, i.e. the 60% of medium performing stocks.
3. **Losers**, i.e. the 20% of worst performing stocks.

From each of these classes, only the best performers (of the last 12 month) were bought. After a holding period of one month, the three stocks were re-sold and three new ones were bought, choosing again the best performers from the three classes and so forth. Doing that (in simulation) for many years (sometimes over a hundred years), luck or coincidence could be excluded. The results were impressive, the returns of
the three classes in almost all cases showed the same pattern: Excellent performance of the winners, mediocre performance of the in-betweens and lousy performance of the rest.

Nonetheless, Dimson et al. (2008) did not discover a recipe for becoming rich. In reality, prices will adjust due to the buying and selling of many people. But, at first glance, the outcome is puzzling from some other point of view: It seems to prove that there is something like a "moment of inertia" in value. One may even find the optimum observation period. Say observing for 10 months only and buying and selling every 25 days. From this, one may get something like a fictive mass. And one may create something like a Newtonian equation of value, similar to the real Newtonian equation for the position of a mass point.

The whole thing turns into the conundrum mentioned above, if one realizes that a stock is a piece of a company. A real company consists of a very complicated network of buying, producing and selling. In the end, it (hopefully) delivers worth, i.e. *added value*. Management science tries to map this complicated arrangements into even more complicated equations. The success however is pretty limited due to complexity. And all this can be condensed in the above mentioned Newtonian-like equation? Indeed that looks like standing in the eye of discovering something as fundamental as quantum mechanics. This justifies people's enthusiasm when seeing hints for a momentum effect.

In actuality, there is a fundamentally wrong assumption in the line of argumentation above. It is supposed that the stock price has a good correlation (at least in the long run) with the performance of the underlying company. In a recent paper, Appel and Grabinski (2011) however showed clearly that there is no such correlation: There is *intrinsic value*, which is a conserved quantity. It is essentially given by the cash the company will generate in the future. In contrast, there is *market value*. It is not conserved and it may fluctuate chaotically in the mathematical sense. (A more detailed summary of the author's findings is given in chapter 2. It explains the relatively new tenet of conserved and non-conserved quantities in management sciences). All of momentum's mystery vanishes without a trace, if (future) investment decisions are based on non-conserved (historic) market prices, given that the latter can fluctuate chaotically under certain circumstances. It becomes obvious that the momentum effect is easily explained as a big self-fulfilling prophecy. For centuries people bet on the lately winning horse. This is especially true for the stock market. There are even so called finance advisers advertising such strategies. (More details will be stated in chapter 3).

Having taken away the mystery of momentum effects, there is even another (statistical) explanation of the experiment of buying and selling stocks based on their last performance: Each time a stock is bought, it is not bought in a fixed number. Rather a fixed amount of money buys as many stocks as possible. At each transaction, the investor gains due to the average cost effect. This extra gain is proportional to the square of the fluctuation in price. The fluctuations of good performing (interesting) stocks tend to be much higher than the fluctuations of low performing (boring) ones. At least partly the results of Dimson et al. (2008) are explained by this special version of the well-known average cost method. Of course it delivers real extra money, which however is (usually) consumed by trading fees. (The average cost effect is covered in more detail in chapter 4).

2. **Conserved values versus chaotically fluctuating market prices**

The essence of Gutenberg's systemic approach (1998) is that a business situation can be described by a function of certain variables. The systemic approach was borrowed from the natural sciences. It has three ingredients:

1. The *existence of a function is hypothesized*, which potentially reflects the outcome of a system (= business).

2. *Proper variables* are to be identified.

3. Given the fulfillment of these two steps, one may try to *find the function and discuss its behavior*. This third step is the main subject of management science. Arguably it is its very definition.

While the first step can just be assumed, the second one - finding proper variables - requires further investigation: In management sciences, up to our knowledge, Appel and Grabinski (2011) initially addressed this issue. They showed *conserved quantities* being the only proper variables for describing the system performance, no matter whether or not the system's characteristic is natural
scientific or managerial. Though, from a pure mathematical point of view, the behavior of non-conserved quantities is completely deterministic, they may change unpredictably. This effect is called "chaos" (cf. Schuster (1984)). It is the reason why non-conserved quantities are improper for describing anything (cf. Grabinski (2007)). Non-conserved quantities namely tend to step-ups, i.e. marginal changes at the outset are amplified throughout the system and thereby may lead to drastic deviations towards the expected outcome (cf. "butterfly wing effect").

Researching chaos in management or economics is relatively new (cf. Ferreira et al. (2010), Filipe et al. (2010) and Grabinski (2007, 2008)). Yet the market or exchange value has been proven to reflect the archetype of a non-conserved economic quantity (cf. Grabinski (2007), Appel and Grabinski (2011)). Therefore building a business on observing and predicting (trends of) non-conserved market values is as ludicrous as accepting the calculation of next week's lottery numbers as a business (cf. Grabinski (2007 and 2008)).

Proper variables have been found indeed if both requirements are fulfilled. Being conserved, they will not change without notice; macroenvironmental catalysts affecting the requirement like political, economic, socio-logical, technological, legal, or environmental conditions have to change before (cf. Appel and Grabinski (2011), Hax and Majluf (1984)). Hence, by applying conserved quantities, the description of a system's (= business') future state can be accomplished in line with Guttenberg's approach (1998).

The discrimination of conserved and non-conserved quantities was tested by analyzing the cash generation of several listed companies to calculate their historic intrinsic firm value. It was compared with their historic share price development. The share of the SAP AG (worldwide number 4 software company) showed a typical pattern. It is a good example, because: SAP has the advantage of being big enough to attract speculators. Changes in value are not distorted by big machines or other non-operational reasons. In actuality, SAP's value is essentially given by its future cash flow determined by "real" customer requirements for the software's utilities. This is because in reality, nearly nobody buys a SAP system in course of speculation. That is the reason why SAP's intrinsic value - as defined and calculated by Appel and Grabinski (2011) - did not change very much, though the rest of the world lived through much turbulence.

During the period under consideration, SAP's (intrinsic) firm value never showed such extreme turning points as the market capitalization. In between, the share prices often followed considerable up- and downward trends being long enough to be exploited. Such trends lifted the market capitalization above the intrinsic firm value by multiples ranging from 1.9x to 7.2x. In other words, the conserved part of the daily market price on average amounted to just 24.5% and ranged from 7.2% to 68.4%. It seems appropriate to conclude that SAP's operations could not match the speculators' expectations! (Hence there seems to be no such thing as market values but only market prices). The Compound Annual Growth Rate.
("CAGR") of the firm value was 10.2% per annum ("p.a."), the one of the market capitalization just 3.4% p.a. Since any investor has to pay (most likely) overvalued market prices, comparing the underlying intrinsic value of a stock is inevitable in order to detect actually cheap shares instead of being fooled by the noise in the market (cf. Appel and Grabinski (2011), The Economist (2011)).

Pure momentum strategies involve sorting stocks into winners and losers, based on past returns over a ranking period. Then winners are bought and losers are sold over a holding period. In well-functioning markets, it should be impossible to rip off profits simply from smart timing of buying and selling assets dependent on their past performance. Yet the most comprehensive momentum study provides extensive evidence that momentum profits were large and pervasive across time and markets. Covering over 108 years of the top 100 stocks, which at today's measure amount to about 85% of the world's equity market capitalization, Dimson et al. (2008) verified that the return of the winners beats to one of the losers by about 10%-points p.a.: Starting 1900 by investing £1 in the winners, more than £4¼ million (14.1% p.a.) could have been gained. Investing £1 in the losers would have grown to £111 (4.36 % p.a.) only. The medium 60% show a 9.01% p.a. So the spread between medium to upper 20% is just around four percentage points.

3. **Trading non-conserved quantities removes the mystery from the momentum effect**

Momentum is the commonly observed propensity for trending in market prices. In the most extreme form, it leads to bubbles and - at times of major reversal - crashes. It has been described as the "premier unexplained anomaly" in asset pricing (Fama and French (1993)). The reason is that, according to theory, the past performance of share prices is no guide to the future; the practice however proves otherwise (The Economist (2011)).

Figure 2: Market capitalization (outstanding shares) vs. intrinsic firm value (10 year rolling forecast), applying the example of SAP
Figure 3: Value-weighted momentum portfolio returns for the Top 100 UK equities, annually from 1900 to 2007 (cf. Dimson et al. (2008))

Figure 4: Return on winners minus losers for Top 100 UK equities, annually from 1900 to 2007 (cf. Dimson et al. (2008))

Figure 5: Extremes of equity market history from 1900 to 2007 (Dimson et al. (2008))
Two material limitations however attend trading on momentum:

1. **Transactions costs can seriously dent performance**, because with rebalancing, the turnover can be very high. For example, a 12/1/1 strategy ranks returns over the past 12 months, waits 1 month and then holds for 1 month until rebalancing. For that strategy, winner and loser turnover averaged 31% and 33% per month. (The opposing impact of frequent rebalancing, which benefits momentum returns, is discussed in chapter 4).

2. **Winners underperformed losers in numerous periods**, sometimes by a dramatic margin (cf. Dimson et al. (2008), The Economist (2011)).

Momentum works off the proven premise that stocks having just risen in price are likely to keep on doing so, at least for an exploitable while. But this means, when performing value investing, i.e. picking stocks having low prices compared with intrinsic value of the underlying companies, a large part of the value portfolio will be at variance to fair value at any one time (cf. Bright (2009)). Not surprisingly, momentum strategies were not only reversed and falsified numerously, but also in each episode of turbulence, the losses experienced in the worst affected market were disastrous. Interestingly, the three great bear markets damaged the "value" - or rather the price - of the world equity portfolio far more than the world wars (cf. Dimson et al. (2008))! Given that the world wars for sure resulted in more severe breaks of the real (intrinsic) value creation of companies, it is completely unreasonable to assume that any bear market could result in more severe value destruction.

Considering chaos sheds some additional light on both the large performance gap between winners and losers and the "value" destruction in turbulent periods. Up to now, it should be clear that trends in market prices are nothing else than temporary fluctuations of non-conserved quantities. Hence they cannot be foreseen and may be irrational, like demonstrated by Figures 4 and 5. Building on that, our alternative explanation to momentum is:

1. Fundamentals - like the conserved operating cashflow of companies' businesses - add to intrinsic value. Dependent on the market's mood and expectations, they however not necessarily add the same amount to market prices. (Non-conserved) share prices therefore trade regularly above (conserved) intrinsic values.

2. Given expectations drove share prices far beyond intrinsic values, the prices have no fundamental fixture anymore. In such cases, market prices can change chaotically in either direction.

3. **The outperformance of the winner portfolio therefore can be mostly explained by the spreads between intrinsic values and share prices, because they regularly leave ample room for chaotic behavior.** And, because trends in prices may continue unreasonably, the rational advice to any tradesman to buy low and sell high becomes (temporarily) obsolete in the context of trading on momentum.

In a nutshell, in cases of momentum traders outperforming value investors, this is possible mostly because momentum bases on the potentially chaotic behavior of non-conserved market prices. Hence, ultimately, good luck!

### 4. Average Cost in trading

If somebody buys a certain amount of something at a regular basis, it will amount to N times that amount after N periods. Assuming an average price \( <p> \) per mentioned amount, one will have spent \( N \times <p> \) for it. In contrast one may spend exactly \( <p> \) each time. The total spending will also be \( N \times <p> \) each time. The total spending will also be \( N \times <p> \). However, because one bought more when the price was low and less when the price was high, the total amount will be bigger. Exactly this is called the average cost effect. It is a useful and well-known way if someone is investing regularly in a certain asset. Normally the effect is small, because each time one gains a certain percentage in the order of the squared fluctuation. A similar thing happens by the buying and selling simulated by Dimson et al. (2008). But its effect may be much bigger. First of all, over the very long period, buying and selling happens many times. Second, the fluctuations are big because the average is taken over a long period of time. In order to see how it works quantitatively, we will give a mathematical description of the statement above.

Let's assume to have two stocks \( i \) and \( j \). Their corresponding prices are:
Their (time dependent) fluctuation is denoted by $\Delta$. The exponential function in front is due to the compound interest rate $p$. (For simplicity, we assume the same average interest rate for both stocks. But this is no real limitation). Starting with say $p_i$ at $t = 0$ and investing one currency unit, one has at $t = \Delta t$:

$$\frac{p_{oi}e^{p\Delta t} + \Delta_i(\Delta t)}{p_{oi} + \Delta_i(0)} \prod_{n=1}^{N-1} \left( \frac{1 + \frac{\Delta_i((2n-1)\Delta t)}{p_{oi}e^{p(2n-1)\Delta t}}}{1 + \frac{\Delta_i((2n-1)\Delta t)}{p_{oj}e^{p(2n-1)\Delta t}}} \right)$$

Eq. (2) can be written as:

$$\frac{p_{oi}e^{p\Delta t} + \Delta_i(N\Delta t)}{p_{oi} + \Delta_i(0)} \prod_{n=1}^{N-1} \left( \frac{1 + \Delta_i((2n-1)\Delta t)}{1 + \frac{\Delta_i((2n-1)\Delta t)}{p_{oj}e^{p(2n-1)\Delta t}}} \right)$$

The factor in front (before the product $\Pi$) is the value by holding stock $i$ without exchanges for a time $t = N\Delta t$. The second factor (with the product $\Pi$) denotes the "gain" for the exchanging. Because the relative fluctuation $\Delta$ can be negative or positive, it is not clear whether this factor is bigger (gain) or smaller (loss) than 1. However, fluctuations as defined in (1) in connection with (3) are symmetric. Taking the additional (admittedly non-trivial) assumption that the fluctuations of stock $i$ and $j$ are uncorrelated, one can show that the second factor is always bigger than 1. In other words, there is a gain due to the average cost method. The simplest way to see how it works is to make a Taylor expansion in the $\Delta_i$'s in (4). Of course, all odd powers of $\Delta_i$ will vanish (on average). Then one will get in lowest order in $\Delta_i$ the following:

$$\frac{p_{oi}e^{p\Delta t} + \Delta_i(\Delta t)}{p_{oi} + \Delta_i(0)} \prod_{n=1}^{N-1} \left( 1 + \sum_{n=1}^{N-1} \left( \frac{\Delta_i^{2n-2}}{1 + \Delta_i^{2n-2}} \right) \right)$$

The next term will be fourth power. It is neglected here. In order to estimate the magnitude, one may define an average quadratic fluctuation by:

$$\Delta_r^2 \equiv \frac{1}{N-1} \sum_{n=1}^{N-1} (\Delta_i^{2n-1})^2 + (\Delta_j^{2n})^2$$

The gain due to the exchange can be expressed by an extra interest $a$ (in addition to $p$). Using the definition (6) in (5), the additional interest $a$ can be derived from equating:

$$1 + (N - 1) \cdot \Delta_r^2 = e^{aN\Delta t}$$

Solving for $a$ leads to:

$$a = \frac{\ln(1 + (N - 1) \cdot \Delta_r^2)}{N \cdot \Delta t}$$

Equation (8) is the additional interest from the $N$ exchanges. In the experiment described in chapter 3, the monthly exchanges went on for 108 years ($N = 1296$). With this the plot of (8) is given below in figure 6.

Figure 6: Additional interest rate "a" due to exchanges over 108 years

Figure 6 shows it is easily possible to gain a couple of percentage points due to the average cost method.
Please note that the entire spread from medium 60% to upper 20% was just four percentage points in the simulated performed by Dimson et al. (2008). This is easily explained by an (extra) relative monthly fluctuation of little over 20%.

Of course our extra interest due to the average cost method should be tested with real date. Unfortunately, we do not have access to the particular data of the stocks over 108 years. Just to see how it works in reality, we have taken two quite independent stocks, namely AFL (American Family Life Assurance Company) and GD (General Dynamics).

Both gained in prices by a factor of around nine between January 1985 and January 1995. So it would have been totally irrelevant which stock to choose over the ten years. In Figure 7, we have displayed what happened with both stocks individually, and what would have happened, if we had exchanged both stocks monthly over the ten year period. The gain in annual interest is over four percentage points per year. That is an average cost effect. (Please note that the period of ten years considered here is much too short. Though we have 120 monthly values, the major changes are within a few months. Therefore statistical assumptions as taken above are by no means justified).

We close with a short note on whether or not the gain from the average cost method is a real one. Where does it come from? It is real and it comes from all people not dealing in the same way. So if everybody used the average cost method, the market would be distorted and there would not be the purely statistical fluctuations. The same is true, if some people knew about the future market (for whatever reason). Again, the fluctuations would not be by chance any more.

5. Conclusions and next steps

We have clearly shown that the so-called "momentum effect" is by no means a surprise. Because market prices are non-conserved quantities, they may fluctuate chaotically. With it, the momentum effect is easily explained as a giant self-fulfilling prophecy. Assuming that top stocks fluctuate more, at least part of the effect may be due to a generalized average cost effect. (Top stocks fluctuate more, because they are more interesting than the boring middle 60% or the pathological 20% at the bottom. Another line of argumentation is that fluctuation is synonymous with uncertainty here. And uncertainty demands a premium).

As a further proof of our theory, one should take the original stock data of Dimson et al. (2008). Two tests should be performed:

1. Though the time span was long, the question is whether or not particular occasions determined the entire picture more than the time span of a hundred years. As a suggestion, one may take the five (one) percent best and/or worst months out of the data applied for simulation. What happens to the general picture? This test is about the statistics.

2. One should quantify the average cost effect as described in chapter 4. How big is it exactly with the data of Dimson et al. (2008)?

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