A Cash Flow Oriented EOQ Model of Deteriorating Items with Time-Dependent Demand Rate under Permissible Delay in Payments

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Abstract - In this study inventory model is developed to determine an optimal ordering policy for deteriorating items and time-dependent demand rate with permissible delay in payments. The effects of inflation and time discounting are taken into account. Mathematical models have been derived under two different situations, i.e. Case I: The permissible delay period is less than cycle time for settling the account, and Case II: The permissible delay period is greater than cycle time for settling the account. In this study, we determine the optimal cycle time and optimal payment time for item so that the annual total cost is minimized. The numerical example is given to support the development of the mathematical model.

Key words: Inflation, Cash flow, Inventory, Deterioration, Time-dependent, Permissible, Demand, Delay in payments.

1. Introduction
In practice, suppliers offer their customers a fixed period without interest during the permissible delay time period. This fixed period is said to be credit period. Allowing a delay in payment to the supplier is a form of price-discount. Such a convenience is to motivate customers to order more quantities because paying later reduces the purchase cost indirectly. On the other hand, a deteriorating item such as green vegetable, fruit, electronic item, volatile liquid and photographic film gradually losses its potential. Deterioration is applicable to many inventories in practice. During the past few years, many research papers/articles dealing with deteriorating inventory problems have appeared in various research journals. The classical inventory models have considered demand rates were either constant or depended upon a single factor only, like, stock, time, etc. but changing market conditions a demand cannot depend on a single parameter. A combination of two or more factors grant more accurately to the formulation of the model. During the past decade, many articles dealing with deteriorating EOQ problems have appeared in various research journals. Most of the research articles assume a constant deterioration over time.

An EOQ model with permissible delay in payments was developed by Goyal [1], where he did not consider the difference between the selling price and purchase cost. This model was improved by Dave [2] under the assumption that the selling price is higher than its purchase price. Recently, Hou and Lin [3] considered an ordering policy with a cost minimization procedure for deterioration items under trade credit and time discounting. Davis and Gaither [4] developed EOQ models for firms offering a one time opportunity to delay payments by their suppliers for the order of a commodity. Shah et al. [5] studied the model when a delay in payments of order and shortages are permitted. Mandal and Phanjdar [6] studied the same situation by considering the interest earned from sales revenue. Aggarwal and Jaggi [7], Shah [8], Hwang and Shinn [9] extended Goyal’s [1] model to consider the deterministic inventory model with a constant deterioration rate. Chung [10] presented the discounted cash flow (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Dye [11] in their paper considered the stock-dependent demand for deteriorating items with partial backlogging and condition of permissible delay in payment. They assumed initial stock-dependent demand function. Chang et al. [12] considered an inventory model for deteriorating items with instantaneous stock-
dependent demand and time-value of money when credit period is provided. Recently, Huang [13] presented an inventory model assuming that the retailer also offers a credit period to his/her customer which is shorter than the credit period offered by the supplier, in order to stimulate the demand. Jamal et al. [14] and Sarker et al. [15] addressed the optimal payment time under permissible delay in payments with deterioration. Huang and Chung [16] extended Goyal’s model [1] to cash discount policy for early payment. Liao [17] assumed on an EPQ model for deteriorating items under permissible delay in payments.

In above cited references, the demand is taken to be deterministic and constant. However, the demand of the seasonal product (like mango, apple, cold drink, orange, etc. is decreases with time. Hou and Lin [18] developed a cash flow oriented EOQ Model with deteriorating items under permissible delay in payments and minimum total present value of the costs is obtained by considering constant net discount rate of inflation. Agrawal et al. [19] developed a model on integrated inventory system with the effect of inflation and credit period. In this model the demand rate is assumed to be a function of inflation. This EOQ is applicable when the inventory contains trade credit that supplier give to the retailer.

Jaggi et al. [20] developed a model retailer’s optimal replenishment decisions with credit- linked demand under permissible delay in payments. Tripathi and Misra [21] developed EOQ model credit financing in economic ordering policies of non- deteriorating items with time dependent demand rate in the presence of trade credit using a discounted cash flow (DCF) approach. An EOQ model with time-dependent demand rate and time- dependent holding cost function is developed by Tripathi [22].

In this study demand is considered to be time dependent. Shortages are not allowed and effect of inflation rate, deterioration rate and delay in payments are discussed. Mathematical models are derived under two different situations i.e. Case I: The credit period is less than or equal to the cycle time for settling the account, and Case II: The credit period is greater than the cycle time for settling the account. Also, expressions for inventory systems total cost are derived for these two cases. Moreover, an algorithm is proposed to obtain optimal solution. Finally, a numerical example demonstrates the applicability of the proposed model.

The rest of the paper is organized as follows: In section 2, notation and assumptions are given. In section 3, we develop the mathematical formulation for the solution of the total present value of the cost over the time horizon H, with regard to two different cases. An algorithm is developed in section 4, followed by numerical example in section 5. Finally, we draw the conclusions and future research in section 6.

2. Notations and Assumptions

The following notations are used throughout the manuscript:

- \( Q \) : Order quantity, units/cycle
- \( R(t) \) : Demand rate per unit time, and \( R(t) = a + bt \), \( a > 0, 0 < b < 1 \)
- \( H \) : Length of planning horizon
- \( T \) : Replenishment cycle time
- \( n \) : Number of replenishment during the planning horizon, \( n = H/T \)
- \( A_0 \) : Ordering cost at time zero, \$/order
- \( c \) : Per unit cost of the item, \$/unit
- \( h \) : Inventory holding cost per unit per unit time excluding interest charges, \$/unit/unit time
- \( r \) : Discount rate represent the time value of money
- \( f \) : Inflation rate
- \( k \) : The net discount rate of inflation (\( k = r - f \))
- \( I_e \) : The interest earned per dollar per unit time
- \( I_e \) : The interest charged per dollar in stocks per unit time by the supplier, \( I_e \geq I_e \)
- \( m \) : The permissible delay in settling account
- \( Z_1(n) \) : The total present value of the costs over the time horizon \( H \), for \( m \leq T = H/n \)
- \( Z_2(n) \) : The total present value of the cost for \( m > T = H/n \)
- \( E \) : The interest earned during the first replenishment cycle
- \( E_1 \) : The present value of the total interest earned over the time horizon
- \( E_2 \) : The interest earned in the first cycle (case II)
- \( E_3 \) : The present value of the total interest earned over the time horizon \( H \) (case II)
- \( I_r \) : The total interest payable over the time horizon \( H \)

The following assumptions are being made:

1. The demand \( R(t) \) for the item is a downward sloping function of the time. For simplicity, we assume that demand is a function of time i.e. \( R(t) = a + bt \), where \( a > 0 \) and \( b > 0 \).
3. Mathematical Formulation

The inventory \( I(t) \) at any time \( t \) is depleted by the effects of demand and deterioration. Thus the variation of inventory \( I(t) \) with respect to \( t \) is governed by the following differential equation:

\[
\frac{dI(t)}{dt} + \theta(t) = -(a + bt), \quad 0 \leq t \leq T \tag{1}
\]

With the boundary condition \( I(T) = 0 \). The solution of equation (1) can be represented by

\[
I(t) = \left( \frac{a}{\theta} - \frac{b}{\theta^2} \right) e^{\theta(T-t)} + \frac{b}{\theta} \left( T e^{\theta(T-t)} - t \right), \quad 0 \leq t \leq T = H/n \tag{2}
\]

The order quantity (or initial inventory) after replenishment becomes

\[
Q = I(0) = \left( \frac{a}{\theta} - \frac{b}{\theta^2} \right) e^{\theta T} + \frac{b}{\theta} T e^{\theta T} - 1, \quad 0 \leq t \leq T = H/n \tag{3}
\]

The present value of the total replenishment cost is given by:

\[
C_1 = A_o \sum_{j=0}^{n-1} e^{-jkT} = A_o \left( \frac{1 - e^{-kH}}{1 - e^{-kT}} \right), \quad (T = H/n) \tag{4}
\]

The present value of the total purchasing costs is given by

\[
C_2 = c \sum_{j=0}^{n-1} Q e^{-jkT} = cQ \left( \frac{1 - e^{-kH}}{1 - e^{-kH/n}} \right) \tag{5}
\]

and the present value of the total holding costs over the time horizon \( H \) is given by

\[
A = h \sum_{j=0}^{n-1} e^{-jkT} \int_0^T I(t) e^{-kt} dt, \quad (T = H/n) \tag{6}
\]

\[
h \left[ \frac{a - b}{\theta^2} \left( e^{\theta T} - e^{-kT} \right) + \frac{e^{-kT} - 1}{k} + \frac{b}{\theta} \left( T e^{\theta T} - e^{-kT} \right) \right] = \frac{1 - e^{-kT}}{1 - e^{-kT}} \tag{6}
\]

Case I. \( m \leq T = H/n \)

The present value of the interest payable during the first replenishment cycle is:

\[
i_p = cl_c \int_0^T I(t)e^{-kt} dt \]

\[
= cl_c \left[ \left( \frac{a}{\theta} - \frac{b}{\theta^2} \right) e^{\theta(T-t)} + \frac{b}{\theta} T e^{\theta(T-t)} - 1 \right] \left( \frac{e^{-km} - e^{-kT}}{\theta + k} + \frac{e^{-kT} - e^{-km}}{k} \right) \]

\[
+ \frac{b}{\theta} \left( e^{-kT} - e^{-km} \right) \left( \frac{1 - e^{-kT}}{k^2} \right) \tag{7}
\]

Hence, the present value of the total interest payable over the time horizon \( H \) is:

\[
I_p = \sum_{j=0}^{n-1} ip e^{-jkT} = i_p \left( \frac{1 - e^{-kH}}{1 - e^{-kT}} \right), \quad (T = H/n) \tag{8}
\]

The present value of the interest earned during the first replenishment cycle is

\[
E = cl_c \int_0^T R(t) e^{-kt} dt \]

\[
= cl_c \left\{ - \frac{(aT + bT^2)}{k} e^{-kT} + \frac{2b}{k^3} (1 - e^{-kT}) \right\} \tag{9}
\]

Hence the present value of the total interest earned over the time horizon \( H \) is

\[
E_1 = \sum_{j=0}^{n-1} E e^{-jkT} = E \left( \frac{1 - e^{-kH}}{1 - e^{-kT}} \right), \quad (T = H/n) \tag{10}
\]
Therefore, the total present value of the costs over the time horizon $H$ is

$$Z_1(n) = C_1 + C_2 + A + I_p - E_1$$  \hspace{1cm} (11)

**Case II.** $m > T = H/n$

The interest earned in the first cycle is the interest earned during the time period $(0, T)$ and the interest earned from the cash invested during the time period $(T, m)$ after the inventory is exhausted at time $T$ and it is given by

$$E_2 = c I_e \left[ \int_0^T R(t)e^{-kt}dt + (m-T)e^{-kT} \int_0^T R(t)dt \right]$$

\hspace{1cm} (T = H/n)

$$= c I_e \left\{ \frac{-(a + bT)e^{-kT}}{k} + \left( \frac{a}{k^2} + \frac{2b}{k^2} \right)(1-e^{-kT}) \right\}$$

\hspace{1cm} (12)

Hence, the present value of the total interest earned over the time horizon $H$ is:

$$E_3 = \sum_{j=0}^{n-1} E_2 e^{-jkT} = E_2 \left( \frac{1-e^{-kT}}{1-e^{-kT}} \right), \hspace{1cm} (T = H/n)$$

\hspace{1cm} (13)

Thus the total present value of the costs $Z_2(n)$ is given by

$$Z_2(n) = C_1 + C_2 + A - E_3$$ \hspace{1cm} (14)

At $m = T = H/n$, we find $Z_1(n) = Z_2(n)$

Consequently, we have

$$Z(n) = \begin{cases} Z_1(n), & \text{if } T = H / n \geq m \\ Z_2(n), & \text{if } T = H / n \leq m \end{cases}$$

where $Z_1(n)$ and $Z_2(n)$ are given in equations (11) and (14) respectively.

To derive the optimal $T$, $Q$ and $Z(n)$ value, we develop the following algorithm:

**4. Algorithm**

Step 1: Choose a discrete variable $n \geq 1$.

Step 2: If $T = H/n \geq m$ for different integer $n$ values, derive $Z_1(n)$ from (11) if $T = H/n \leq m$ for different integer $n$ values derive $Z_2(n)$ from (14).

Step 3: Repeat step 1 and 2 for all possible $n$ values with $T = H/n \geq m$ until the minimum $Z_1(n)$ is found from equation (11) and let $n_1^* = n$. For all possible values of $n$ with $T = H/n \geq m$ until the minimum $Z_2(n)$ is found from equation (14) and let $n_2^* = n$. The $n_1^*$, $n_2^*$, $Z_1(n_1^*)$ and $Z_2(n_2^*)$ values from the optimal solution.

Step 4: Select the optimal number of replenishment $n^*$ such that

$$Z(n^*) = \min \begin{cases} Z_1(n_1^*), & \text{if } H / n_1^* \geq m \\ Z_2(n_2^*), & \text{if } H / n_2^* \leq m \end{cases}$$

Hence optimal order quantity $Q^*$ is obtained by putting $n^*$ into (3) and optimal cycle time $T^*$ is $T^* = H / n^*$.

**5. Numerical Example**

To illustrate the results of the model developed in this study an example is given with the following data: $a = 600$ unit, $b = 0.8$ unit/yr, $A = \$70$/order, the holding cost excluding interest charges, $h = \$2.5$/unit/year, the per unit item cost, $c = \$10/unit; the constant rate of deterioration $\theta = 0.15$, the net discount rate of inflation, $k = 0.10/$/year, the interest charged per $ in stock per year by the supplier, $I_c = \$0.20$/year, the interest earned per $ per year, $I_r = \$0.15$/year and the planning horizon, $H = 6$ year. The permissible delay in settling account, $m = 90$ days $= 90/360 = 0.25$ years (assumed 360 days per year). Using the algorithm, we have the computational results shown in Table 1. We find the case 1 is optimal option in credit policy. The minimum total present value of costs is obtained when the number of replenishment, $n$ is 32. With 32 replenishments, the optimal (minimum) cycle time $T$ is 0.188 year, the optimal (minimum) order quantity, $Q = 114.42$ units and the optimal (minimum) total present value of costs, $Z = \$29429.28$.

**Table 1. Numerical results**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Order Quantity ($Q$)</th>
<th>Cycle Time ($T$)</th>
<th>Order Total costs ($Z(n)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.600</td>
<td>376.85</td>
</tr>
<tr>
<td>11</td>
<td>0.545</td>
<td>340.86</td>
<td>30842.62</td>
</tr>
<tr>
<td>12</td>
<td>0.500</td>
<td>311.64</td>
<td>30561.45</td>
</tr>
<tr>
<td>13</td>
<td>0.462</td>
<td>287.12</td>
<td>30338.38</td>
</tr>
</tbody>
</table>
6. Conclusion and Future Research

This study develops an inventory model for deteriorating items over a finite planning horizon, when the supplier provides a permissible delay in payments. The model considers the effects of inflation and permissible delay in payments. In this study we have presented an optimal solution procedure to obtain the optimal number of replenishment cycle time and order quantity to minimize the total present value of costs. Numerical example is given to illustrate the model.

The model proposed in this paper can be extended in several ways. For instance, we may extend the time dependent deterioration rate. We could also consider the demand as a function of quantity as well as quadratic time varying. Finally, we could generalize the model to allow for shortages and quantity discount etc.

Note: This paper is the extension of Kuo-Lung Hou and Li-Chiao Lin (2009) in which the deterioration and demand rate both constant.

References


