EPQ Model for Deteriorating Items with Quadratic Demand, Dynamic Production Rate and Production Time Dependent Selling Price: A Risk Analysis Approach

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***Abstract*— In this article, a production inventory model with dynamic production rate and production time dependent selling price has been presented. We consider the product with constant deterioration rate which is a very realistic approach. It is also considered that the production rate is a decreasing function of the inverse efficiency of the system. Here, time dependent quadratic demand is deliberated which is suitable for the products whose demand increases initially and subsequently it starts to decrease. Industries like fashion and electronics most probably deals with this type of demand. The main objective is to maximise the total profit with respect to inverse efficiency and total production period. The model is supported with numerical example. Sensitivity analysis is done to derive insights for decision-maker. Graphical results, in two and three dimensions, are exhibited for optimality of the model with supervisory decisions.**

Keywords— *Inventory, deterioration, quadratic demand, production rate, production period, selling price, inverse efficiency*

1. Introduction:

In industrial and corporate operation, the inventory problems are common features. In any production inventory system, uncertainties are usually linked with demand, several relevant costs, raw materials, life of the production services, machine maintenances and preservation time. When these uncertainties are immaterial then these can be possibly calculated by classical Economic Order Quantity (EOQ) or Economic Production Quantity (EPQ) modelling. EOQ modelling was firstly developed by [1]. Through realistic features, various methods have been developed. The extension of this model is EPQ modelling which also has been served several years to control the optimal lot size in a production system. These models depend on some basic traditions such as demand, production cost, cost parameters, selling price etc. Also when these are random in nature, they are usually controlled by probabilistic methods.

Most of the researchers consider constant production rate in their articles. Optimal ordering policy for deteriorating items of fixed-life under quadratic demand and two-level trade credit in which production rate is in proportion with demand

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rate was established by [2]. An integrated production-inventory model with preservation technology investment for time-varying deteriorating item under the effect of time and price sensitive demand was developed by [3]. Now a days, the concept of dynamic production rate also catches little attention from researchers. Risk analysis of production inventory model with fuzzy demand and variable production rate under the impact of production time dependent selling price was studied by [4].

Because of the effect of fundamental environmental variations, most of the items losses its utility over time, called as deterioration. Effect of deterioration in inventory model was considered first time by [5]. Some important research papers on deteriorating products for inventory system focused on the role of deterioration are analysed by [6], [7], [8] and [9]. The citations in the review papers enclose constant deterioration rate, weibull distributed deterioration etc. Optimal down – stream credit period and replenishment time under the impact of deterioration in a supply chain was considered by [10]. Flexible deterioration rate was accepted by [11]. Optimal policies of three players for fixed life-time and two-level credit limit under the effect of time and credit dependent demand was studied by [12]. Optimal strategies for time-varying deteriorating item under preservation technology with selling price and trade credit dependent quadratic demand in a supply chain was recognised by [13]. Economic order quantity model for non-instantaneously deteriorating items under order-Size-dependent trade credit for price-sensitive quadratic demand was developed by [14].

In the business, demand barely remains constant for infinite horizon. A coordinated decision with two-part credit limit when demand is quadratic which is observed in fashion goods, seasonal products etc was analysed by [15]. It is detected in the market that with the launch of latest generation product, market demand of the former brand decreases extremely. Optimal shipments, ordering and payment policies for integrated vendor-buyer inventory system with price-sensitive trapezoidal demand and net credit was considered by [16]. Inventory model under trade credit and customer return under the effect of price-sensitive quadratic demand was established by [17]. The model on the impact of future price increase on ordering policies for deteriorating items under quadratic demand was presented by [18].

We have analysed a production inventory model with dynamic production rate which decreases with time under the impact of production time dependent selling price in this article. Here, demand function initially increases with linear rate of change and after some time it starts to decrease quadratically. The concept of inverse efficiency of production is also debated here. Main purpose of this article is to obtain optimum total profit of manufacturer with respect to inverse efficiency and total production time. A numerical example with sensitivity analysis and three dimension graph has been demonstrated to estimate the risk value for the offered production inventory system.

The rest of the paper is organised as follow. Section 2 presents the notations and the assumptions that are used. Section 3 develops the mathematical model of the inventory problem. Section 4 establishes the proposed inventory model with numerical example. This section also provides some managerial insights. Finally, Section 5 provides conclusion and future research directions

2. Notations and Assumptions:

The proposed inventory problem is based on the following notations and assumptions.

2.1 Notations:

A	Production setup cost (in $)
$P(t)$	Production rate per unit time

$p(t)$	Selling price per unit (in \$)
P_0	Constant production rate
C	Production cost per unit (in \$)
h	Holding cost per unit item per unit time (in \$)
a	Total market potential demand; $a \geq 0$
b	Linear rate of change of demand; $0 \leq b < 1$
c	Quadratic rate of change of demand; $0 \leq c < 1$
$R(t)$	$= a\left(1+bt-ct^2\right)$; quadratic demand rate at time t
θ	Constant deterioration rate; $0 \leq \theta \leq 1$
T	Cycle time (in years)
t_1	$= k\,t_2$; Duration of constant production, $k > 0$
t_2	Total production period (decision variable)
λ	Inverse efficiency (decision variable)
C_P	Rate of efficiency cost
Q_1	Inventory level at time t_1 (units)
Q_2	Inventory level at time t_2 (units)
$I_1(t)$	Inventory level at time t (units), $0 \leq t \leq t_1$
$I_2(t)$	Inventory level at time t (units), $t_1 \leq t \leq t_2$
$I_3(t)$	Inventory level at time t (units), $t_2 \leq t \leq T$
SR	Sales Revenue
HC	Holding Cost
OC	Setup Cost
PC	Production Cost
EFC	Efficiency Cost
$TP(\lambda, t_2)$	Total profit per unit time (in \$)

2.2 Assumptions:

1. The model is developed for single item.
2. Lead time is negligible or zero.
3. Shortages are not allowed.
4. The demand rate is $R(t) = a\left(1+bt-ct^2\right)$, where $a > 0$ is the scale demand, b and c are linear and quadratic demand rates. The functional form of demand rate suggests that demand increases linearly and decreases quadratically. This demand is justified for electronic gadgets, fashion goods, medicines during epidemics.
5. The cycle period T is constant.
6. Initially, in the factory upto certain time limit, production rate remains constant, as all factors associated with production are afresh such as machines are in well conditions, all specialists are afresh in observance and physic. Usually as time of production increases, all elements related to the production procedure face some exhaustion such as machinery fault, labour complications, deficiency of enough raw materials etc. Consequently the production will decrease. Now, if there is no planning of proper production to fulfil the consumer's total demand, stock of production will be in shortage, i.e., finally consumer's demands as time increases, will not be fulfilled. Since any producer will not anticipate such situation, so in such conditions supervisor takes certain decisions to increase the efficiencies of all elements to produce and to preserve the appropriate production to satisfy the consumers' demand, paying some extra cost which is recognised as efficiency cost, denoted by EFC .

7. At the opening of any business, the rate of production will be same for some period. Subsequently the production rate will start to decrease due to some inherent complications in the system. As the total cycle time (T) is fixed, then to satisfy the total consumer's demand during the cycle time (T), some efficiency (E) of different elements in the system must be increased for additional production. Seeing this fact in the production inventory system, production rate, $P(t)$, which is considered as a function of a new variable λ known as inverse efficiency, is offered as follows:

$$P(t)=\begin{cases} P_0, & \text{for } 0\le t\le t_1 \\ P_0\, e^{-\lambda(t-t_1)}, & \text{for } t_1\le t\le t_2 \end{cases},$$

where, $\lambda=\dfrac{1}{E}$

8. After a certain time t_2, the production will be stopped in such a way that the system gives the optimum profit.

9. Selling price of an item depends upon the production time, since primarily in the production everything is fresh, so each produced item will be of good value. But after certain time, the quality of the item will be declined slowly with respect to time upto the end of production due to machinery liability, lack of proficiency of the resource persons for continuous working, etc. Therefore the selling price $p(t)$ of an item has been considered as:

$$p(t)=\begin{cases} p_0, & \text{for } 0\le t\le t_1 \\ p_0-p_1(t-t_1), & \text{for } t_1\le t\le t_2 \end{cases}$$

where, $[0,t_2]$ is the production period. p_0 and p_1 are selling prices.

3. Mathematical Model:

We have analyzed one cycle. The cycle starts with production at a constant production rate P_0 and continues up to time $t=t_1$. During $[0,t_1]$, the rate of change of inventory is governed by following differential equation

$$\frac{dI_1(t)}{dt}=P_0-R(t)-\theta I_1(t), \qquad 0\le t\le t_1$$

with the initial condition $I_1(0)=0$.

The solution of the differential equation is given by,

$$I_1(t)=-\frac{\begin{pmatrix} -act^2\theta^2+abt\theta^2+e^{-\theta t}P_0\theta^2 \\ +e^{-\theta t}ab\theta-e^{-\theta t}a\theta^2 \\ +2act\theta+2e^{-\theta t}ac \\ -P_0\theta^2-ab\theta+a\theta^2-2ac \end{pmatrix}}{\theta^3}$$

At $t=t_1$, the order quantity $I_1(t_1)=Q_1$ can be obtain by following equation

$$Q_1=-\frac{\begin{pmatrix} -act_1^2\theta^2+abt_1\theta^2+e^{-\theta t_1}P_0\theta^2 \\ +e^{-\theta t_1}ab\theta-e^{-\theta t_1}a\theta^2 \\ +2act_1\theta+2e^{-\theta t_1}ac \\ -P_0\theta^2-ab\theta+a\theta^2-2ac \end{pmatrix}}{\theta^3}$$

Then production rate $P(t)$ decreases and production stops at the time t_2.

So, during $[t_1, t_2]$, the rate of change of inventory is governed by following differential equation

$$\frac{dI_2(t)}{dt}=P(t)-R(t)-\theta I_2(t), \qquad t_1\le t\le t_2$$

with the boundary condition $I_1(t_1)=Q_1$.

The solution of the differential equation is given by,

$$I_2(t)=\begin{pmatrix} Q_1-\frac{act_1^{\ 2}}{\theta}+\frac{2act_1}{\theta^2}+\frac{abt_1}{\theta} \\ -\frac{2ac}{\theta^3}-\frac{ab}{\theta^2}-\frac{P_0}{-\lambda+\theta}+\frac{a}{\theta} \end{pmatrix} e^{-\theta(t-t_1)} + \frac{P_0e^{-\lambda(t-t_1)}}{-\lambda+\theta}-\frac{2act}{\theta^2}-\frac{abt}{\theta} + \frac{act^2}{\theta}+\frac{2ac}{\theta^3}+\frac{ab}{\theta^2}-\frac{a}{\theta}$$

Using boundary condition $I_2(t_2)=Q_2$, the order quantity defined by following equation

$$Q_2=\begin{pmatrix} Q_1-\frac{act_1^{\ 2}}{\theta}+\frac{2act_1}{\theta^2}+\frac{abt_1}{\theta} \\ -\frac{2ac}{\theta^3}-\frac{ab}{\theta^2}-\frac{P_0}{-\lambda+\theta}+\frac{a}{\theta} \end{pmatrix} e^{-\theta(t_2-t_1)} + \frac{P_0e^{-\lambda(t_2-t_1)}}{-\lambda+\theta}-\frac{2act_2}{\theta^2}-\frac{abt_2}{\theta} + \frac{act_2^{\ 2}}{\theta}+\frac{2ac}{\theta^3}+\frac{ab}{\theta^2}-\frac{a}{\theta}$$

After this time period, production will stop and inventory level will reach to zero at $t=T$. In this period $[t_2, T]$, the rate of change of inventory is governed by following differential equation:

$$\frac{dI_3(t)}{dt}=-R(t)-\theta I_3(t), \qquad t_2\le t\le T$$

with the boundary condition $I_3(T)=0$.

The solution of the differential equation is given by,

$$I_3(t)=\frac{a\begin{pmatrix}-T^2c\theta^2+Tb\theta^2+2Tc\theta \\ -b\theta+\theta^2-2c\end{pmatrix}e^{\theta(T-t)}}{\theta^3} - \frac{a\begin{pmatrix}-t^2c\theta^2+tb\theta^2+2tc\theta \\ -b\theta+\theta^2-2c\end{pmatrix}}{\theta^3}$$

For the derivation of the total profit, the mathematical expressions of setup cost, holding cost, production cost, sales revenue, efficiency cost are calculated as follows.

- Ordering Cost: $OC=\frac{A}{T}$
- Holding Cost:

$$HC=\frac{h}{T}\left(\int_0^{t_1} I_1(t)\,dt+\int_{t_1}^{t_2} I_2(t)\,dt+\int_{t_2}^{T} I_1(t)\,dt\right)$$

- Production Cost:

$$PC=\frac{C}{T}\left(\int_0^{t_1} P(t)\,dt+\int_{t_1}^{t_2} P(t)\,dt\right)$$

- Sales Revenue:

$$SR=\frac{1}{T}\left(\int_0^{t_1} p(t)P(t)\,dt+\int_{t_1}^{t_2} p(t)P(t)\,dt\right)$$

- Efficiency Cost

$$EFC=\frac{C_P}{T}\left(\int_{t_1}^{t_2} P(t)\,dt\right)$$

As a result, the total profit per unit time is:

$$TP(\lambda,t_2)=SR-OC-HC-PC-EFC$$

4. Numerical Example and Sensitivity Analysis

4.1 Numerical Example: The data for the numerical example is given in Table 1. The optimum solution is exhibited in Table 2.

Table 1. Input parameters for the numerical example

Inventory parameters	**Values**
a	45
b	0.05
c	0.2
h	5.45
C	20
A	90
P_0	105
p_0	29
p_1	5.35

C_P	5
k	0.2
θ	0.1
T	1

Table 2. Optimal Solution

Total Profit $TP(\lambda, t_2)$ **($)**	**Total Production Time** t_2 **(years)**	**Inverse Efficiency** λ
110.27	0.935	0.543
Q_1 **(units)**	Q_2 (units)	t_1 (years)
9	32	0.186

The concavity of the total profit is exhibited in Figure 1.

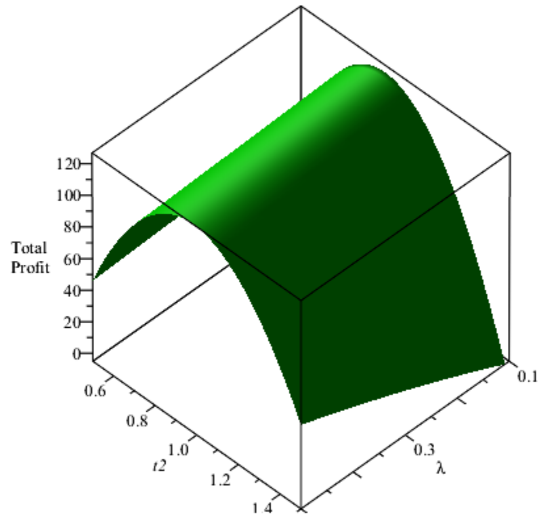

Figure 1. Concavity of total profit Vs inverse efficiency λ and total production time t_2

4.2 Sensitivity Analysis for the Inventory Parameters:

For the different inventory parameters, the sensitivity analysis of numerical example is carried out in table 3 by changing one variable at a time.

Table 3. Sensitivity Analysis

Parameters	**Values**	**Inverse Efficiency** λ	**Total Production Time (years)** t_2	**Total Profit** $TP(\lambda, t_2)$ **($)**
a	44	0.903	0.935	96.67
	47	0.716	0.935	103.47
	50	**0.543**	**0.935**	**110.27**
	53	0.384	0.935	117.08
	56	0.237	0.935	123.88
b	0.040	0.558	0.935	109.92
	0.045	0.551	0.935	110.10
	0.050	**0.543**	**0.935**	**110.27**
	0.055	0.536	0.935	110.45
	0.060	0.529	0.935	110.63
c	0.16	0.505	0.935	110.93
	0.18	0.524	0.935	110.60
	0.2	**0.543**	**0.935**	**110.27**
	0.22	0.563	0.935	109.95
	0.24	0.583	0.935	109.61
h	4.905	0.573	0.970	120.46
	5.45	**0.543**	**0.935**	**110.27**
	5.995	0.499	0.903	100.58
	6.54	0.441	0.873	91.27
C	19.8	0.702	0.982	126.29
	19.9	0.626	0.958	118.29
	20	**0.543**	**0.935**	**110.27**
	20.1	0.456	0.911	102.25
	20.2	0.364	0.888	94.23
A	72	0.543	0.935	128.27
	81	0.543	0.935	119.27
	90	**0.543**	**0.935**	**110.27**
	99	0.543	0.935	101.27
P_0	90	0.258	0.935	101.58
	95	0.403	0.935	105.93
	100	**0.543**	**0.935**	**110.27**
	105	0.679	0.935	114.62
	110	0.810	0.935	118.96
p_0	28.8	0.364	0.888	94.23
	29	**0.543**	**0.935**	**110.27**
	29.2	0.704	0.982	126.29
p_1	4.85	0.726	0.994	121.07
	5.35	**0.543**	**0.935**	**110.27**
	5.85	0.369	0.881	99.68
C_P	4.8	0.748	0.982	122.36
	4.9	0.648	0.958	116.37
	5.0	**0.543**	**0.935**	**110.27**
	5.1	0.434	0.911	104.08

	5.2	0.320	0.888	97.78
k	0.16	0.650	0.890	80.22
	0.18	0.594	0.912	95.01
	0.20	**0.543**	**0.935**	**110.27**
	0.22	0.498	0.959	126.03
	0.24	0.457	0.984	142.29
θ	0.08	0.556	0.934	109.63
	0.09	0.550	0.934	109.95
	0.1	**0.543**	**0.935**	**110.27**
	0.11	0.537	0.935	110.60
	0.12	0.531	0.936	110.92
T	0.95	0.677	0.935	116.57
	1.00	**0.543**	**0.935**	**110.27**
	1.05	0.420	0.935	104.03

1. From table 3, it is observed that constant production rate $\left(P_0\right)$ and selling price $\left(p_0\right)$ increases inverse efficiency $\left(\lambda\right)$ rapidly whereas quadratic rate of change of demand $\left(c\right)$ increases inverse efficiency $\left(\lambda\right)$ slowly. On the other hand, total market potential demand $\left(a\right)$, production cost per unit $\left(C\right)$, selling price $\left(p_1\right)$, rate of efficiency cost $\left(C_P\right)$ and total cycle time $\left(T\right)$ has huge negative impact on inverse efficiency $\left(\lambda\right)$. Whereas, linear rate of change of demand $\left(b\right)$, holding cost $\left(h\right)$, constant deterioration rate $\left(\theta\right)$ and k decreases inverse efficiency $\left(\lambda\right)$ gradually.

2. From table 3, it is detected that selling price $\left(p_0\right)$ and k has huge positive impact on total production period $\left(t_2\right)$. Conversely, holding cost $\left(h\right)$, production cost per unit $\left(C\right)$, selling price $\left(p_1\right)$ and rate of efficiency cost $\left(C_P\right)$ decreases total production period $\left(t_2\right)$ slowly.

3. From table 3, it is noticed that total market potential demand $\left(a\right)$, selling price $\left(p_0\right)$ and k increases total profit $TP\left(\lambda, t_2\right)$ rapidly whereas linear rate of change of demand $\left(b\right)$ has small positive impact on total profit $TP\left(\lambda, t_2\right)$. Conversely, holding cost $\left(h\right)$, production cost per unit $\left(C\right)$, selling price $\left(p_1\right)$ and rate of efficiency cost $\left(C_P\right)$, total cycle time $\left(T\right)$ and production setup cost $\left(A\right)$ has huge negative impact on total profit $TP\left(\lambda, t_2\right)$ whereas constant production rate $\left(P_0\right)$ and quadratic rate of change of demand $\left(c\right)$ decreases total profit $TP\left(\lambda, t_2\right)$ slowly.

5. Conclusion

In this article a production inventory model has been established with a dynamic production rate which decreases slowly with respect to time. Here selling price has been considered as a function of production time and demand is considered quadratic. As a result total optimum profit has been worked out which is a function of inverse efficiency and total production time. Finally, a numerical example is evaluated and sensitivity analysis is carried-out with respect to inventory parameters. For future scope, it is anticipated to extend the proposed model by considering the product with maximum fixed life-time, quantity discount, preservation technology, advertisement.

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