

A Deterministic Inventory Routing Model for the Single-period Problems with Finite Time Horizon

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Abstract— This paper considers the problem of managing inventory and routing problems in a two-stage supply chain system under a Vendor Managed Inventory (VMI) policy. VMI is an inventory management policy in which the supplier assumes the responsibility of maintaining the inventory at the customers while ensuring that they will not run out of stock. The delivery times to the customers are no longer agreed in response to customers' orders; instead the supplier indicates when each delivery takes place. Under the VMI policy, the planning is proactive as it is based on the available information rather than reactive to customers' orders. Thus, in this research, we assumed that the demand at each customer is stationary and the warehouse is implementing a VMI. The objective of this research is to minimize the inventory and the transportation costs of the customers for a two-stage supply chain system. The problem is to identify the delivery quantities, delivery times and routes to the customers for the single-period deterministic inventory routing problem (SP-DIRP) system with finite time horizon. As a result, a linear mixed-integer program is established for the solutions of the SP-DIRP problem.

Keywords— *single-period, deterministic model, inventory routing problem, vendor managed inventory.*

1. Introduction

Vendor Managed Inventory (VMI) is an incorporated inventory management policy in which the supplier assumes the responsibility of maintaining the inventory at the customers while ensuring that they will not run out of stock-outs at both warehouse and customers. Under the VMI policy, the customer has no longer to dedicate

resources to inventory management. In this policy, the service level increases, as the supplier exactly knows the inventory level at the customers and at the warehouse.

In this paper, we developed a model for the solution of customers' demands on inventory and routing management for a two-stage supply network. The main objective of this paper is to optimize the overall inventory and transportation costs while assuming that the demand at each customer is known and deterministic. Therefore, we concentrated for a single-period deterministic inventory routing problem (SP-DIRP) with finite time horizon. Precisely, we study a distribution system in which a fleet of homogeneous vehicles is used to allocate the product from a single warehouse to a set of customers. Based on the formulation of the multi-period and the single-period inventory routing problem (IRP) [1] and [2], we formulated a linear mixed-integer model for the SP-DIRP.

The remaining of this paper is structured as follows. Section 2 reviews some papers on the modelling of deterministic IRP. Section 3 formulates a linear mixed-integer program for the SP-DIRP. Section 4 includes an example of supply chain. Lastly, conclusions will be defined in Section 5.

2. Literature Review

In the early 1980s, some studies have started to incorporate inventory concerns within the existing vehicle routing literature. These were mostly variations of vehicle routing problem (VRP) models and heuristics developed to accommodate inventory costs. Most of the papers considered

consumption rate at the customers as known and deterministic. In a general setting, the IRP has a finite time horizon and a one-warehouse multiple-customers inventory system dealing with a single product. The warehouse has enough goods to supply the customers whose demands are known to the supplier at the beginning of the planning period. A homogeneous fleet of vehicles is available for the distribution of the problem and neither the warehouse nor the customer faces any ordering or inventory costs. The objective is to minimize the distribution costs during the planning period without causing stock-outs at any of the customers.

Dror et al. [3] is among the earliest paper to address the IRP, and propose a short term solution approach to take into account what happens after the single day planning period. They described this problem over a short planning period, e.g. one week, and proposed a mixed integer programming model to display effects of present decisions on later periods. The solution is based on the assignment of customers to their so called optimal replenishment period t , and then calculating the expected increase in cost if the customer is visited in another period.

Dror and Ball [4] presented a procedure for reducing the annual optimization problem and selecting the set of customers replenished in a short operational time period, with the objective to minimize annual costs subject to no customer shortages. The relationship between the annual distribution cost, the fixed delivery cost, and the amount delivered to the customers are examined and the customers to be visited on a given day are selected according to these costs.

Bertazzi et al. [5] addressed a multi-period model with a deterministic case in which a set of products is shipped from a common supplier to several customers. For each product, a starting level of the inventory is given both for the supplier and for each customer and the level of the inventory at the end of the time horizon can be different from the starting one. Therefore, each customer should determine a lower and an upper level of the inventory of each product and can be visited several times during the time horizon. Every time a customer is visited the quantity delivered is such that the maximum level of inventory is reached. This inventory policy is called the OU policy, decreasing the flexibility of the decision

maker, but simplifying the set of possible decisions of the problem.

Campbell and Savelsbergh [6] studied a multi-period IRP motivated by the application in the industrial gas industry, PRAXAIR, which is a large industrial gas company with about 60 production facilities and more than 10,000 customers across North America. The authors propose a two-phase solution approach. In the first phase, they determine which customers receive a delivery on each day of the planning period and decide on the size of the deliveries. In the second phase, they then determine the actual delivery routes and schedules for each of the days.

Abdelmaguid and Dessouky [7] proposed a genetic algorithm (GA) approach for solving the integrated inventory distribution problem with multiple planning periods, in which backorders are permitted. Backorder decisions are generally established in two cases. In the first case is when there is insufficient vehicle capacity to deliver to a customer, while the second case is when there is a transportation cost saving that is higher than the incurred backorder cost by a customer. The authors designed a suitable genetic representation that focuses on the delivery schedule represented in the form of a two-dimensional matrix and leave the vehicle routing part to be solved using any efficient polynomial time heuristic such as the savings algorithm.

Archetti et al. [8] proposed an exact algorithm to the deterministic IRP over a given time horizon. Each customer defines a maximum inventory level. The supplier monitors the inventory of each customer and determines its replenishment policy, guaranteeing that no stock-out occurs at the customer. These authors considered the case with only one vehicle, no backlogging and using the OU inventory policy, which is the quantity delivered by the supplier to the visited customers is such that it reached the maximum inventory level. They developed a branch-and-cut algorithm and derived several valid inequalities to strengthen the linear relaxation of the model. The authors then compared the optimal solution of this problem, with the optimal solution of two problems obtained by relaxing in different ways the deterministic OU policy.

Yu et al. [9] considered a multi-period deterministic inventory routing problem with split delivery (IRPSD) where the customers' demands in

each period over a given planning horizon are assumed to be constant and must be satisfied without backorder. The distribution to each of the customers in every period of time can be split and performed by multiple vehicles. In order to solve large scale instances, the authors proposed an approximate model for the multi-period IRP.

Archetti et al. [10] considered on the multi-period deterministic IRP in discrete time, where a supplier has to serve a set of customers over a time horizon. A capacity constraint for the inventory is given for each customer and the service cannot cause any stock-out situation. Two different replenishment policies are considered, the order-up-to level (OU) and the maximum level (ML) policies.

For recent study, Rahim et al. [11] studied a single-period deterministic inventory routing problem (SP-DIRP) where the customers' demands in one period of time are assumed to be constant and must be satisfied without backorder. Thus, the delivery to the customers in each period cannot be split and must be replenished by a single vehicle. The summary of classification of main papers on the single item deterministic IRP with finite time horizon can be shown in Table 1 below.

The SP-DIRP involves of a single warehouse using a fleet of homogeneous vehicles to deliver a single product to a set of geographically circulated customers over a finite time horizon. It is expected that customer-demand rates are deterministic, and that travel-times are constant over time. The objective of this SP-DIRP is to find optimal quantities and delivery time, and vehicle delivery routes, so that to minimize the total inventory and transportation costs.

Let τ_i be the size in time units of period 1, in this case we are used 8 working hours per day. Let S be the set of customers indexed by i and j ; and $S^+ = S \cup \{r\}$, which represents the warehouse. A fleet of vehicles V is used to serve these customers. The parameters and the variables of the model are given below:

- ϕ_j : the fixed handling cost per delivery at location $j \in S^+$ (customers and warehouse);
- η_j : the per unit per period holding cost of the product at location $j \in S^+$;
- ψ^v : the fixed operating cost of vehicle $v \in V$;
- δ_v : travel cost of vehicle $v \in V$;
- κ^v : the capacity of vehicle $v \in V$;
- v_v : average speed of vehicle $v \in V$;

Table 1. Classification of main papers on the single item deterministic IRP with finite time horizon

Authors	Year	Routing			Inventory policy			Fleet composition		Fleet size		
		Direct	Multiple	Continuous	Lost sales	Backlogging	Non-negative	Homogeneous	Heterogeneous	Single	Multiple	Unconstrained
Dror et al. [3]	1985		✓		✓			✓			✓	
Dror and Ball [4]	1987		✓		✓			✓			✓	
Bertazzi et al. [5]	2002		✓				✓			✓		
Campbell and Savelsbergh [6]	2004		✓		✓			✓			✓	
Abdelmaguid and Dessouky [7]	2006		✓			✓			✓		✓	
Archetti et al. [8]	2007		✓				✓	✓		✓		
Yu et al. [9]	2008		✓		✓			✓			✓	
Archetti et al. [10]	2012		✓				✓	✓		✓		
Rahim et al. [11]	2016		✓				✓	✓		✓		

3. Modelling and formulating SP-DIRP

- θ_{ij} : duration of a trip from customer $i \in S^+$ to customer $j \in S^+$;
- d_j : the stationary demand rate at customer j .

- I_{j0} : the initial inventory levels at each customer $j \in S$.
- Q_{ij}^v : the quantity of product remaining in vehicle $v \in V$ when it travels directly to location $j \in S^+$ from location $i \in S$. This quantity equals zero when the trip (i,j) is not on any tour of the route travelled by vehicle $v \in V$;
- q_j : the quantity delivered to location $j \in S$, and 0 otherwise;
- I_j : the inventory level at location (customers and warehouse) $j \in S^+$;
- x_{ij}^v : a binary variable set to 1 if location $j \in S^+$ is visited immediately after location $i \in S^+$ by vehicle $v \in V$, and 0 otherwise;
- y^v : a binary variable set to 1 if vehicle $v \in V$ is being used, and 0 otherwise.

Thus, if we let I_{j0} be the initial inventory level at the warehouse, the linear mixed-integer formulation for the SP-DIRP is given as follows:

SP-DIRP: Minimize

$$CV = \sum_{v \in V} \left[\psi^v y^v + \sum_{i \in S^+} \sum_{j \in S^+} (\delta_{i,v} \theta_{ij} + \phi_j) x_{ij}^v \right] + \sum_{j \in S^+} \eta_j I_j \quad (1)$$

Subject to:

$$\sum_{v \in V} \sum_{i \in S^+} x_{ij}^v \leq 1, \quad \forall j \in S \quad (2)$$

$$\sum_{i \in S^+} x_{ij}^v - \sum_{k \in S^+} x_{jk}^v = 0, \quad \forall j \in S^+, v \in V \quad (3)$$

$$\sum_{i \in S^+} \sum_{j \in S^+} \theta_{ij} x_{ij}^v \leq \tau_i, \quad v \in V \quad (4)$$

$$\sum_{v \in V} \sum_{i \in S^+} Q_{ij}^v - \sum_{v \in V} \sum_{k \in S^+} Q_{jk}^v = q_j, \quad \forall j \in S, \quad (5)$$

$$Q_{ij}^v \leq k^v x_{ij}^v, \quad \forall j \in S^+, v \in V \quad (6)$$

$$I_j + q_j - I_j = d_j \tau_i, \quad \forall j \in S, \quad (7)$$

$$I_{j0} \leq I_j, \quad \forall j \in S \quad (8)$$

$$x_{ij}^v \leq y^v, \quad \forall j \in S^+, v \in V \quad (9)$$

$$x_{ij}^v, y^v \in \{0,1\}, I_{j0}, I_j \geq 0, Q_{ij}^v \geq 0, q_j \geq 0, \quad \forall j \in S^+, v \in V$$

The objective function (1) includes four cost components, which are total fixed operating cost of the vehicles, total transportation cost, total delivery handling cost and total inventory holding cost at the

warehouse and customers. Constraints (2) confirm that each customer is visited at most once. Constraints (3) guarantee that if a truck arrives at a customer, it must leave after it has served it to a next customer or to the warehouse. Constraints (4) ensure that trucks complete their routes within one travel period, so the total travel time of a truck should not exceed the total working hours. Constraints (5) determine the quantity delivered to a customer. The truck capacity constraints are given by (6) and assure that the variables Q_{ij}^v cannot carry any cumulated flow unless x_{ij}^v equals 1. Constraints (7) are the inventory balance equations at the customers. Constraints (8) indicate that the final inventory level at customer j at the end of period is of the same magnitude as its initial inventory. Constraints (9) indicate that a truck cannot be used to serve any customer unless it is selected.

4. Analysis of supply chain example

In this paper, we present an example case for the single-period deterministic inventory routing problem (SP-DIRP) to illustrate the behaviour of our proposed model. In this particular case, we consider 10 customers as illustrated in Figure 1. These customers are scattered around the warehouse with the coordinates (x, y) and have demand rates that are assumed to be stable. We assume that a fleet of vehicles is available for product replenishment from the warehouse.

Figure 1. An example case with 10 customers

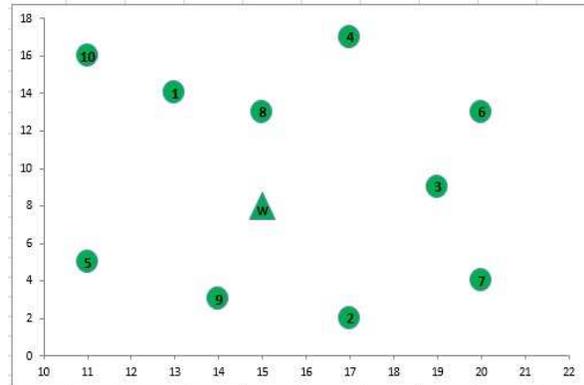


Figure 2. A VRP tour solution with vehicle capacity of 100 tons

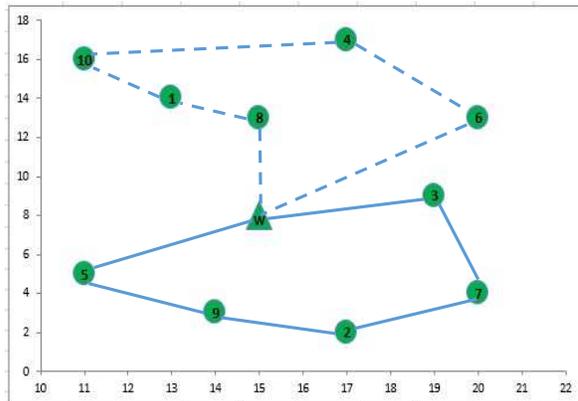
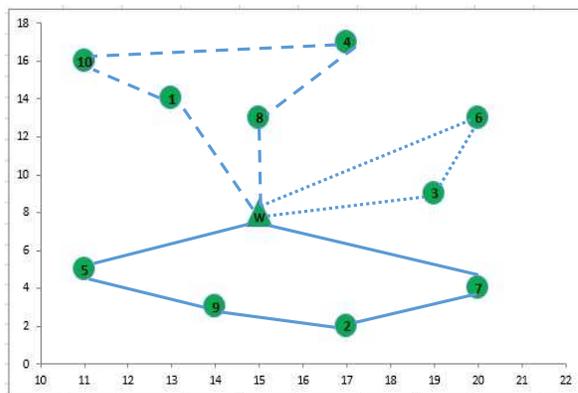


Figure 2 above shows the results for the distribution pattern. In the solution, we can see that only one vehicle is used to replenish the product to each customer. For example, the vehicle 1 with 100 tons capacity makes the tour $V_1 = (0-3-7-2-9-5-0)$ and tour $V_2 = (0-8-10-4-6-0)$.

Figure 3. A VRP tour solution with vehicle capacity of 80 tons



In addition, Figure 3 gives the results for the distribution pattern of 80 tons vehicle's capacity. In Figure 3, a fleet of homogeneous vehicles with a capacity of truck 80 tons is available for the distribution of the product. In this solution, we can see also only one vehicle is used to replenish the product to each customer, nevertheless the tour has increased to three tours compared to the first case. For instance, the vehicle 1 makes the tour $V_1 = (0-1-10-4-8-0)$, tour $V_2 = (0-6-3-0)$ and tour $V_3 = (0-7-2-9-5-0)$.

From the figure above, we also can evaluate the effect of the vehicle storage capacity restrictions. In this case, capacities of 80 tons and 100 ton are used for delivering the product to each of customers. The vehicle capacity factor is used to show that our solution approach not only helps to decide on the fleet size, but can also be used to select the most appropriate vehicle type for a particular problem instance.

For the size of 10 customers, the result shows that the average total cost rate is RM359.50 when using a small vehicle of 80 tons and RM351.50 when using a larger vehicle of 100 tons. Therefore, we can realize that the smaller the delivery quantities to each of the customers are, the less customers are replenished per tour and more tours are made.

5. Conclusion

In this paper, we investigated the single-period deterministic inventory routing problem (SP-DIRP) in which a single warehouse is distributing a single product to a set of customers consuming it at deterministic demand rates, using a fleet of homogeneous vehicles with finite time horizon.

The objective of this research is to find the optimal quantities to be delivered to each customer, the delivery time, and to design vehicle delivery routes, so that the total inventory and transportation costs are optimized. Then, the SP-DIRP is formulated as a linear mixed-integer program.

Therefore, further research approach needs to be explored in which consist of adapting the existing model and solution to some numerical experiments and to the real life application problems, including a large set of customers. Finally, it is worthwhile to investigate how the approach can be extended from the *single-period* setting to the *multi-period* setting which are deterministic and stochastic cases. We will be concentrating this research to the stochastic case in the future research.

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