Deterministic Inventory Routing Problem (DIRP): A Literature Review

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***Abstract*— Vendor managed inventory (VMI) is a management policy in which the supplier implements the responsibility of maintaining the inventory at the retailers to confirm that they will not run out of stock. Under the VMI policy, the supplier takes responsibility for managing the retailers. Moreover, delivery times to the retailers are no longer set by responding to retailers' orders, instead the supplier establishes when each delivery takes place. Therefore, this study reviewed the main publications on single item (vehicle) related to deterministic inventory routing problem (DIRP). Under this approach, the order delivery is in congruence with deterministic demand rates. The objective of DIRP is to resolve a flow strategy that can minimise operational and inventory holding costs without having a stock-out according to a retailer planned schedule. In this study, the focus in on the single-warehouse, multiple-retailer vendor managed inventory (SWMR-VMI), where all retailers face a deterministic and constant demand rate.**

Keywords*—Vendor Managed Inventory, Inventory Routing Problem, Deterministic Demands*

1. Introduction

The inventory routing problem (IRP) is one of the challenging optimisation problems in logistics and supply chain management. It focuses on optimally integrating inventory control and vehicle routing operations in a supply network. Its main goal is to determine an optimal distribution policy, consisting of a set of vehicle routes, delivery quantities needed, and delivery times that can minimise the total inventory holding and transportation costs. Implementing policies such as vendor managed inventory (VMI) has proven to considerably improve the overall performance of a supply network. However, this does not guarantee that implementing VMI elements such as visibility of demand, inventory location, and replenishment decision will impact or improve VMI performance (service and cost) [1].

More specifically, previous studies focused on the single-warehouse, multiple-retailer vendor managed inventory (SWMR-VMI), in cases where all retailers face a deterministic, constant demand rate. However, in real-world issues, demand rates are not usually constant and are normally stochastic. Instead of going further into the stochastic case, the focus will be on the multi-period deterministic inventory routing problem (MP-DIRP), where retailers consume the product at a constant demand rate. Therefore, this paper shall elaborate in detail some publications on the deterministic inventory routing problem (DIRP).

2. Literature Review

In this section, we present the literature review of several variants of the IRP. This paper also includes the review of relevant literature on the problem arising in inventory management, as long as it is connected and relevant to IRP. Some methods that have been used in vehicle routing are also described, which could also be applied in the IRP.

Logistics is a very challenging area. From a cost centre, logistics is now seen as a value adding centre, controlling the delivery process and inventory, and managing the demand. IRP is a viable alternative that can be employed. A classification of IRP models by Andersson et al. (2010), is shown in Table 1 below.

Table 1. Variant Classification of IRP

Criteria	Possible options		
Time	Finite	Infinite	
Demand	Deterministic	Stochastic	Dynamic
Structure	One-to-one	One-to-many	Many-to-many
Routing	Direct	Multiple	Continuous
Inventory policy	Lost sales	Back-order	Non-negative
Fleet composition	Homogeneous	Heterogeneous	
Fleet size	Single	Multiple	Unconstrained

Source: Adapted from Andersson et al. [2]

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Table 1 refers to the options for IRP. For the first criteria, time can be a finite or an infinite horizontal planning period. The demand from retailers is the major problem in IRP. It consists of deterministic, stochastic, and dynamic demands. In logistics, the number of supplier and retailers may change, where it could entail one-to-one, one-to-many, and sometimes many-to-many relationships. The routing options can be direct when one vehicle serves only one retailer in a route, multiple when there are several retailers on the same route, and continuous where there is no central depot, just like in maritime applications. An inventory policy is defined as rules to replenish retailers. Through results found in the literature, there are either fixed or uncontrolled policies, such as the just-in-time method. Managing the inventory shows how the model can be determined. If the inventory cannot supply the demand, then back-ordering would occur, which will restock the lack of inventory through newly delivered shipments. If there is no back-order, then the extra demand is considered lost sales and stock-out may occur. Meanwhile, fleet composition and size can be homogeneous or heterogeneous. Homogeneous is when the vehicle delivers the inventory using fixed capacity in a route. Heterogeneous refers to when the delivery use and the number of vehicles available may be fixed at one, fixed at many, or unconstrained.

Table 2. Classification of main papers on single item deterministic IRP

Authors	Year	Time horizon		Routing			Inventory policy			Fleet composition		Fleet size		
		Finite	Infinite	Direct	Multiple	Continuous	Lost sales	Backlogging	Non-negative	Homogeneous	Heterogeneous	Single	Multiple	Unconstrained
Dror et al. [3]	1985	✓			✓		✓			✓			✓	
Dror and Ball [4]	1987	✓			✓		✓			✓			✓	
Anily and Federgruen [5]	1990		✓		✓			✓		✓				✓
Adelman [6]	2004		✓		✓		✓			✓			✓	
Campbell and Savelsbergh [7]	2004	✓			✓		✓			✓			✓	
Moin and Salhi [8]	2007		✓		✓		✓			✓			✓	
Aghezaff [9]	2008	✓		✓			✓			✓		✓		
Savelsbergh and Song [10]	2008	✓				✓	✓			✓			✓	
Bertazzi, et.al [11]	2011	✓		✓					✓	✓		✓		
Rahim, et al. [12]	2016	✓		✓			✓			✓		✓		
Rahim, et al. [13]	2017	✓		✓			✓			✓		✓		
Rahim, et al [14]	2017	✓		✓			✓			✓		✓		

Table 2 above shows the classification on single item deterministic IRP. IRP is the important process in order to prevent the inventory from having stock-outs. There is some earlier paper focusing on IRP as mentioned in Table 2.

Dror et al. [3] studied vehicle scheduling and varied versions of the IRP have been outlined. A large variety of solution approaches had also been proposed to solve these problems. The IRP can be modelled and approached in many different ways depending on the characteristics of its parameters. Different models can be obtained for example, when retailers consume the product at a stable or at a variable rate; when retailer-demands are assumed deterministic or stochastic; when the planning horizon is finite or infinite, and so on.

Dror and Ball [4] decomposed a multi-period IRP into a series of single period problems. They studied the problem of constant demands and then proposed and compared two solution approaches for the resulting single period problem.

Anily and Federgruen [5] broke down the distribution systems with a depot or warehouse, where the demand is assumed constant and deterministic, but the retailer has their own rates. In their paper, the inventory is kept at the retailer. They determine feasible replenishment strategies where the inventory rules and routing patterns are used in minimising the transportation and inventory cost in the systems.

Campbell and Adelman [6] compared stock-out costs with replenishment policies, choosing the one that can maximise the value. A linear programming method is derived from the value function, and its optimal dual prices are used to calculate the optimal policy of the semi-Markov decision process.

Campbell and Savelsbergh [7] also worked on multi-period IRPs where the decisions are executed over a finite horizon. This paper studied on a variation of the IRP that arises when all the decisions are made by the vendor. They employed a two-phase approach based on decomposing the set of decisions.

Moin and Salhi [8] stressed on the supply chain management in inventory routing. The aim was to provide the method used in this area to come up with a very useful model. They sorted the results according to the planning (single period, multi period, and infinite horizon), but classified SIRP separately.

Aghezzaf et al. [9] considered the case of a cyclic IRP where retailer demand rates and travel times are stochastic, but stationary and proposed a model that generates optimal robust distribution plans. All these contributions assumed a stationary demand rate for the product(s).

Savelsbergh and Song [10] discussed in detail the typical IRP from a single facility using a single product with an unlimited supply to distribute to the set of customers. They focused on the inventory routing problem with continuous moves by considering the product to be fixed and at different depots. They also developed the algorithm in solving the low size instance to minimising the cost of transportation and inventory cost.

Bertazzi et al. [11] studied IRP in where the supplier has to serve a set of retailers. They assumed that the maximum inventory level is defined. This paper formulated the stochastic IRP as a dynamic program and solved it by means of a hybrid rolling horizon algorithm. An order-up policy was applied to each retailer. An inventory cost was applied to any positive inventory level and the penalty cost was charged when the inventory level became negative. The problem was to determine a shipping strategy that minimises the expected total cost, sum of the expected total inventory cost, and penalty cost at the retailers in the expected routing cost.

Routing options can be direct distribution, which can be a single client in a route, multiples when there are a few clients being served by one vehicle on a similar route, and continuous in situations where there is no central depot. Inventory policies defined pre-established rules to replenish retailers. The alternatives found in the writing were either unconstrained approach or fixed policies, called the request to level (order-up-to level). Inventory decisions decide how to demonstrate the inventory management. Finally, the last two criteria refer to fleet composition and size. The fleet can either be homogeneous or heterogeneous, and the quantity of vehicles accessible might be at one, settled at numerous, or unconstrained. Every last one of the accompanying specified papers exists in some of these classifications, and they will be ordered accordingly.

Rahim et al. [12] discussed the problem of efficiently managing inventory and routing problems in a two-level supply chain system. They assumed that the demand at each retailer is deterministic and the warehouse is implementing a VMI. The objective of their paper was to minimise the inventory and the transportation costs of the customers for a two-level supply chain. The problem was to determine the delivery quantities, delivery times, and routes to the retailers for the single-period deterministic inventory routing problem (SP-DIRP) system. As a result, a linear mixed-integer program was developed for the solutions of the SP-DIRP problem.

Rahim et al. [13] focused more on the single-warehouse, multiple-retailer vendor managed inventory (SWMR-VMI), in which all retailers face a deterministic and constant demand rate. They considered a two-stage supply system where a supplier serves a set of retailers from a single warehouse using a fleet of vehicles which has limited capacity. The aims were to minimise the overall inventory and transportation costs of the SWMR system, under a VMI policy. They proposed a two-phase optimisation approach for coordinating the shipments in this VMI system. The proposed solutions for the SWMR-VMI problem assumed that retailer demand rates were constant.

The latest version of the research was also studied by Rahim et al. [14]. Their paper investigated the deterministic IRP model for the single period focus on a finite time horizon. They assumed that supply chain has two stages under the VMI policy. In their paper, they assumed that the demand at each retailer is stationary and the warehouse is implementing the VMI. The aim was to minimise the inventory and transportation cost for the two-stage supply chain system.

The purpose of this section was to develop a comprehensive review of the related IRP literature. It contributes to the latest knowledge by giving some new ideas. For the other sections of this paper, some examples that can be used in solving the IRP problem are presented.

3. Theoretical Study

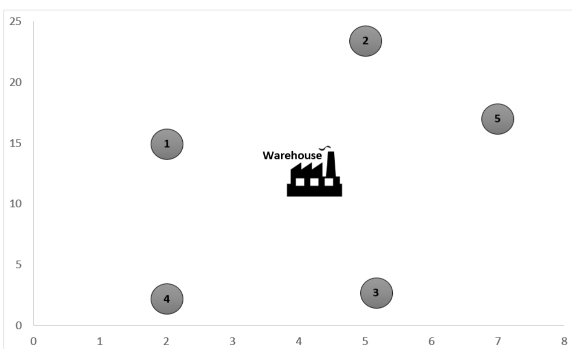
This section contains the initial study that covers problem definition, literature review which describes the terminology, and previous studies related to the multi-period deterministic inventory routing problem (MP-DIRP) model.

As mentioned above, the MP-DIRP discussed in this paper, consists of a single distribution centre using a fleet of homogeneous vehicles to distribute a single product to a set of geographically dispersed retailers over a given planning horizon. It is assumed that retailer demand rates are stationary and that travel times are constant over time. The objective of this MP-DIRP is to determine optimal quantities to be delivered to the retailers, delivery time, and vehicle delivery routes, so that the total distribution and inventory costs are minimised, while preventing stock-outs from occurring at all retailers and assuring some predetermined service level during the entire planning horizon.

4. Example of Planning Distribution Process

In this paper, an example case for the inventory routing problem (IRP) is presented to illustrate the behaviour of the routing model using a fleet of vehicles. Figure 1 below shows that a warehouse will distribute to five retailer centres. The numbered dot represents the place for retailers.

Figure 1. Example of retailer locations around the warehouse



For example, the retailers are placed around the warehouse with coordinates (x, y). It is assumed that the demand rates are constant with a fleet of vehicles available in order to distribute the products.

Figure 2. Distribution planning routes with vehicle capacity 50 tons

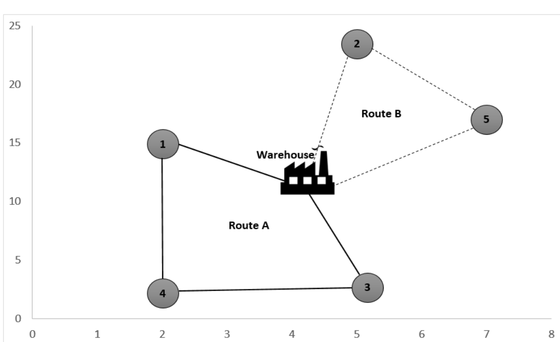


Figure 2 shows the following routes that should be taken by the vehicle to transport the order to each retailer, with 0 representing the warehouse. The figure shows that the vehicle needs to decide which route is the best in minimising the transportation and inventory cost. In that case, it can be observed that the vehicle should take two routes to distribute the order. For route A, the vehicle should make A = (0-1-4-3-0) and for route B = (0-2-5-0). This is how the distribution planning works in order to keep the operations running smoothly.

Figure 3. Distribution panning routes with vehicle capacity 30 tons

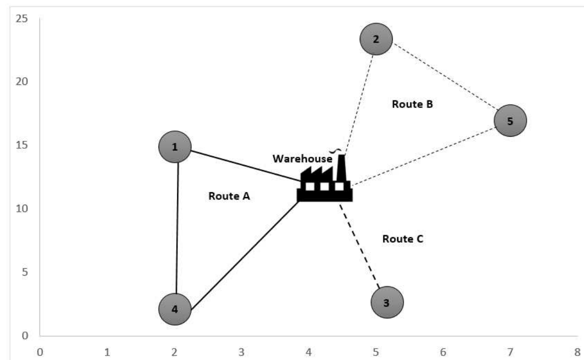


Figure 3 shows when the vehicle capacity is less compared to Figure 2, which is 30 tons. The vehicle needs to serve five retailers, but the inventory carried is not enough. In that case, the route has to extend to become three routes in order to fulfil the requirements. The vehicle has to take route A = (0-1-4-0), route B = (0-2-5-0), and route C = (0-3-0).

From the examples above, the effect of the vehicle storage capacity has a significant impact. Figure 2 and Figure 3 show that the capacity of 50 and 30 tons are used in order to serve each retailer, respectively. In this case, it does not only show that the vehicle capacity is important, but also the most appropriate vehicle needs to be selected in order to reduce the overall cost. For example, the total average cost for the 50 tons is RM200 and the total amount for 30 tons is RM150. Therefore, it can be assumed that the smaller the capacity, the lesser retailers are served, and the more routes are required.

5. Conclusion

In this paper, the deterministic inventory routing problem (DIRP) was examined in which a single warehouse distributes a single product to retailers, focusing on stationary demand rates, utilising a fleet of homogeneous vehicles over a given limited time horizon. The goal is to decide ideal amounts to be conveyed to the retailers, delivery time, and design vehicle delivery routes, so that the total distribution and inventory costs are minimised, while some service level quality is ensured at every retailer within every time deadline of the planning horizon.

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