Parametric Approaches for Spare Parts Demand

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***Abstract*— Forecasting of the spare parts needs is an important operational question. The problem in the needs forecast is that the demand is intermittent. Besides, in stock management, there are several forecasting tools based on demand history such as linear regression, basic and modified Croston methods, simple and weighted moving average, exponential smoothing, and finally the bootstrap method.**

The aim of this paper is to deal the bootstrap method through three sections: literature review, the procedure of the method and last section dedicated to the application of this method in comparison with the simple exponential smoothing and with hybrid method where the last comparison have never been made before.

***Keywords*—** *spare parts; forecast; Resampling; request; Markov chain, hybrid method.*

1. Introduction

Spare parts are goods to replace a corresponding good, either a defective or degraded part of an asset in operation. Besides, the term spare part refers to several types of goods ranging from a very simple elementary component in order to complete subsets or devices. Hence, spare parts widely used in many industries, due to their main in this process. Moreover, to have a sufficient stock of components in dependency with the manufacturer's specifications allowing them to be replaced in minimum time later.

The needs prediction is a fundamental aspect of the management of a supply chain and stock as well as an essential operational issue. The problem confronted in the forecasting of needs is that the intermittent demand is very difficult to predict. Furthermore, this problem is a consequence of the intermittent demand that contains a proportion of zero values with non-zero values randomly, and its models are frequent among spare parts. Besides, the intermittent items dominate inventory of repair parts those are multiple at risk of obsolescence.

In stock management, there are numerous forecasting mechanisms based on request history, such as linear regression, basic and modified Croston methods, simple and weighted moving average, exponential smoothing, and finally the bootstrap method. Nevertheless, the latter method consists in re-sampling an initial sample chosen from the initial population a lot of times and gives better results than the exponential smoothing and the Croston base method [1, 2].

In this paper, we will treat the bootstrap method. Moreover, The first section is divided into two parts where the first part is a state of the art that gives a good idea about the application of the method in the management of spare parts, and the second presents its procedure used in the forecast of the demand based on the Markov chain. Finally, we will try to apply and compare it with the simple exponential smoothing and the hybrid method in the second part.

2. Bootstrap method

2.1. State of the art

The bootstrap method is a statistical resampling method that uses, in general, the data history and demand history in the case of stock management to predict the desired parameter. The classic bootstrap involves consecutive sampling, with replacement, from a set of available data, to construct an empirical distribution of the data concerned. A large number of replications (e.g. 10,000) are generally used, and although this procedure is demanding in terms of calculation [3].

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The samples, obtained through bootstrap, may be different from each other in the population and are used to construct a histogram of the distribution of inventory requests during the delay. Statistics such as the mean and variance of demand in realization time are calculated directly from the histogram rather than deduced from a theoretical distribution. Further, the bootstrapping is now relatively easy to apply and gives the recent progress in the calculation [3].

The bootstrap method is a very practical statistical inference technique. Nevertheless, the simulation of this method and its types could be parametric, a nonparametric and semi-parametric. Furthermore, the difference between parametric and non-parametric bootstrap is that in the first one, a parametric model is assumed for the data [4]. Besides, other researchers such as the article [5], the authors used the bootstrap method to determine the total demand distribution and establish control levels with a target fill rate.

In order to predict the cumulative distribution of the intermittent demand which is characteristic for service parts and capital goods inventories, the authors have developed in the reference [2] a patented algorithm, shown that the bootstrap method produces more accurate forecasts. Accurate distribution of demand over a fixed delivery time as the exponential smoothing and the Croston method. Alternatively, the bootstrap method was the most accurate method of prediction of the three methods. Despite its ability to provide estimates and clarification of average demand per period, Croston's method was slightly less accurate and unfavorable.

According to the results of the paper [6], the bootstrap method works better with randomly generated data sets, where there is a large amount of historical (simulated) data to generate the distribution. On the other hand, with the actual industrial data sets, the parametric method have a better performance than the bootstrap method. According to the same reference, computational experiments show that the improved bootstrap method gives better results. The key element of the newly proposed bootstrap method is the use of a Markov chain with a two states.

The parametric methods used in [6, 7] are the basic Croston method and simple exponential smoothing. According to the reference [6], the parametric methods are simpler, and the simplest method of all is simple exponential smoothing. Thus, the parametric methods require less computing power, which is important when the demand for a large number of stock units to stocks is expected. Also, they require less specialized knowledge and are more transparent and resistant to potentially harmful interventions.

An advanced form of bootstrap cited in article [7] captures the autocorrelation between the realizations of the demand and has advantages, notably the possibility of simulating the demand values that have not appeared in the history.

Thus, the bootstrap method can be combined with other tools. For example, a hybrid approach is proposed in [8] for the probabilistic forecast of the price of electricity. It is based on neural networks, the bootstrap method that is used to quantify the uncertainties to the model specification and a maximum likelihood estimate that is applied to estimate the noise of the residuals. This approach can be more than a hundred times faster than the traditional approach.

In reference [9], to determine the control point s in a system (s, Q) for the desired service levels, Bookbinder and Lordahl have found that it is best to use the bootstrap procedure in any situation through discussion in terms of cost model on the relative performance of bootstrap and normal approaches. They performed variance analyzes to compare estimated replenishment points with different levels of distribution, coefficient of variation, sample size and service level.

According to the article [3], the bootstrap approach has received considerable attention in terms of nonparametric prediction in the academic literature. An important underlying assumption in such applications is that the past behavior of the data relates equally to the future. Thus, there is the possibility of combining a Markov process and bootstrapping in a heuristic method to simulate an entire distribution for the demand. Hence, to determine the appropriate values for the inventory control parameters, Snyder proposes a parametric bootstrap method in his paper [10] that integrates the demand forecast with inventory control.

Moreover, Bacchetti cited in his article [11] that the bootstrap method does not require a hypothesis on the distribution of demand. Thus, it represents an interesting alternative to Croston variants, especially in cases where the small data history limits the reliability of time series methods.

Finally, it can be said that the bootstrap approach using the Markov chain is the best method for forecasting the intermittent demand for spare parts.

2.2. Procedure

According to the literature review, there is a variation in the use of the bootstrap method. Although, we have opted to show the steps of the bootstrap method based on the Markov chain. This method gives better results compared to the traditional methods. It contains 8 steps. Besides, according to [1, 2], the concise summary of the steps is represented as follows:

Step 0: Obtain request history in the selected time buckets.

Step 1: Estimate the transition probabilities for a 2-state Markov model (one state for null requests and another state for non-zero requests).

Step 2: Conditional to the last observed demand, use the Markov model to generate a sequence of periods with zero or non-zero values over the forecast horizon under consideration.

Step 3: At each non-zero period, give a randomly selected demand value in the set of non-zero requests in the history.

Step 4: Add a random variation to non-zero values.

Step 5: Add up all forecasts over the time horizon to obtain the expected value of the application.

Step 6: Repeat steps 2 through 5 a large number of times.

Step 7: Classify the requests obtained and generate a demand distribution over the time horizon. The mathematical expectation of this distribution is the expected average demand.

Hence, the bootstrap approach is handled with a library in language R and another in Matlab. Therefore, we will try to apply the steps of the bootstrap approach cited in this paragraph to estimate the spare parts demand. Hence, this application is the objective of the following paragraph.

3. Application

3.1. Bootstrap method

In this section, we will try to summarize the application of the bootstrap approach already mentioned in RStudio software, in order to estimate the spare parts demand for a given horizon.

We have worked with a request history that takes random values between 0 and 5 as a maximum value. In other words, the history contains 6 states of demand value: 0, 1, 2, 3, 4, 5. These states have been transformed into two states: zero state for null value and non-zero state for demand values from 1 to 5. A horizon of 20 periods was chosen as a length of the generated sequence using the Markov model.

The expected average demand is determined using the distribution generated by the expected demand values over the horizon. These values, shown in Figure 1, are constructed by the sum of the forecasts during the periods of the horizon and the repetition of the forecasting steps several times. We chose to repeat the resampling 1500 times.

Finally, the expected average demand obtained at the end of the application is 56 pieces during the considered horizon. Thus, we have been observed that the density of demand during the horizon under consideration follows a normal distribution, whose mean and standard deviation are the mean and the standard deviation of the results obtained, as shown in Figure 2, and the re-sampling repeat number increases more than the demand density function approximates the associated normal distribution.

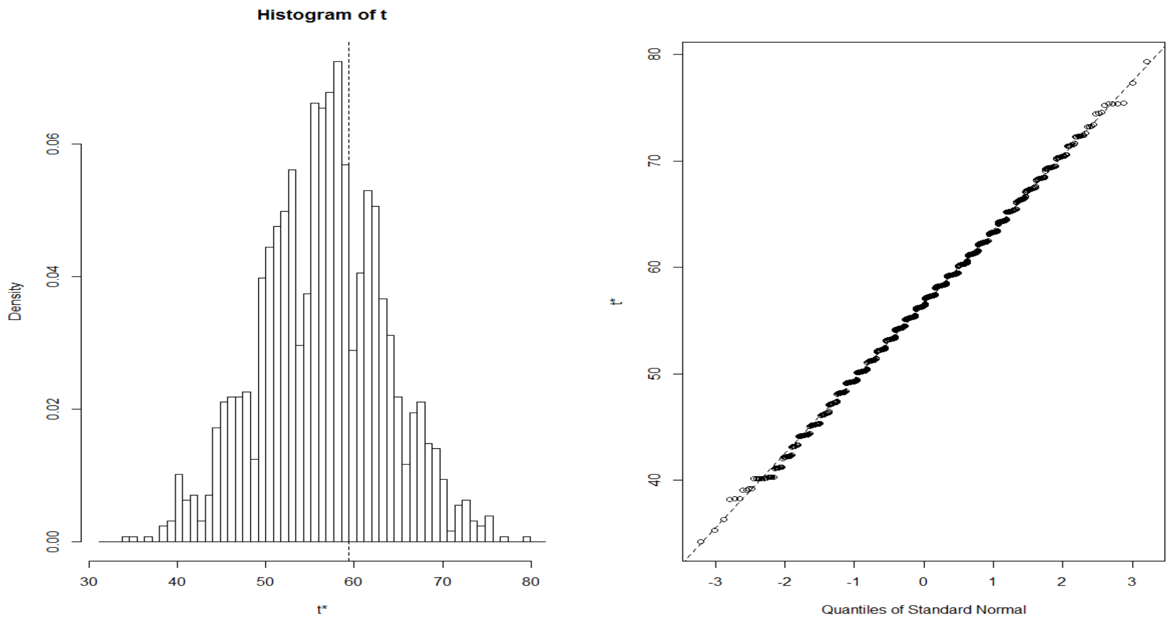

Figure 1. Results obtained by the bootstrap approach

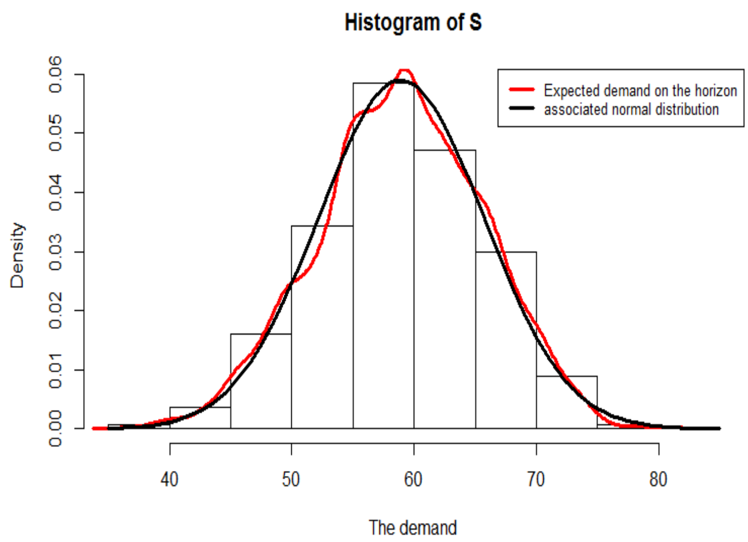

Figure 2. The demand distribution over the horizon

The 95% confidence interval is calculated using four methods, the results are summarized in Table 1. The methods used are defined as follows:

- ✓ Ordinary method: Interval matrix calculated using the normal approximation. It will have 3 columns, the first being the level and the other two being the upper and lower ends of the intervals.
- ✓ Basic method: the intervals calculated using the basic bootstrap method.
- ✓ Percentile method: Percentile intervals use only ordered bootstrap values corresponding to confidence interval percentiles.
- ✓ Bias Corrected percentile method BCa: Intervals are calculated using the adjusted bootstrap percentile and the most accurate adjusted intervals.

From Table 1, it has been observed that the confidence intervals obtained through the methods: normal, basic and percentile have almost the same length. On the other hand, the BCa method gave an interval with a length less than the others.

Table 1. The confidence interval calculated using different methods

Level	Ordinary	Basic
95%	[40.43, 67.45]	[40.27, 67.39]
	Percentile	**BCa**
	[43.23, 70.35]	[41.18, 67.44]

Finally, we summarize what is obtained during the application as follows: from a demand history, which has 6 states between 0 and 5, we have obtained 56 pieces as an expected average demand value of has been over the horizon of 20 periods with a confidence interval [41.18, 67.44].

As regards the rate of zero values in the demand history, it influences the transition matrix and the estimated demand. In other ways, more than this rate increases, so the total demand during the horizon in the history and the estimated demand decrease. In our case, the last observed demand (zero or non-zero) did not greatly influence the estimated demand during the horizon, it is perhaps because of the low proportion of zero values.

3.2. Exponential smoothing

The simple exponential smoothing is written:

$$S_t = \alpha X_t + (1-\alpha)S_{t-1} \quad (1)$$

With α smoothing parameter, X_t is the observed value of the null and non-zero demand, $S_t$is the smoothed mean as a forecast for the next period.

Simple exponential smoothing is widely used to predict the intermittent demand. The method has significant limitations and weighs more heavily on recent data, which can produce biased forecasts. We have applied this method to the same history used in the bootstrap method. We have varied the smoothing parameter to predict demand over the horizon of 20 periods. The results obtained are presented in the following figure:

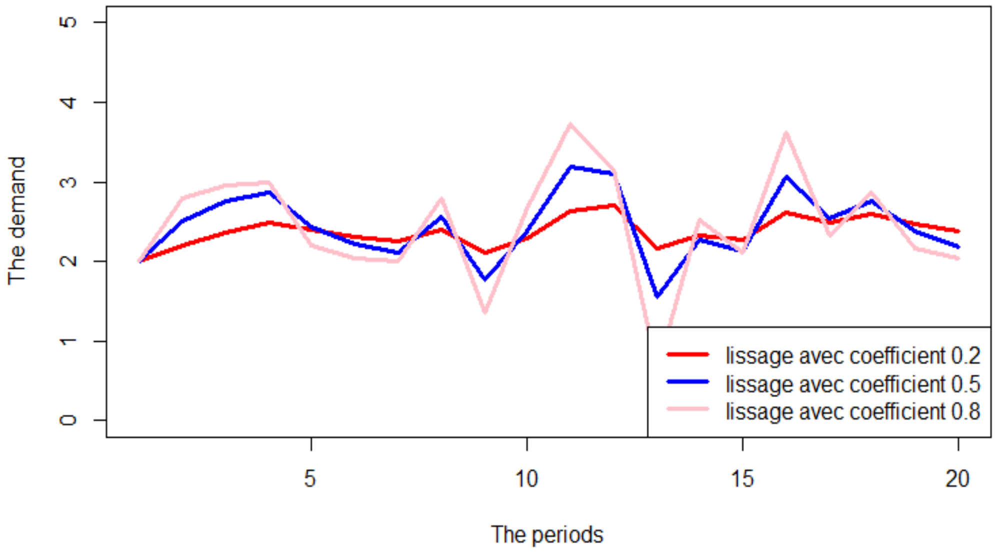

Figure 3. Exponential smoothing with 3 coefficients

From this figure, we have concluded that the smoothing parameter plays an important role in smoothing. When the parameter is close to 1, it gives importance to observed values in the history. In other cases, we will have null values of the demand, according to their frequency in the history. On the other hand, a smaller smoothing parameter, which gives importance to the smoothed means, makes it possible to minimize the frequency of the zero values.

3.3. The hybridization method

In order to improve the spare parts forecasting process, Lazrak proposed in his paper [12] the hybrid methods that allow a combination of the characteristics of the parametric methods used in the prediction. So, this combination based on his work helps strengthen the best method by the performance of others since there is not one method that completely outperforms the others.

Lazrak proposed in [12] and illustrated two methods of hybridization of N methods by simple examples. He noted that there is a decrease in underestimation or overestimate trends and an improvement in the quality of the results. This is done through an analysis of the results and an evaluation of the methods compared to the parametric forecasting methods considered in his work, for example MSE (Mean Squared Error). Thus, he noticed that the hybrid method 2 has adapted to the variations of the demand and is more efficient than the hybrid method 1 [12]. For that, we chose the hybrid method to apply it in our case in order to compare it with the bootstrap method.

3.3.1. *Hybrid method 2*

This method consists of defining transition probabilities from one method to another, taking into account for each period and the previous period, thus calculating the transition frequency from one method to another. According to [12], the prediction by hybrid method 2 is calculated by equation (2) based on the best-evaluated method j over period H.

$$F_{hyb2}(H+t) = \sum_{i=1}^{N} P_{ji} F_i^{(H)}(H+t) \quad (2)$$

H: Number of periods of history used.

N: Number of prediction methods to hybridize.

P_{ji} : Transition probability of methods j to i.

$F_i^{(H)}(H+t)$: Forecast made on date H for the forecasting method i.

In this paper, we will apply the hybrid method in our case. Then, we will hybridize two methods: the Croston SBA method and double exponential smoothing during the horizon of 20 periods that is similar to the size of the history used. Table 2 summarizes the results of the hybrid method 2.

Table 2. The results of the hybrid method 2

Periods	1	2	3	4	5	6	7	8	9	10
P_{11}	1	1	1	1	0.9	0.833	0.857	0.875	0.833	0.8
P_{12}	0	0	0	0	0.1	0.167	0.143	0.125	0.167	0.2
P_{21}	0	0	0	0	0.1	0.167	0.143	0.125	0.167	0.2
P_{22}	1	1	1	1	0.9	0.833	0.857	0.875	0.833	0.8
Previsions	2	2.48	2.76	2.9	1.89	2.12	2.02	2.47	1.31	2.27

Periods	11	12	13	14	15	16	17	18	19	20
P_{11}	0.818	0.833	0.807	0.785	0.8	0.812	0.823	0.833	0.842	0.85
P_{12}	0.182	0.167	0.193	0.215	0.2	0.188	0.177	0.167	0.158	0.15
P_{21}	0.182	0.167	0.193	0.215	0.2	0.188	0.177	0.167	0.158	0.15
P_{22}	0.818	0.833	0.807	0.785	0.8	0.812	0.823	0.833	0.842	0.85
Previsions	3.11	3.05	0.92	1.86	1.7	2.95	2.30	2.145	2.02	1.85

The demand, predicted by the hybrid method 2 during the 20-period horizon, takes a value around 44 parts. We found that the expected demand of the hybrid method 2 during the horizon is close to the lower limits of the confidence intervals found by the bootstrap method.

3.4. Synthesis

Simple exponential smoothing is the simplest method of all the parametric methods and works well. It depends on the selected smoothing parameter. The results obtained by simple exponential smoothing in our case are lower than the bootstrap estimated average demand but they are in the 95% confidence interval.

In this case, the history contains random values with a low proportion of zero values. This proportion influences demand forecasting in both methods. In other ways, when the proportion increases, it affects simple exponential smoothing more than the bootstrap method at the level of values obtained in the two methods. Also, the frequency of the null values generally will be bigger than our case (low proportion) and more precisely in the simple exponential smoothing if we give importance to the history.

The null values of the demand cause the obsolescence risk which is caused by the dead stock. The risk of having a dead stock and a zero demand will push to make an urgent order. So, in this case, the costs which will be increased, are: storage costs including the cost of obsolescence, cost of purchase and cost of risk of shortage. The lead time is the main factor influencing the risk of stock-out, but the case of the dead stock and a zero demand can also cause the shortage.

We noticed that the hybrid method was able to eliminate the null values of the demand. So, this can decrease the risk of obsolescence which causes to have parts obsolete at the moment of intervention and then to have a stock-out of the spare parts. But, in general, the risk of having an overstock exists. Overstocking can cause long-term dead stock and generates additional cost. By calculating one of the statistical indicators mentioned in [12]: Mean Squared Error (MSE) for the hybrid 2 and bootstrap methods, we found: $MSE_{bootstrap} = 0.679$ and $MSE_{hybrid\ 2} = 0.354$. According to this criterion, the quality of the forecasts is slightly improved by the hybrid method compared to the bootstrap forecast.

In summary, the bootstrap method based on the Markov chain is the most accurate method of prediction. It is more efficient than simple exponential smoothing at the level of the minimization of the risk of obsolescence. Thus, it allows to determine the distribution of the demand during the horizon in question.

4. Conclusion and perspectives

In this paper, we have presented the use of the bootstrap method in the literature for demand prediction in stock management. Thus, we have been able to apply the procedure found in the references for the prediction of the intermittent demand using a Markov model. Besides, we have obtained the result at the end of the application where 56 parts as an expected average demand over the horizon of 20 periods with a confidence interval [41.18, 67.44].

Nevertheless, the researchers have been compared the bootstrap method with the two methods: exponential smoothing and the basic Croston method. Hence, in this paper we have compared the bootstrap method with the simple exponential smoothing method. Moreover, the results shown that the bootstrap method gives the better results.

Also, we have compared the results of bootstrap method with the results of hybrid method and we have found that the hybrid method slightly improves the quality of predictions compared to the bootstrap method.

In general, a single indicator is not enough to compare methods. For this purpose, we aim to make a comparative analysis through several indicators, since they have close to the level of the quality of forecasts. In future, we envisage in future work to compare these approaches with other methods.

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