A Fuzzy Economic Order Quantity (EOQ) Model with Consideration of Quality Items, Inspection Errors and Sales Return

Mahdi Fadaei Eshkiki[1], Nima Saberifard[2], Mehdi Ajalli[3]
[1.] Department of Management, Roudbar Branch, Islamic Azad University, Roudbar, Iran

[2.] Department of Management, Rasht Branch, Islamic Azad University, Rasht, Iran
[3]Faculty of Management, University of Tehran, Tehran, Iran
[1] Fadaei@iaurasht.ac.ir
[2] N.saberi@ut.ac.ir
[3] ajalli@ut.ac.ir

Abstract— **In this paper, we develop an economic order quantity model with imperfect quality, inspection errors and sales returns, where upon the arrival of order lot, 100% screening process is performed and the items of imperfect quality are sold as a single batch at a lessen price, prior to receiving the next shipment. The screening process to remove the defective items may involve two types of errors. In this article we extend the Khan et al. (2011) model by considering demand and defective rate in fuzzy sense and also sales return in our model. The objective is to determine the optimal order lot size to maximize the total profit. We use the signed distance, a ranking method for fuzzy numbers, to find the approximate of total profit per unit time in the fuzzy sense. The impact of fuzziness of fraction of defectives and demand rate on optimal solution is showed by numerical example.**

So, this paper suggested a fuzzy model for an inventory problem with imperfect quality items. In this model the defective rate and the annual demand represented by a fuzzy number. The prior researches on fuzzy inventory often employed the centroid method to obtain the approximate of total cost in the fuzzy sense.

Keywords: *Imperfect quality, inventory model, screening errors, signed distance.*

1. Introduction

Ref. [1] deals with quality-related problem, as well as the ambiguous demand. In recent years, several researchers have used the fuzzy sets theory and approach to extend and solve the manufacturing/inventory problems. For example, Ref. [2] and Ref. [3] extended the classical EOQ model by representing the fuzziness of ordering cost and holding cost.

Ref. [4] contemplated human error in inspection. They came up with one of the first inspection schemes with misclassifications for multi-typical critical elements.

Ref. [5] instigated a model that shows a dominant relationship through quality and lot size. He integrated the impact of defective items into the basic EOQ model and instigated the alternative for expending in process quality improvement by means of lessening the probability that the process moves out of control.

Ref. [6] instigate the problems of joint control of manufacturing cycles or manufacturing quantities, and retention by inspections. They answered the problem of simultaneous designation of economic manufacturing quantity (EMQ) and the inspection plan by using an approximation method to the cost function. They also enquire the cost penalties of using the classical EMQ and the effects of different parameters of the system on the value of these penalties.

Ref. [7] extended the EOQ model by adjoining the presumptions that defective items of a known ratio were present in incoming lots and that fixed and variable inspection cost were created in finding and eliminating these defective items. It was also presumed that the supplier shelled out the buyer for any defective items found and eliminated.

Ref. [8] showed an EOQ model with demand-dependent unit manufacturing cost and imperfect manufacturing processes. He formulated the inventory decision problem as a geometric program and solved it to gain closed form optimal solutions.

Ref. [9] has presented the economic order quantity (EOQ) model which is the base of contemporary inventory models. A considerable amount of attempts has been dedicated in the inventory

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management literature. In order to overcome the limiting assumption of the EOQ models, all the efforts were done to expand lot sizing models. Although, some weaknesses have been found in the analysis which have done to submit an economic order quantity. One easy to notice is several unreasonable assumptions which oriented many researchers to investigate the EOQ widely under real-life circumstances. An unrealistic presumption in using the EOQ is that all units manufactured are of good quality. But, it may not always be the case. Due to imperfect manufacturing process, natural disasters, breakage in transit, or for many other reasons, the lot sizes manufactured/received may accommodate some defective items. To capture the real situations better, many researchers contemplate the above scenarios in formulating the manufacturing/inventory models and investigate the impact of imperfect quality on lot sizing policy.

Ref. [10] introduced a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity.

Ref. [11] studied a fuzzy model for inventory with backorder, where the backorder quantity was fuzzified as a triangular fuzzy number.

Ref. [12] fuzzified the demand quantity and production quantity per day, and Ref. [13] Lin and Yao (2000) fuzzified the production quantity in each cycle, demeaning all as the triangular fuzzy numbers.

Ref. [14] investigated the impact of imperfect manufacturing processes on the economic lot sizing policy.

Ref. [15] extended an economic order quantity model in which each ordered lot evolves a stochastic ratio of defective items. They presumed that received items are subject to 100% inspection with no inspection errors and that poor-quality items are retained in stock and sold as a single batch at a lessened price at the end of the 100% inspection process. Such Ref. [15] presumed a model that there was no human error in the screening process.

Ref. [16] corrected an error in their paper.

Ref. [17] showed a simple approach for making allowance the economic manufacturing quantity of an item with imperfect quality. They illustrated that near-optimal results are gained by using simple approach. The model designated in their note is simpler to implement.

Ref. [18] suggested a common inspection scheme for quality assurance of critical multi-typical elements. They in Ref. [4] extended inspection scheme for the case of six types of misclassification errors, where an inspector could categorized an item to be good, rework or scrap.

Ref. [19] contemplates the economic manufacturing quantity model with the rework process of imperfect quality items. He presumed that not all of the defective items are rework able; ratios of them are scrap and are discarded.

Ref. [20] expanded an economic production quantity (EPQ) model where the imperfect items could be sold at a lessen price and the defective items could be either repaired or rejected.

Ref. [21] carried out a sensitivity analysis to contemplate the statistical and economic impact of the many types of misclassification errors on the rendition measures of the inspection plan.

Ref. [22] proposed a profit-maximizing EPQ model that merged both imperfect manufacturing quality and two-way imperfect inspection. They also investigated rework and rescue in the discarding of screened and returned items, and solved the model optimally and dispensed numerical sensitivity analyses to supply important managerial insights into practices.

Ref. [23] contemplated imperfect quality items and shortage backordering, and gained closed-form expressions. The research on jointly defining the inventory lot size and sampling policy has received very limited consideration. The recent work of many researchers contemplated lot-sizing inventory models with imperfect quality items. We mention the reader to Ref. [24] for a detailed review of the extension of the EOQ model with imperfect items.

Ref. [25] contemplated a production-inventory model in an imperfect production process, with time-varying manufacturing rate and demand. The unit manufacturing cost is presumed to be a ramification of the manufacturing fraction and production reliability. The optimal product reliability and manufacturing rate are specified, taking into account the effect of inflation and time value of money.

Ref. [26] intended the economic order and manufacturing quantity with imperfect quality items.

Ref. [27] investigated a joint lot sizing and inspection policy under an EOQ model with stochastic supply, and contemplated acceptance sampling under the average extrovert quality limit constraint.

Other works that contemplate inventory models with imperfect manufacturing processes encompass in Ref. [28] and Ref. [29].

Ref. [30] addressed some errors in paper of Ref. [24]. Ref. [31] extended two economic production quantity models with imperfect manufacturing processes, inspection errors, scheduled backorders and sales returns.

From literature survey, we note that some of the prior researchers presumed that the defective ratio in lot sizes manufactured/received is a fixed constant, while others presumed it as a stochastic variable with a known probability distribution to designate the uncertainty of imperfect quality. In real situations, while we know that the defective fraction may have a little change from one lot to another, nevertheless, it may loss historical data to approximate the probability distribution, basically, for the items dispensed by the new manufacturers/suppliers. As stated in Ref. [32] an effectual way to appear for factors including flexibility, quality of products, raised reply to market demand, and decrease in inventory, which can neither be intended on by crisp values nor stochastic processes, is using linguistic variables or fuzzy numbers. In this paper, we shall then use the fuzzy sets concept first introduced by Ref. [33] suggested the EOQ model in the fuzzy sense, where both order quantity and total demand were fuzzified as the triangular fuzzy numbers. Ref. [34] contemplated a blend inventory model with variable lead-time, where the demand per year was fuzzified as the triangular fuzzy number and as the statistic fuzzy number. Ref. [35] suggested two fuzzy models for an inventory problem with imperfect quality items. In first model, the defective rate is appeared by a fuzzy number, while the demand per year ratio is treated as a fixed constant. In the second model, not only the defective rate but also the ratio of demand per year is considered by a fuzzy number. Ref. [36] dealt with the order inventory problem with shortages and imperfect items. The annual demand and cost parameters are presumed to be trapezoidal fuzzy numbers. Ref. [37] inquired the EOT model with fuzzy arrival rate and fuzzy cost parts.

Ref. [38] Hsu and Hsu (2013a) established a closed-form solution for an EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns, where the customers who return the defective items will receive full price refunds; i.e., the returned items are not replaced with good items.

Ref. [39] extend Hsu and Hsu & apos; s (2013a) work to consider the case that returned items are replaced with good items. A closed-form solution is developed for the optimal order size and the maximum shortage level. Numerical examples are provided to show the differences in the optimal solutions when returned items are replaced, and when they are not.

Ref. [40] propose an inventory model where items are inspected through multiple screening processes on varies quality characteristics before delivery to customers. Each screening process on a single quality characteristic has independent screening rate and defective percentage. Defective items screened out are stored and then returned to supplier. Shortage backordering are also allowed in the model. Two approaches are used to obtain the closed-form optimal order size and the maximum backordering quantity. Numerical examples are also provided to demonstrate the use of the model

Ref. [41] take a step in this line of thought and revisits some economic order quantity (EOQ) models with imperfect quality from a sustainable point of view. First, an EOQ model with imperfect quality items and emission costs, which are the result of warehousing and waste disposal activities, is formulated. Next, the model is extended to account for the situations where the buyer considers different areas for stocking the imperfect and good quality items, learning occurs in imperfect quality and the inspection process at the buyer's end contains error. The developed models are tested numerically and compared to investigate the optimal policies considering emission costs.

Ref. [42] deals with an economic order quantity (EOQ) model in which a certain percentage of a lot size is of imperfect quality products. This percentage follows a uniform distribution function. During the inspection of the total lot-size, a stock-out situation may occur. In a stock-out situation, a partial fraction of the demand is adjusted by partial back ordering and the rest of the demand is considered as a case of lost sales. Also, three special cases of the general model are studied. A suitable numerical example is provided to illustrate the model and the solution procedure. Comparison between the general and special cases are also shown with the help of numerical examples. Sensitivity analysis of the optimal solutions with respect to all individual parameters of the general model is carried out.

Ref. [43] formulate EOQ model as a calculus of variations. This new extended problem is a simple optimal control problem with an unknown initial state. By solving this problem we generalize EOQ formula.

Ref. [44] develops an inventory model for items of imperfect quality with deterioration under trade-credit policies with price dependent demand. Shortages are allowed and fully backlogged. A mathematical model is developed to depict this scenario. The aim of the study is to optimize the optimal order level, backorder level and selling price so as to maximize the retailer's total profit. Findings are validated quantitatively by using numerical analysis. Sensitivity analysis is also performed so as to cater some important decision-making insights.

In this article we extend the Ref. [24] model by considering demand and defective rate in fuzzy sense and also sales return in our model. This paper is organized as follow. In section 2 some definition about fuzzy numbers and fuzzy sets is presented. In section 3 a mathematical model is developed. Section 4 prospers numerical example to show the main aspect of the model. Finally in section 5, the general conclusion of study is presented.

2. Preliminaries

Before we discuss modelling in fuzzy sense, first we introduce the fuzzy numbers and rules of these numbers. Then we instigate the signed distance, a methodology for ranking fuzzy numbers. This method first introduced by Ref. [45]. Most of definitions and rules in this section are derived from Ref. [45].

Definition 1. For $0 \le \alpha \le 1$ the fuzzy set $\tilde{a}_\alpha$ define on $R = (-\infty, \infty)$ is called an α-level fuzzy point if the membership function of $\tilde{a}_\alpha$ is given by

$$\mu_{\tilde{a}_\alpha} = \begin{cases} \alpha, x = a \\ 0, x \ne a \end{cases}$$

Definition 2. The fuzzy set $\tilde{A}$= (a,b,c) where $a < b < c$ and introduce on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{(x-a)}{(b-a)}, a \le x \le b, \\ \dfrac{(c-x)}{(c-b}, b \le x \le c, \\ 0, otherwise \end{cases}$$

Definition 3. For $0 \le \alpha \le 1$ the fuzzy set $\left[a_\alpha, b_\alpha\right]$ define on R is called an α-level fuzzy interval if the membership function of $\left[a_\alpha, b_\alpha\right]$ is given by

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, a \le x \le b \\ 0, otherwise \end{cases}$$

Definition 4. Let $\tilde{B}$ be a fuzzy set on R and $0 \le \alpha \le 1$. The α-cut B(α) of $\tilde{B}$ comprise of points x such that $\mu_{\tilde{B}}(x) \ge \alpha$, that is, $B(\alpha) = \left\{x \mid \mu_B(x) \ge \alpha\right\}$

Definition 5. For each $a, b \in R$, define the signed distance d^* of a,b by $d^*(a,b) = a - b$. Since $d^*(a,0) = 0$ then $d^*(a,b) = d^*(a,0) - d^*(b,0)$. $d^*(a,0)$ Defines the signed distance of a from 0. If $a > 0$, then $d^*(a,0) = a$ connotes that a is on the right-hand side of 0 with distance a. if $a < 0$ then $d^*(a,0) = a$ connotes that a is on the left-hand side of 0 with distance $-d^*(a,0) = -a$. Thus, we have the following way to define the rank of any two numbers on R. for each $a, b \in R$:

$$d^*(a,\mathrm{b}) > 0, \mathrm{iff}, \mathrm{d}^*(a,0) > d^*(b,0), iff\ , a > b$$

$$d^*(a,\mathrm{b}) < 0, \mathrm{iff}, \mathrm{d}^*(a,0) < d^*(b,0), iff\ , a < b$$

$$d^*(a,\mathrm{b}) = 0, \mathrm{iff}, \mathrm{d}^*(a,0) = d^*(b,0), iff\ , a = b$$

Let Ω be the family of all fuzzy sets $\tilde{B}$ define on R with which the α-cut $B(\alpha) = \left[B_L(\alpha), B_U(\alpha)\right]$ exists for every $\alpha \in [0,1]$, and both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on $\alpha \in [0,1]$. Then for any $\tilde{B} \in \Omega$, we have:

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \left[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha\right]$$

From definition 5, the signed distance of two end points, $B_L(\alpha)$ and $B_U(\alpha)$, of the α-cut $B_\alpha = \left[B_L(\alpha), B_U(\alpha)\right]$ of $\tilde{B}$ to the origin 0 is

$d_0(B_L(\alpha),0) = B_L(\alpha)$ and $d_0(B_U(\alpha),0) = B_U(\alpha)$, respectively. Their average, $(B_L(\alpha)+B_U(\alpha))/2$, is taken as the signed distance of α-cut $(B_L(\alpha),B_U(\alpha))$ to 0. That is, the signed distance of interval $[B_L(\alpha),B_U(\alpha)]$ to 0 is defined as:

$$d_0([B_L(\alpha),B_U(\alpha)],0) = [d_0(B_L(\alpha),0)+\mathrm{d}_0(B_U(\alpha),0)]/2 = (B_L(\alpha)+B_U(\alpha))/2$$

Furthermore, for every $a \in [0,1]$, there is a one-to-one mapping between the α-level fuzzy interval $[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$, and the real interval $[B_L(\alpha),B_U(\alpha)]$, that is, the following correspondence is on-to-one mapping:

$$[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \leftrightarrow [B_L(\alpha),B_U(\alpha)]$$

Also, the 1-level fuzzy point $\tilde{0}$ is mapping to the real number 0. Therefore, the signed distance of $[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ to $\tilde{0}$ can be defined as:

$$d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha],0) = d([B_L(\alpha),B_U(\alpha)],0) = (B_L(\alpha)+B_U(\alpha))/2$$

Moreover, for $\tilde{B} \in \Omega$, since the above function is continuous on $0 \le \alpha \le 1$, we can use the integration to gain the mean value of the signed distance as follows:

$$\int_0^1 d([B_L(\alpha)_\alpha, \mathrm{B}_U(\alpha)_\alpha],0)d\alpha = \frac{1}{2}\int_0^1 (B_L(\alpha)+B_U(\alpha))d\alpha$$

According to definition, we gain the following property:

Property 1. For the triangular fuzzy number $\tilde{A}$=(a,b,c), the α-cut of $\tilde{A}$ is $A(\alpha) = [A_L(\alpha),A_U(\alpha)]$, $\alpha \in [0,1]$, where $A_L(\alpha) = a+(b-a)\alpha$ and $A_U(\alpha) = c-(c-b)\alpha$. The signed distance of $\tilde{A}$ to $\tilde{0}$ is:

$$d(A,0) = \frac{1}{4}(a+2b+c)$$

3. Notation, assumption and model delineation

We contemplate a buyer with an annual demand rate. The buyer places an order of size Q for each procurement cycle. It is presumed that the vendor's manufacturing processes are imperfect and may manufacture defective items. Once the buyer captured the lot, a 100% screening process is conducted. The screening process and the demand process simultaneously. The screening process is also imperfect in that an inspector may incorrectly classify a non-defective item as defective, or a defective item as non-defective. Items that are defective or returned from the market are sold at a lessen price at the end of each cycle. The optimum operating inventory doctrine is captured by trading off total revenues per unit time, procurement cost per unit time, the inventory carrying cost per unit time and item screening cost per unit time so that their sum will be maximum.

Q order lot size

K the ordering cost per order

S the unit selling price of a non-defective item

v the unit selling price of a defective item

C the unit variable cost

d the unit screening cost

x the screening rate

h the holding cost per unit per unit time

T the cycle length

C_r the cost of rejecting a non-defective item

C_a the cost of accepting a defective item

B_1 the number of items that are classified as a defective in one cycle

B_2 the number of defective items that are returned from the market in one cycle

$\tilde{p}$ the rate of defective item in Q, which is a triangular fuzzy number

$\tilde{D}$ the number of demanded per year, which is a triangular fuzzy number

By definition, the number of items that are classified as defective consist of those that are non-defective, $Q(1-p)$, and incorrectly classified as defective (with probability m_1), and those that are defective, Qp, and classified as defective (with probability $1-m_2$); therefore we have :

$$B_1 = Q(1-p)m_1 + Qp(1-m_2)$$

The number of defective items returned from the market are those that are defective, Qp, and incorrectly classified as non-defective (with probability m_2); so:

$$B_2 = Qpm_2$$

Now define TR (Q) and TC (Q) as the total revenue and the total cost per cycle, respectively. TR (Q) is the sum of total sales volume of good quality and imperfect quality items and is given as:

$$TR(Q) = sQ(1-p)(1-m_1) + vQ(1-p)m_1 + vQp$$

TC (Q) is the sum of procurement cost per cycle, screening cost per cycle, rejecting a non-defective item cost per cycle, accepting a defective item cost per cycle and holding cost.

The cost of rejecting a non-defective item and the cost of accepting a defective item per cycle is given as:

$$C_r Q(1-p)m_1 + C_a Qpm_2$$

The holding cost per cycle is given as:

$$C_h = h(\frac{Q^2((1-p)^2(1-m_1)^2 + 3(p-p^2)(1-m_1)m_2 + 2p^2m_2^2)}{2D} + \frac{Q^2}{x})$$

Then TC (Q) is given as:

$$TC(Q) = K + cQ + dQ + C_r Q(1-p)m_1 + C_a Qpm_2 +$$

$$h(\frac{Q^2((1-p)^2(1-m_1)^2 + 3(p-p^2)(1-m_1)m_2 + 2p^2m_2^2)}{2D} + \frac{Q^2}{x})$$

The total profit per cycle is the total revenue per cycle less the total cost per cycle, TP (Q) = TR (Q) – TC (Q), and it is given as:

$$TP(Q) = sQ(1-p)(1-m_1) + vQ(1-p)\mathrm{m}_1 + vQp -$$

$$(K + cQ + dQ + C_r Q(1-p)m_1 + C_a Qpm_2 +$$

$$h(\frac{Q^2((1-p)^2(1-m_1)^2 + 3(p-p^2)(1-m_1)m_2 + 2p^2m_2^2)}{2D})$$

To forbear shortages, we use the order overlapping scheme as proposed by Ref. [46] to extend an EOQ model for extreme case. That is an order is placed when the inventory position is just enough to cover the demand during the lead-time plus the demand during the screening process. The demand during the screening process of an order is faced from the inventory of the previous order. In the other words, a safety stock of $Dt = D(Q/x)$ units is kept in inventory.

We fuzzify D to be a triangular fuzzy number, $\tilde{D}$= $(D-\Delta_1, D, D+\Delta_2)$, where $0 < \Delta_1 \le D$, and $0 < \Delta_2 \le 1-D$. And p to be a triangular fuzzy number, $\tilde{p}$= $(p-\Delta_3, p, p+\Delta_4)$, where $0 < \Delta_3 \le p$ and $0 < \Delta_4 \le 1-p$. These parameters are specified by decision makers. therefore, the total profit per cycle is a fuzzy value also, which is represent as:

$$\tilde{\tilde{TP}}(Q) = sQ(1-\tilde{\tilde{p}})(1-m_1) + vQ(1-\tilde{\tilde{p}})m_1 + vQ\tilde{\tilde{p}} -$$

$$(K + cQ + dQ + C_r Qm_1(1-\tilde{\tilde{p}}) + C_a Qm_2\tilde{\tilde{p}} +$$

$$h(\frac{Q^2((1-\tilde{\tilde{p}})^2(1-m_1)^2 + 3(\tilde{\tilde{p}}-\tilde{\tilde{p}}^2)(1-m_1)m_2 + 2m_2^2\tilde{\tilde{p}})}{2\tilde{\tilde{D}}}$$

Now we defuzzify $\tilde{\tilde{TP}}(Q)$, using the signed distance method, from property 1, the signed distance of $\tilde{\tilde{TP}}$ to $\tilde{\tilde{0}}$ is given by:

$$d(\overset{\approx}{TP},\overset{\approx}{0}) = sQ(1-m_1)(1-d(\overset{\approx}{p},\overset{\approx}{0})) + vQm_1(1-d(\overset{\approx}{p},\overset{\approx}{0})) + vQd(\overset{\approx}{p},\overset{\approx}{0}) -$$
$$(K + cQ + dQ + C_rQm_1(1-d(\overset{\approx}{p},\overset{\approx}{0})) + C_aQm_2d(\overset{\approx}{p},\overset{\approx}{0}) +$$

$$h(\frac{Q^2((1-m_1)^2(1-d(\overset{\approx}{p},\overset{\approx}{0}))^2 + 3(1-m_1)m_2(d(\overset{\approx}{p},\overset{\approx}{0}) - d(\overset{\approx}{p}^2,\overset{\approx}{0})) + 2m_2^2d(\overset{\approx}{p},\overset{\approx}{0})}{2d(\overset{\approx}{D},\overset{\approx}{0})})$$

Where $d(\overset{\approx}{D},\overset{\approx}{0})$, $d(\overset{\approx}{p},\overset{\approx}{0})$ and $d(\overset{\approx}{p}^2,\overset{\approx}{0})$ are measured as follow:

$$d(\overset{\approx}{D},\overset{\approx}{0}) = \frac{1}{4}[(D-\Delta_1) + 2D + (D+\Delta_2)] = D + \frac{1}{4}(\Delta_2 - \Delta_1)$$

$$d(\overset{\approx}{p},\overset{\approx}{0}) = \frac{1}{4}[(p-\Delta_3) + 2p + (p+\Delta_4)] = p + \frac{1}{4}(\Delta_3 - \Delta_4)$$

$$d(\overset{\approx}{p}^2,\overset{\approx}{0}) = \frac{1}{2}\int_0^1 (p_L^2(\alpha) + p_U^2(\alpha))d\alpha = p^2 + \frac{\Delta_3^2 + \Delta_4^2}{6} + \frac{p}{2}(\Delta_4 - \Delta_3)$$

$\overset{\approx}{TP}(Q)$ is attended as the approximate of total profit per unit time in the fuzzy sense. Now we specify the optimal order lot size Q^* to maximize the total profit function $TP(Q)$. It can be illustrated that $TP^*(Q)$ is concave in Q, by solving the first order necessary condition, $\frac{d(TP^*(Q))}{dQ} = 0$, we obtain the optimal lot size.

$$Q^* = \sqrt{\frac{2K(D + \frac{1}{4}(\Delta_2 - \Delta_1))}{AB + C}}$$

Where:

A=
$$h((1-(p+\frac{1}{4}(\Delta_4 - \Delta_3))^2(1-m_1)^2 + 3(1-m_1)m_2$$

B=
$$\left[(p + \frac{1}{4}(\Delta_4 - \Delta_3) - (p^2 + \frac{\Delta_3^2 + \Delta_4^2}{6} + \frac{p}{2}(\Delta_4 - \Delta_3))\right]$$

C=
$$2m_2^2(p^2 + \frac{\Delta_3^2 + \Delta_4^2}{6} + \frac{p}{2}(\Delta_4 - \Delta_3)) + 2(D + \frac{1}{4}(\Delta_2 - \Delta_1))/x$$

Note that if $\Delta_1 = \Delta_2 = 0$, it is clear the fuzzy annual demand reduces to $\overset{\approx}{D} = (D, D, D)$, also when p=0 and we have no error in inspection processes, then Q^* reduces to the traditional:

$$Q^* = \sqrt{\frac{2KD}{h}}$$

4. Numerical example

To illustrate the results of the proposed models, we consider an inventory system with the data in Ref. [15] and Ref. [35]: purchase cost, C=25$ per unit; ordering cost, K=100$ per ordering; selling price of good-quality items, s=50$ per units; selling price of imperfect quality items, v =20$ per unit; screening rate x=175200 unit per year; screening cost, d=0.5$ per unit; holding cost, h=5$ per unit; proportion of non-defective items are classified to be defective, m_1=0.2; proportion of defective items are classified to be non-defective, m_2=0.2; cost of accepting a defective item, C_a=500$ per unit; cost of rejecting a non-defective items, C_r=100$ per unit; the rate of defective items, $\tilde{p}$, is about 0.02; the number of items demanded per year, $\widetilde{D}$, is about 50000 unit per year. The effects of the fuzziness of fraction of defectives and demands on the optimal solutions are analysed and shown in table I:

Table 1: Optimal solution for the model

Δ_1	Δ_2	Δ_3	Δ_4	$d^*(\tilde{\tilde{p}},\tilde{\tilde{0}})$	$d^*(\tilde{\tilde{p}}^2,\tilde{\tilde{0}})$	$d^*(\tilde{\tilde{D}},\tilde{\tilde{0}})$	Q^*
500	350	0.001	0.015	0.0235	0.0006	49962.5	1543.228
	500			0.0235	0.0006	50000	1544.085
	650			0.0235	0.0006	50037.5	1544.941
1000	750			0.0235	0.0006	49937.5	1542.657
	1000			0.0235	0.0006	50000	1544.085
	1250			0.0235	0.0006	50062.5	1545.512
1500	1000			0.0235	0.0006	49875	1541.23
	1500			0.0235	0.0006	50000	1544.085
	2000			0.0235	0.0006	50120	1546.939
500	**350**	**0.005**	**0.005**	**0.02**	**0.0004**	**49962.5**	**1545.021**
	500			0.02	0.0004	50000	1545.878
	650			0.02	0.0004	50037.5	1546.734
1000	750			0.02	0.0004	49937.5	1544.45
	1000			0.02	0.0004	50000	1545.878
	1250			0.02	0.0004	50062.5	1547.305
1500	1000			0.02	0.0004	49875	1543.022
	1500			0.02	0.0004	50000	1545.878
	2000			0.02	0.0004	50125	1548.733
500	350	0.015	0.001	0.0165	0.0003	49962.5	1546.807
	500			0.0165	0.0003	50000	1547.666
	650			0.0165	0.0003	50037.5	1548.521
1000	750			0.0165	0.0003	49937.5	1564.235
	1000			0.0165	0.0003	50000	1547.664
	1250			0.0165	0.0003	50062	1549.092
1500	1000			0.0165	0.0003	49875	1544.807
	1500			0.0165	0.0003	50000	1547.664
	2000			0.0165	0.0003	50125	1550.52

To clarify the formula of Q^* obtained in this study, we demonstrate a detailed calculation of one of the situation illustrated in table I.

According to Chang (2004), consider the situation were Δ_1=500 and Δ_2=350. Based on Salameh and Jaber (2000), D=50000, so the signed distance between $\tilde{D}$ and $\tilde{0}$ calculated as follow:

$$d(d,0) = d + \frac{1}{4}(\Delta_2 - \Delta_1) = 50000 + \frac{1}{4}(350 - 500) = 49962.5$$

Also, if $\Delta_3 = \Delta_4$=0.005, based on Yao and We (2004), the signed distance between $\tilde{p}$ and $\tilde{p}^2$ and $\tilde{0}$ calculated as follow:

$$d(p,0) = p + \frac{1}{4}(\Delta_4 - \Delta_3) = 0.02 + \frac{1}{4}(0.005 - 0.005) = 0.02$$

$$d(p^2,0) = p^2 + \frac{\Delta_3^2 + \Delta_4^2}{6} + \frac{p}{2}(\Delta_4 - \Delta_3) =$$

$$(0.02)^2 + \frac{(0.005)^2 + (0.005)^2}{6} + \frac{0.02}{2}(0.005 - 0.005) = 0.0004$$

Also, according to Salameh and Jaber (2000) and Chang (2004), we have:

m_1=m_2=0.2 K=100$ h=5$ x=175200 unit per year

by placing above numbers in Q^* formula, we have :

$$Q^* = \sqrt{\frac{2K(D+\frac{1}{4}(\Delta_2-\Delta_1))}{AB+C}}$$

Where

A=

$$h((1-(p+\frac{1}{4}(\Delta_4-\Delta_3))^2(1-m_1)^2+3(1-m_1)m_2$$

B=

$$\left[(p+\frac{1}{4}(\Delta_4-\Delta_3)-(p^2+\frac{\Delta_3^2+\Delta_4^2}{6}+\frac{p}{2}(\Delta_4-\Delta_3))\right]$$

And

C=

$$2m_2^2(p^2+\frac{\Delta_3^2+\Delta_4^2}{6}+\frac{p}{2}(\Delta_4-\Delta_3))+2(D+\frac{1}{4}(\Delta_2-\Delta_1))/x$$

So Q^*= 1545.021

This case is specified in in bold type in table I.

From table I, as the value of Δ_1 and Δ_2 are increase, the optimal order quantity increase. Also the value of Δ_3 increase and Δ_4 decrease, the optimal order quantity is increase.

5. Conclusion

This paper suggested a fuzzy model for an inventory problem with imperfect quality items. In this model the defective rate and the annual demand represented by a fuzzy number. For this fuzzy model, a method of defuzzification, namely the signed distance is employed to find the approximate of the total profit per cycle in the fuzzy sense, and the corresponding optimal order lot size is derived to maximize the total profit. Numerical example is accomplished to determine the behavior of our suggested model. We would like to point out the performance of using signed distance method for difuzzification in this study. The prior researches on fuzzy inventory often employed the centroid method to obtain the approximate of total cost in the fuzzy sense. To achieve this task, the membership function of fuzzy total cost has to be found first using the expansion principle, while the derivations are very complex and serious, specially, for the case where the fuzzy number is located in the denominator.

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