The Model for Determining Rational Inventory in Occasional Demand Supply Chains

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***Abstract* – In order to ensure continuous demand for products in cases where it is impossible or economically inexpedient to fully combine the product supply volume and terms with the moments of appearing demand for it in the occasional demand supply chains, the necessity for inventory appears. High cost of maintaining inventory and significant losses due to its deficit cause an important issue of determining the rational inventory in occasional demand supply chains.**

The article proposes a model for determining a rational inventory with a fixed period of its replenishment and periodic demand. The main difference between the proposed model and the known models consists in taking into account the finite, but a priori unknown quantity of consumers of products, characteristic for real demand. Accounting for these factors required the use of a mathematical apparatus for determining the distribution function of a random finite number of random terms, as well as using a mathematical apparatus of a small sample to determine the parameters of the model. Previously, such a mathematical apparatus was not used in supply chain models. Its use complicates the model, but substantially increases its adequacy to the actual supply process.

***Keywords*—** *model, demand, random, rational, supply*

1. Introduction

Creation of inventory in supply chains is reasonably conditioned by the need to continuously ensure the demand for products and the impossibility or economic inexpediency to fully combine the volume and terms of the product supplies with the moments of appearing demand for it. At the same time, creation of inventory is associated with additional costs for constructing appropriate warehouses, creating reserves for storage and maintenance.

Therefore, there is an arising issue of determining the rational volumes of inventory. In particular, this issue is manifested in the sphere of trade and, above all, of retail trade. The process of circulation of goods ends in retail trade, and they are transferred to the sphere of personal consumption. Consequently, it is in retail trade that the efficiency of the goods production is ultimately manifested. To ensure the effective transfer of goods to the personal consumption sphere, the following basic functions shall be implemented in retail trade:

– consideration of the demand for goods, their market offer and maintenance of the supply-demand balance;

– formation of the commercial assortment that satisfies population's need for goods;

– organization of goods circulation, bringing goods to customers;

– stimulation of production in terms of expanding the range and increasing the volume of goods;

– formation of inventory and maintaining its necessary level (commodity supply);

– implementation of promotional and PR activities of retail trade enterprises;

– implementation of trade and technological operations with the goods, such as storage, industrial refinement (dispensing, packing, etc.);

– providing customers with a set of services that facilitate purchase and use of good (taking pre-orders, selling individual goods on credit, delivering purchased goods to the customer, assembling and installing purchased goods at buyer's place, making the rules of operation of technically complex products clear to the customer, providing gift wrapping of purchased goods), etc.

Commodity supply is the most important function of retail trade. It is a complex of commercial and technological activities carried out by industrial, trade and transport organizations aimed at bringing goods from manufacturing enterprises and wholesale bases to retail enterprises (stores). The main objective of commodity supply is to ensure rational volumes and range of inventory in stores. Their rationality consists in full provision of effective consumer demand and is manifested in the financial and economic performance of retailers. The issue

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of commodity supply rationality is especially pressing in determining the volumes and range of inventory of perishable goods. This is due to the peculiarities of storage, short shelf life and high losses thereof, both in case of a shortage, and in case of an excess of such products. Shortage-caused losses are manifested in lost profits, and excess-caused losses – in failure to sell the expired goods. Consequently, generation of decisions on the supply of perishable goods shall be based on a sufficiently correct consideration of these losses. Such consideration is based on modeling the commodity supply process.

2. Literature Review

Such modeling issues were considered in works [1, 2, 3, 4, 5, 6, 7]. The models and methods for solving the problem in question proposed in the works are based on the various variants of formalizing the demand process in the form of limit distributions for sums of the independent random variables. At the same time, real demand is related to the summation of a finite (that is, not limiting), but a priori unknown number of random variables [8, 9]. Herewith, characterization of the demand model parameters, as a rule, shall be carried out based on small sampling. This imposes significant restrictions on the use of traditional models to determine the rational volume of products. The specified circumstances are taken into account in the model proposed in the article for determining the rational volume of inventory in occasional demand supply chains.

The solution of the optimization problems of the model under consideration can be based on the approaches considered in [5, 6, 7, 8, 9].

3. The Methodology and Model

Let us introduce the notation as follows:

T – duration of the inventory replenishment period;

J – number of product types (inventory list);

Nj – a random variable characterizing the number of consumers who applied for the j-th product during period Tj;

βrj – a random variable characterizing the volume of goods of the j-th type requested by the r-th (r=1,2,…,Nj) customer;

Xj – volume of the j-th type goods inventory.

With the notation introduced, the volume of the j-th type goods requested over the T period is characterized with a random value:

$$Y_j = \sum_{r=0}^{N_j} \beta_{rj}, \quad j = 1,2,...,J \qquad (1)$$

Taking into account (1), the rationality of the j-th type goods volume is characterized with the probability Fj(Xj) of event consisting in $(Y_j \le X_j)$. Consequently, it is necessary to construct a function of distribution of the random value Yj of this product demand volume to determine the rational volume of the j-th goods inventory volume at a fixed replenishment period and occasional demand.

To construct it, the probability of the event consisting in the Nj random variable taking n value; let us set it as pnj. Moreover, let us consider that the random variables βrj (r=1,2,…,Nj, j=1,2,…J) of the j-th type goods volume demanded by consumers are independent, have the distributions similar for each product type and are independent of n. Then, we may use the mathematical apparatus of the characteristic functions to determine the functions of distribution $F_j(y) = F_j(Y_j < y), \quad j = 1,2,...,J$ of the sums of random numbers Nj (j=1,2,…,J) of the random values βrj (r=1,2,…,Nj).

Let the characteristic function of the random values βrj (r=1,2,…,Nj, j=1,2,…,J) be set as $\varphi_{0j}(\xi)$. Then, based on the multiplicative properties of the characteristic functions, the characteristic function of the random value Yj (j=1,2,…,J) may be written as [5]:

$$\varphi_j(\xi) = \sum_{n=0}^{\infty} p_{nj} \varphi_{0j}^n(\xi), \quad j = 1,2,...,J \qquad (2)$$

Taking into account (2), the probability density $f_j(y) = \dfrac{dF_j(y)}{d(y)}$ of the random value Yj (j=1,2,…,J) is written as:

$$f_j(y) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-i\xi y} \sum_{n=0}^{\infty} p_{nj} \varphi_{0j}^n(\xi) d\xi, \quad j = 1,2,...,J \qquad (3)$$

In view of the finiteness of the equation

$$\sum_{n=0}^{\infty} p_{nj} \varphi_{0j}^n(\xi) \le \sum_{n=0}^{\infty} p_{nj} \varphi_{0j}^n(0) < \infty, \quad j = 1,2,...J$$,

(3) suggests:

$$f_j(y) = \frac{1}{\pi} \sum_{n=0}^{\infty} p_{nj} \int_{-\infty}^{\infty} e^{-i\xi y} \varphi_{0j}^n(t) d\xi = \sum_{n=0}^{\infty} p_{nj} f_{nj}(y), \quad j = 1,2,...J \qquad (4)$$

where $f_{nj}(y)$ is the probability density of the sum n of the random values βrj (r=1,2,…) (4) suggests that the probability of no j-th (j=1,2,…,J) type goods deficiency at the volume Xj of its inventory is determined by the equation.

$$F_j(y \le X_j) = \sum_{n=0}^{\infty} p_{nj} \int_0^{X_j} f_{nj}(y) dy, \quad j = 1,2,...,J \qquad (5)$$

Taking into account (5), the probability of no deficiency through the entire inventory list is determined by the equation:

$$R(X) = \prod_{j=1}^{J} F_j(y \le X_j) \quad (6)$$

and the probability of deficiency of at least one inventory item – by the equation

$$Q(X) = 1 - \prod_{j=1}^{J} F_j(y \le X_j) \quad (7)$$

The equations (5) – (7) are the basis of a number of models for optimizing the goods inventory volume with the fixed period replenishment period and occasional demand. To construct them, let us introduce the notation as follows:

cj – cost of maintenance of one inventory product of the j-th type;

gj –weight of one product of the j-th type;

vj – volume of one product of the j-th type.

Then, the total cost of maintenance of the goods inventory is determined by the equation

$$C(X) = \sum_{j=1`}^{J} c_j X_j \quad (8)$$

(8)

The total weight of the inventory of goods is determined by the equation

$$G(X) = \sum_{j=1`}^{J} g_j X_j \quad (9)$$

and their total volume – by the equation

$$V(X) = \sum_{j=1`}^{J} v_j X_j \quad (10)$$

Taking into account the equations (6) – (10), the following objectives of optimization of the inventory volume with the fixed replenishment period and occasional demand may be constructed (Anisimov, Anisimov Rodionova and Saurenko 2016: and Anisimov, Anisimov, Novikov and Ostanin 2016).

Objective 1. Determination of the inventory volume $X^* = \{X_1^*, X_2^*, ..., X_J^*\}$ that provides minimization of the maintenance cost thereof

$$C(X^*) = \sum_{j=1`}^{J} c_j X_j^* = \min_X C(X) \quad (11)$$

at the preset probability $R^*(X)$ of no deficiency

$$R(X) = \prod_{j=1}^{J} F_j(y \le X_j) \ge R^*(X) \quad (12)$$

and limitations as to the allowable weight G* and volume V* of inventory:

$$G(X) = \sum_{j=1`}^{J} g_j X_j \le G^* \quad (13)$$

$$V(X) = \sum_{j=1`}^{J} v_j X_j \le V^* \quad (14)$$

Objective 2. Determination of maximization of the probability of no deficiency through the entire inventory list

$$R(X^*) = \max_X \prod_{j=1}^{J} F_j(y \le X_j) \quad (15)$$

or minimization of the probability of the deficiency at least of one inventory list item

$$Q(X^*) = \min_X [1 - \prod_{j=1}^{J} F_j(y \le X_j)] \quad (15a)$$

with the limitation on the cost C* of the inventory maintenance, its weight and volume:

$$C(X) = \sum_{j=1`}^{J} c_j X_j \le C^* \quad (16)$$

$$G(X) = \sum_{j=1`}^{J} g_j X_j \le G^* \quad (17)$$

$$V(X) = \sum_{j=1`}^{J} v_j X_j \le V^* \quad (18)$$

The constructive methods for solving these problems are obtained by establishing the random values distribution laws characterizing the flow of consumers and the volumes of products they request.

For definiteness, let us assume that:

the number of consumers of the j-th type product over the period T obeys the Poisson law with the parameter μj, that is:

$$p_{nj} = \frac{\mu_j^n}{n!} e^{-\mu_j}, \quad j = 1,2,..., J \quad (19)$$

the volume of goods requested by each consumer has an exponential distribution with the parameter λj:

$$w_j(y) = \lambda_j e^{-\lambda_j}, \quad j = 1,2,..., J \quad (20)$$

The characteristic function of an exponentially distributed random variable takes the form

$$\varphi_{0j}(\xi) = (1 - i\xi\lambda_j^{-1})^{-1} \quad (21)$$

By substituting (19) and (21) in (2), we obtain:

$$\varphi(\xi) = \sum_{n=0}^{\infty} \frac{\mu_j^n}{n!} e^{-\mu_j} (1 - i\xi\lambda_j^{-1})^{-1} \quad (22)$$

By substituting (22) in (4), after transformations, we obtain:

$$f_j(y) = \frac{1}{2\pi} \sum_{n=0}^{\infty} \frac{(\lambda_j \mu_j)^n e^{-\mu_j}}{n!} \int_{-\infty}^{\infty} e^{-i\xi y} (\lambda_j - i\xi)^{-n} d\xi = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^{\,n}}{n!} \frac{(\lambda_j y)^{n-1} e^{-\lambda_j y}}{\lambda_j \Gamma(n)}, \quad j = 1,2,..., J \quad (23)$$

where Γ(n) is a gamma-function.

(23) suggests that the probability of no deficit of the j-th (j=1,2,…,J) type goods with the volume Xj of the inventory, taking into account the assumptions made, is determined by the equation

$$F_j(y \le X_j) = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^n}{n!} \int_0^X \frac{(\lambda_j y)^{n-1} e^{-\lambda_j y}}{\lambda_j \Gamma(n)} dy, \quad j = 1,2,...,J` \quad (24)$$

Substitution of the equation (24) in (12) and (15) provides constructability to the problems of optimization of the inventory volumes at the fixed completion period and occasional demand.

4. The findings

In general, the obtained equations (1) – (18) represent a generalized model for determining the rational inventory volume in the supply chains with occasional demand. The equations (19) – (24) precise this model for the Poisson consumer flow and exponential law of demand volume of each of them.

The Poisson flow of consumers and the exponential law of the needs of each of them are adequate to the actual process, information about which is exhausted by data on the average number of consumers accessing the products per unit of time (for example, per day) and the average volume of products requested by them.
This information situation is more typical for the practice of determining the rational inventory volume in the supply chains with random demand. It has resulted in expedience of constructive introducing the general model (1) - (4) in the form (19 - (24).

In other information situations, the establishment of laws for the distribution of random variables characterizing the flow of consumers and the volumes of products requested by them can be carried out on the basis of the "maximum entropy principle" [8, 9]. Consideration of such situations is a subject of a separate research.

5. Conclusion

Use of the models reviewed in the sphere of retail perishable goods trade provides a more adequate consideration of the facts affecting the rationality of inventory of such goods compared to the known models.

In particular, unlike well-known models, the proposed model takes into account that the number of consumers is limited, but a priori unknown. Accounting for these factors required the use of a mathematical apparatus for determining the distribution function of a random finite number of random terms, as well as using a mathematical apparatus of a small sample to determine the parameters of the model. Previously, such a mathematical apparatus was not used in supply chain models. Its use complicates the model, but substantially increases its adequacy to the actual supply process.

Applying the proposed model allows you to more accurately determine the required amount of stocks and, ultimately, reduce losses due to shortages and surplus products. For example, the application of the proposed model (19) - (24) in a number of retail enterprises in the Moscow region reduced these losses by an average of 11.73%.

The mentioned fact shows that it is reasonable to apply practically the general model in consideration as well as its variant that corresponds to the Poisson consumer flow and exponential law of demand volume of each of them. Meanwhile, this variant corresponds to a wide range of real situations determining the rational inventory volume in supply chains.

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