Application of the Logit Demand Function to Determine the Retail Price and Inventory Policy in a Serial Supply Chain

Lisha Duan[1], José A. Ventura [*2], Bárbara Venegas-Quintrileo[3]

Harold and Inge Marcus Department of Industrial and Manufacturing Engineering
Pennsylvania State University, University Park, PA 16802, USA

[1]lud149@psu.edu
[2]jav1@psu.edu
[3]bbv105@psu.edu

***Abstract*— We consider a serial supply chain with multiple stages in a centralized control scenario. Within this supply chain, the first stage faces a supplier selection decision for a particular product that experiences a price-sensitive demand. The demand is represented as a logit function, which parameters account for the range and price sensitivity factors. Suppliers are capacitated and offer their individual fixed unit price with a corresponding quality level for the product. The buying stage needs to decide which suppliers to choose and how to allocate orders, determining the optimal inventory policy for all stages and the retail price to offer to end customers, while maximizing the total profit of the supply chain. The problem is formulated as a mixed integer nonlinear programming model and a heuristic approach is proposed to generate an approximate solution. Then, we analyze a special case that considers only one uncapacitated supplier and a buyer with price-sensitive demand in a serial supply chain. An efficient heuristic is developed for this case to obtain a near optimal solution in a timely manner. Finally, we provide a series of numerical examples to illustrate our results and analyze the impact of the parameters within a sensitivity analysis.**

Keywords*— *supplier selection, inventory replenishment, price-sensitive demand, logit demand function, serial supply chain.

1. Introduction

A typical supply chain consists of multiple entities such as suppliers, manufacturers, wholesalers, retailers, and the customers. The incurred activities in a supply chain system include supplier selection, order allocation, production, distribution, and inventory management. Given today's competitive global market, it is essential that all the supply chain entities act under one policy to improve supply chain performance as a whole while reducing the cost of the system.

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

As a result, researchers and companies have devoted their efforts in the development of integrated decision models in supply chain optimization for years. Given the prevalence of both purchasing and inventory policies, much of the recent research has been focused in tackling these two problems simultaneously. For instance, Ref. [1] presents a multi-product, multi-period inventory lot-sizing model considering supplier selection. Ref. [2] introduces a multi-period, multi-supplier, and multi-item inventory model with imperfect quality. Ref. [3] proposes a mixed integer nonlinear programming (MINLP) model to determine the order quantities placed to the selected suppliers as well as an optimal inventory policy for a serial supply chain. Subsequently, Ref. [4] presents a dynamic network structure to represent a multi-supplier, multi-stage, and multi-period operational planning problem. Ref. [5] extends the results from [3] by accounting for transportation costs.

Pricing is a crucial strategic decision for a company. The first effort incorporating pricing into supply chain management can be dated back to [6]. Recently, [7] proposes a joint marketing-inventory model in a two-echelon supply chain with discount promotion and price-sensitive demand in order to determine the optimal ordering, shipping, and pricing policies. Ref. [8] investigates a joint pricing and lot-sizing problem in a two-echelon supply chain considering a finite production rate. Researches coordinating pricing and supply chain related strategies for a multiple period supply chain system include [9], [10], and [11].

The supply sourcing process is considered to be of high complexity, usually including multiple stages. Initially, the buying entity may encounter an overwhelming number of suppliers for the needed product, and hence a first analysis of all the options is usually conducted first. There is vast research on how to rank suppliers using more qualitative methods that consider multiple criteria and the personal preferences of decision makers. The result of this first analysis may still finish with a reduced pool of suppliers, where an ultimate selection process must be performed, together with the allocation and lot sizing decisions. For the first stage plenty of methodologies have been applied by previous researches [12]. The main criteria under consideration are unit price, capacity, quality, setup cost, and delivery time. Given the significance of supplier selection in supply chain management, there are several research papers that consider joint models in pricing and supplier selection. Ref. [13] proposes an integrated procurement, production, and pricing model for a manufacturer sourcing from multiple capacitated suppliers, facing price-sensitive market demand for one product. Ref. [14] introduces a problem where a single buyer faces decisions of supplier selection, lot-sizing, and pricing in a multi-product, multi-period setting. Ref. [15] considers a single item EOQ model with multiple capacitated suppliers providing all-unit quantity discounts. Later, [16] extends the model in [15] to a serial supply chain.

Most of the foregoing models in pricing, inventory management, and procurement strategies assume a linear or a power demand function. However, there are very few papers providing a justification for choosing one demand function instead of the others [7], [17]. In fact, the logit demand function is a more precise demand curve, which can serve as a global price-sensitive demand function. Given the gap in the literature, we further investigate the application of the logit demand function to a serial supply chain, aiming to determine the retail price, the sourcing strategies, and the corresponding inventory policy maximizing the whole system's profit. Besides, we generalize the algorithm presented by [16] and apply it to solve an integrated pricing, supplier selection, and inventory replenishment problem with any price-sensitive demand function. In addition, we develop a heuristic algorithm for a special case of a serial supply chain with a single uncapacitated supplier and show its efficiency.

The remainder of this paper is organized as follows. Section 2 presents the problem description and the model formulation. Section 3 introduces the proposed heuristic algorithms. Two numerical examples are presented in Section 4, illustrating the efficiency of our proposed algorithms. Section 5 provides an analysis with respect to the number of preferred suppliers and the pricing parameters. Lastly, Section 6 concludes the paper and provides future research directions.

2. Model Formulation

We consider a serial supply chain where raw materials or final products flow sequentially from potential suppliers through the manufacturing site, the local warehouse, the regional warehouse, and finally to the distribution center (DC). Figure 1 demonstrates the serial supply chain in consideration. The manufacturer makes purchasing decisions on raw materials from various potential suppliers. Multiple orders can be submitted to selected suppliers within one order cycle. However, there is no splitting on any order, which means the manufacturer only takes one order from one supplier at a time. The finished products are transported to the subsequent warehouse and DC as requested. The inventory at every stage is used to replenish the inventory in the succeeding stage without shortages; that is to say, orders are placed by DC to the warehouse, by the warehouse to the manufacturer, and by the manufacturer to the preferred suppliers. There are no storage capacity restrictions at all stages. Lastly, the inventory at DC can be immediately used to meet the demand from the set of customers without backlogging or lost sales. We assume the demand rate at the last stage is dependent on customers' response to the retail price and we represent this price-sensitive demand with the logit demand function. Let D be the demand rate and p be the retail price respectively, the logit demand function can be expressed as:

$$D = C\frac{e^{-(a+bp)}}{1+e^{-(a+bp)}},$$

where $a < -2, b > 0$.

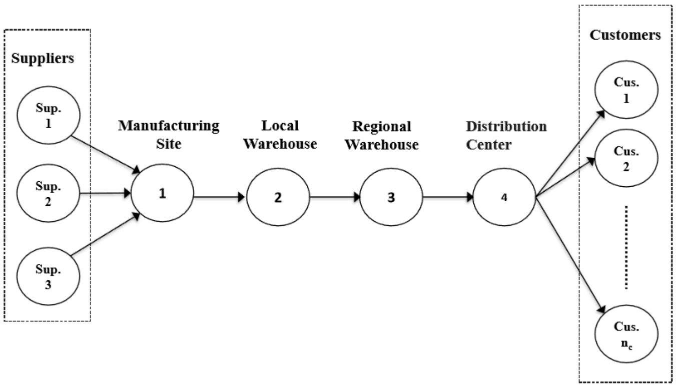

Figure 1. Demonstration of raw material and product flow in a serial supply chain

Given the entire system is managed by a single company, we are looking to find the best pricing policy, procurement strategy, and inventory replenishment policy simultaneously so that the overall profit for this centralized system is maximized over the infinite time horizon. We assume that the lead times between any pair of consecutive stages are fixed or zero. Shortages are not allowed at any stage.

According to previous research, the optimal stationary ordering policy for a serial supply chain must be nested and able to order only when the inventory level is zero. The nested policy refers to the situation that, whenever a stage orders, all its downstream stages also order. Therefore, the order quantity placed at one stage is always an integer multiple of the order quantity placed at the downstream stages. The zero-inventory-ordering policy requires the orders to be placed only when the on-hand inventory drops to zero [18]–[20].

In addition, researchers have also proved the optimality of combining two consecutive stages when the ratio of setup to unit echelon inventory cost for a stage is not greater than that of the immediate downstream stage [19], [20].

Based on these assumptions, we define the corresponding parameters and decision variables as follows:

Sets

i stage, where $i=1,...,r$,

t supplier, where $t=1,...,n.$

Parameters

K_i setup cost of supplier i, where $i=1,...,r$,

c_i unit purchasing cost of supplier i, where $i=1,...,r$,

F_i capacity of supplier i, where $i=1,...,r$,

q_i product quality level from supplier i, where $i=1,...,r$,

q_a required product quality level from the manufacturer,

KS_t setup cost at stage t, where $t=2,...,n$,

h_t unit inventory holding cost at stage t, where $t=1,...,n$,

e_t unit inventory echelon cost at stage t, where $t=2,...,n$,

m overall number of orders within an order cycle,

C market size of the item,

a price range factor,

b price-sensitive factor.

Decision Variables

D DC's demand rate,

p unit selling price,

J_i number of orders from supplier i within an order cycle, where $i=1,...,r$,

QS_t order quantity placed at stage t, where $t=2,...,n$,

Q_i order quantity submitted to supplier i, where $i=1,...,r$,

Q order quantity from all selected suppliers within an order cycle,

N_{1i} multiplicative factor for order quantity submitted to supplier i, where $i=1,...,r$,

N_t multiplicative factor for order quantity submitted to stage t, where $t=2,...,n-1$.

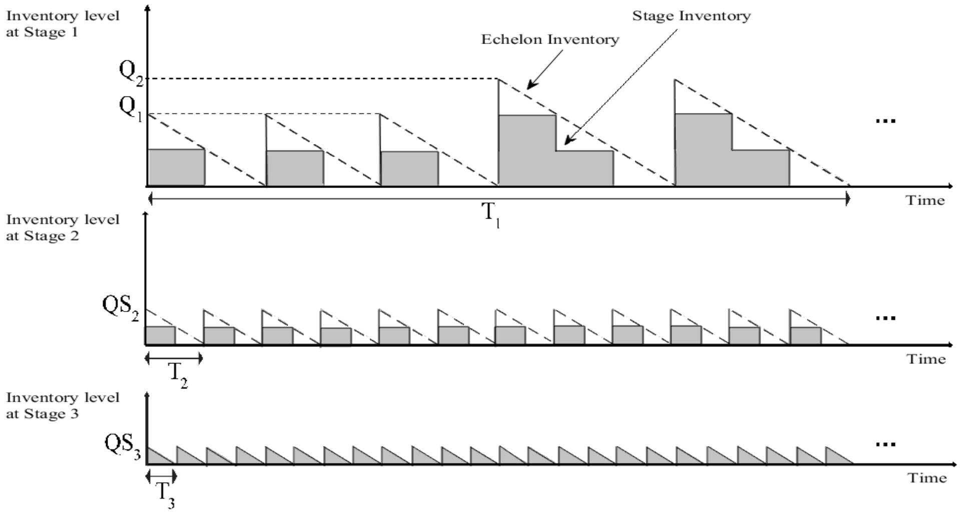

Figure 2. Demonstration of the zero-nested inventory policy in a three-stage serial supply chain

Figure 2 illustrates an example of a three-stage serial supply chain, where two suppliers are chosen as raw material providers. Three orders are placed to supplier 1 while supplier 2 delivers twice during an order cycle. In addition, the value of N_2 is 2 for this example.

The objective function is to maximize the total profit per time unit for this serial supply chain system (Model M_S):

Maximize,

$$Z_s = Dp - \left(\frac{D}{Q}\sum_{i=1}^{r} J_i K_i + \frac{h_1 \sum_{i=1}^{r} J_i Q_i^2}{2Q} + D\frac{\sum_{i=1}^{r} J_i c_i Q_i}{Q} \right) - \left(D\sum_{t=2}^{n} \frac{KS_t}{QS_t} + \frac{1}{2}\sum_{t=2}^{n} QS_t e_t \right) \quad (1)$$

subject to

$$Q = \sum_{i=1}^{r} J_i Q_i, \quad (2)$$

$$\frac{DQ_i J_i}{Q} \le F_i, \quad i = 1,\ldots,r, \quad (3)$$

$$\sum_{i=1}^{r} J_i q_i Q_i \ge q_a \sum_{i=1}^{r} J_i, \quad (4)$$

$$\sum_{i=1}^{r} J_i \le m, \quad (5)$$

$$D = C\frac{e^{-(a+bp)}}{1+e^{-(a+bp)}}, \quad (6)$$

$$Q_i = N_{1i} QS_2, \quad i = 1,\ldots,r, \quad (7)$$

$$QS_t = N_t QS_{t+1}, \quad t = 2,\ldots,n-1, \quad (8)$$

$$N_{1i} \ge 1, \text{integer}, \quad i = 1,\ldots,r, \quad (9)$$

$$N_t \ge 1, \text{ integer}, \quad t = 2,\ldots,n-1, \quad (10)$$

$$J_i \ge 0, \text{integer}, \quad i = 1,\ldots,r, \quad (11)$$

$$Q_i \ge 0, \quad i = 1,\ldots,r, \quad (12)$$

$$QS_t \ge 0, \quad t = 2,\ldots,n, \quad (13)$$

$$D > 0, p > 0. \quad (14)$$

The first term in the objective function represents the revenue per time unit at DC. The second component accounts for the cost per time unit incurred at stage 1, which are setup cost, holding cost, and purchasing cost. The last component in the objective function accounts for the setup and holding costs for the remaining stages. Eq. (2) denotes the overall order quantity from all selected suppliers within an order cycle. Constraint set (3) illustrates the supplier's capacity limit. Constraint (4) ensures that the average quality level from the selected suppliers satisfies the manufacturer's required quality level. Constraint (5) shows the total number of orders submitted to all selected suppliers during an order cycle. Constraint (6) is the logit demand function. Constraint sets (7) - (10) guarantee the nested and zero-inventory ordering policy. Constraint (11) requires the number of orders placed to each selected supplier within an order cycle to be an integer. Constraint sets (12)-(14) ensure the properties of order quantities, DC's demand rate, and the selling price.

3. Algorithms

3.1. Heuristic algorithm for a serial supply chain

The model under consideration becomes complex when the number of suppliers and stages increase. Furthermore, in order to illustrate accurately the price-demand relationship, we propose to use the logit demand function, which results in more complicated computational efforts compared to the linear demand function and the power function. Therefore, to improve the computational efficiency, we develop a heuristic algorithm to obtain near optimal solutions to the proposed model. This heuristic is based on the property of the logit demand function and the power-of-two (POT) inventory policy. The POT policy has been applied by researchers and proved to be computationally efficient in determining near optimal inventory levels in supply chain systems [3], [16], [21]–[23].

Proposition 1. The optimal selling price for Model M_s is greater than the point where the price elasticity of demand is 1.

Proof. See [24].

Proposition 1 can be extended to the development of a lower bound on the optimal price, denoted by p^-, with respect to any demand function except the power function, which has a constant price elasticity of demand at all prices [16].

The algorithm starts by calculating an initial order quantity at each stage and for each supplier using the EOQ formula and the initial selling price, which is a lower bound on the retail price based on Proposition 1. Let Q_i^0 be the initial order quantity submitted to supplier i and QS_t^0 be the initial order quantity submitted at stage t. That is to say, we have:

$$Q_i^0 = \sqrt{\frac{2Ce^{-(a+bp)}K_i}{h_1[1+e^{-(a+bp)})]}},\ i=1,\ldots,r,$$

and $$QS_t^0 = \sqrt{\frac{2Ce^{-(a+bp)}KS_t}{e_t\left[1+e^{-(a+bp)}\right]}} > 0,\quad t=2,3,\ldots n.$$

Then, we adjust the order quantity from stage n-1 to stage 1 in order to make the multiplicative factors be multiples of two using the POT procedure.

POT procedure [3], [16], [23] .

Step 1. If QS_n^{POT} is known, go to step 2. Otherwise, let $QS_n^{POT} = QS_n^0$.

Step 2. For $t = n, n-1, \ldots, 3, 2$, find the integer y, such that $2^y QS_{t+1}^{POT} \le QS_t^0 \le 2^{y+1} QS_{t+1}^{POT}$.

If $\dfrac{QS_t^0}{2^y QS_{i+1}^{POT}} \le \dfrac{2^{y+1} QS_{i+1}^{POT}}{QS_t^0}$, let $x_t = y$ and $QS_t^{POT} = 2^{x_t} QS_{t+1}^{POT}$.

Otherwise, set $x_t = y+1$ and $QS_t^{POT} = 2^{x_t} QS_{t+1}^{POT}$.

Step 3. For $i = 1, \ldots, r$, determine the integer y, such that $2^y QS_2^{POT} \le Q_i^0 \le 2^{y+1} QS_2^{POT}$.

If $\dfrac{Q_i^0}{2^y QS_2^{POT}} \le \dfrac{2^{y+1} QS_2^{POT}}{Q_i^0}$, let $x_{1i} = y$ and $Q_i^{POT} = 2^{x_{1i}} QS_2^{POT}$.

Otherwise, set $x_{1i} = y+1$ and $Q_i^{POT} = 2^{x_{1i}} QS_2^{POT}$.

Once we determine the order quantity at each stage, i.e., QS_t^{POT}, $t = 2, 3, \ldots n$ and the order quantity for each supplier, i.e., Q_i^{POT}, $i = 1, \ldots, r$, it is necessary to find the number of orders to be submitted to each selected supplier, i.e., J_i, $i = 1, \ldots, r$ so that the suppliers' capacity and quality constraints are not violated. Notice that the capacity and quality constraints are only related to the supplier selection problem at stage 1. Therefore, model M_1 is developed and solved given the order quantity from each supplier, i.e., Q_i^{POT}, $i = 1, \ldots, r$.

Maximize,

$$Z_1 = Dp - \left(\frac{D}{Q}\sum_{i=1}^{r} J_i K_i + \frac{h_1 \sum_{i=1}^{r} J_i Q_i^2}{2Q} + D\frac{\sum_{i=1}^{r} J_i c_i Q_i}{Q} \right) \tag{15}$$

subject to constraint sets (2-6), (11-12), and (14).

After obtaining updated p and J_i, $i = 1, \ldots, r$ values by solving Model M_1, we can rewrite the objective function of Model M_s as a function of the order quantity at the last stage, i.e., QS_n.

Maximize,

$$Z_s = C\frac{pe^{-(a+bp)}}{1+e^{-(a+bp)}} - \left(C\frac{e^{-(a+bp)}}{QS_n[1+e^{-(a+bp)}]}\sum_{t=2}^{n}\frac{KS_t}{m_t} + \frac{QS_n}{2}\sum_{t=2}^{n} m_t e_t\right) - \frac{1}{QS_n\sum_{i=1}^{r} J_i m_{1i}}\left(C\frac{e^{-(a+bp)}}{1+e^{-(a+bp)}}\sum_{i=1}^{r} J_i K_i + \frac{h_1 QS_n^2 \sum_{i=1}^{r} J_i m_{1i}^2}{2} + C\frac{e^{-(a+bp)} QS_n \sum_{i=1}^{r} J_i c_i m_{1i}}{1+e^{-(a+bp)}}\right) \quad (16)$$

where $m_{1i} = N_{1i}N_2 \ldots N_{n-1},\ i=1,\ldots,r$ and $m_t = N_t N_{t+1} \ldots N_{n-1},\ t=2,3,\ldots n-1$. Notice that QS_n is independent of the supplier capacity and quality constraints. Therefore, by taking the first derivative and setting it to zero, we are able to derive a closed form expression for QS_n:

$$\frac{dZ_s}{dQS_n} = C\frac{e^{-(a+bp)}}{QS_n^2\left[1+e^{-(a+bp)}\right]}\left(\frac{\sum_{i=1}^{r} J_i K_i}{\sum_{i=1}^{r} J_i m_{1i}} + \sum_{t=2}^{n}\frac{KS_t}{m_t}\right) - \frac{1}{2}\left(\frac{h_1\sum_{i=1}^{r} J_i m_{1i}^2}{\sum_{i=1}^{r} J_i m_{1i}} + \sum_{t=2}^{n} m_t e_t\right) = 0 \quad (17)$$

Then, we can solve Eq. (17) for QS_n as follows:

$$QS_n^* = \sqrt{2C\frac{e^{-(a+bp)}\left(\frac{\sum_{i=1}^{r} J_i K_i}{\sum_{i=1}^{r} J_i m_{1i}} + \sum_{t=2}^{n}\frac{KS_t}{m_t}\right)}{1+e^{-(a+bp)}\left(\frac{h_1\sum_{i=1}^{r} J_i m_{1i}^2}{\sum_{i=1}^{r} J_i m_{1i}} + \sum_{t=2}^{n} m_t e_t\right)}} \quad (18)$$

(1)

where $m_{1i} = N_{1i}N_2 \ldots N_{n-1},\ i=1,\ldots,r$ and $m_t = N_t N_{t+1} \ldots N_{n-1},\ t=2,3,\ldots n-1$. Taking the second derivative of Z_s with respect to QS_n, we get

$$\frac{d^2 Z_s}{dQS_n^{\ 2}} = -2C\frac{e^{-(a+bp)}}{QS_n^3\left[1+e^{-(a+bp)}\right]}\left(\frac{\sum_{i=1}^{r} J_i K_i}{\sum_{i=1}^{r} J_i m_{1i}} + \sum_{t=2}^{n}\frac{KS_t}{m_t}\right).$$

Clearly $\frac{d^2 Z_s}{dQS_n^{\ 2}} < 0$. Hence, we conclude that Z_s is strictly concave in terms of QS_n and therefore Eq. (18) leads to the maximum profit of Model M_s.

Lastly, we use the updated selling price obtained by solving Model M_1 and the refined value of QS_n to repeat the foregoing procedure until the multiplicative factors and the number of orders from each supplier remain unchanged. Hence, we terminate the algorithm and report the corresponding results. Next, we provide the detailed steps of the procedure.

Heuristic algorithm 1

Step 1. Set $k=0$, derive the initial selling price p_0 and D_0 according to Proposition 1.

Step 2. Calculate QS_t^k, $t=2,\ldots,n$ and Q_i^k, $i=1,\ldots,r$ as follows:

$$Q_i^k = \sqrt{\frac{2Ce^{-(a+bp)}K_i}{h_1[1+e^{-(a+bp)})]}},\ i=1,\ldots,r, \text{ and}$$

$$QS_t^k = \sqrt{\frac{2Ce^{-(a+bp)}KS_t}{e_t\left[1+e^{-(a+bp)}\right]}} > 0,\ t=2,3,\ldots n.$$

Step 3. Determine QS_t^{POT}, $t=2,3,\ldots n$ and Q_i^{POT}, $i=1,\ldots,r$ using the POT procedure.

Step 4. Solve Model M_1 given $Q_i^{POT}, i=1,\ldots,r$, to obtain the updated retail price p and the number of orders submitted to each selected supplier, J_i, $i=1,\ldots,r$.

Step 5. Update QS_n using Eq. (18).

Step 6. If $k=0$, go to step 2. Otherwise, check if the number of orders for each supplier J_i, $i=1,\ldots,r$, and the multiplicative factors remain unchanged for two consecutive iterations. If so, terminate the algorithm and report the result as p_k, J_i, Q_i^{POT}, N_{1i}, $i=1,\ldots,r$, QS_t^{POT},

$t = 2,3,\ldots n$, and N_t, $t = 2,3,\ldots,n-1$. Else, let $k = k+1$ and go to step 2.

3.2 Heuristic algorithm for a single uncapacitated supplier in a serial supply chain

In this subsection, let us consider a special case in a serial supplier chain, where there is only one raw material provider without quality and capacity constraints.

3.2.1 An EOQ model with the logit demand function

First, let us consider a two-stage supply chain with one supplier and one retailer. The retailer's demand is price-sensitive, which can be denoted by the logit demand function shown in Eq. (6). Let c denote the unit purchasing price and K represent the setup cost per order. Moreover, let h_1 be the unit holding cost and Q_o be the order quantity placed to the supplier. The retailer places orders periodically for the purpose of maximizing the retailer's profit per time unit, denoted as follows (Model M_o):

$$\text{Maximize } Z_o = D\left(p-c\right) - \frac{DK}{Q_o} - \frac{h_1 Q_o}{2}, \qquad (19)$$

subject to

$$p, D, Q_o > 0. \qquad (20)$$

The first and second derivatives of Z_o with respect to Q_o are $\frac{DK}{Q_o^2} - \frac{h_1}{2}$ and $-\frac{2DK}{Q_o^3}$, respectively. The fact of $\frac{\partial^2 Z}{\partial Q_o{}^2} < 0$ indicates that the objective function is strictly concave with respect to Q_o. Therefore, the optimal order quantity can be derived as:

$$Q_o^* = \sqrt{\frac{2DK}{h_1}}. \qquad (21)$$

When substituting Q_o^* into (19), we have:

$$\text{Maximize } Z_o = D\left(p-c\right) - \sqrt{2DKh_1}. \qquad (22)$$

Next, we derive an expression to compute the optimal price p^* considering the logit demand function in Eq. (6). The corresponding proposition is proposed as follows:

Proposition 2: The following three statements hold true for the EOQ model with the logit demand function:

(1) The point p^-, where the price elasticity of demand equals 1, is a lower bound of the optimal price p^*.

(2) The optimal price p^* is the lowest price which satisfies the following equation:

$$1 + e^{-(a+bp)} - bp + b\sqrt{\frac{Kh_1}{2D}} = -bc. \qquad (23)$$

(3) Let p^m be a price point equal to $-\frac{a}{b}$. Then,

i. if $2 + a + b\sqrt{Kh_1/C} < -bc$, then $p^* \in \left(p^-, -\frac{a}{b}\right)$,

ii. if $2 + a + b\sqrt{Kh_1/C} = -bc$, then $p^* = -\frac{a}{b}$,

iii. and if $2 + a + b\sqrt{Kh_1/C} > -bc$, then $p^* \in \left(p^-, -\frac{2a}{b}\right)$.

Proof.

Statement (1) follows from Proposition 1. Now, differentiating (22) with respect to p, we get

$$\frac{dZ_o}{dp} = \frac{D}{1+e^{-(a+bp)}}\left[1 + e^{-(a+bp)} - bp + bc + b\sqrt{\frac{Kh_1}{2D}}\right] \qquad (24)$$

By setting $\frac{dZ_o}{dp} = 0$, we have:

$$G\left(p\right) = 1 + e^{-(a+bp)} - bp + b\sqrt{\frac{Kh_1}{2D}} + bc = 0. \qquad (25)$$

Eq. (25) proofs statement (2). Since

$$\frac{dG\left(p\right)}{dp} = \frac{b}{1+e^{-(a+bp)}}\left\{b\sqrt{\frac{Kh_1}{2D}} - \left[1 + e^{-(a+bp)}\right]^2\right\} \qquad (26)$$

and

$$\frac{d^2G(p)}{dp^2} = b^2e^{-(a+bp)} + \frac{b^3\sqrt{\frac{Kh_1}{2D}}}{2\left[1+e^{-(a+bp)2}\right]}\left[e^{-(a+bp)} + \frac{b^2}{4D}\right] > 0 \quad (27)$$

then $G(p)$ is strictly convex.

At p^-, the price elasticity of demand is equal to 1:

$$\frac{bp^-}{1+e^{-\left(a+bp^-\right)}} = 1,$$

which implies that $G\left(p^-\right)$ is positive. In addition, the price point $-2a/b$ is an upper bound for p^* [24]. Clearly, as p approaches $-2a/b$, $G(p) \to +\infty$, and hence $p^* \in \left(p^-, -\frac{2a}{b}\right)$. At the point p^m, we have

$$G\left(p^m\right) = 2 + a + b\sqrt{Kh_1/C} + bc .$$

If this is negative, since $G(p)$ is strictly convex, we conclude that $p^* < -a/b$. On the other hand, if $G\left(p^m\right)$ is equal to zero, then p^m satisfies Eq. (23), and hence $p^* = -a/b$. This proves statement (3). □

3.2.2 A pricing and inventory replenishment model in a serial supply chain

When extending Model M_o in Subsection 3.2.1 to multiple stages, with known setup cost and holding cost for each stage, we are able to write down the corresponding MINLP formulation as follows (Model M_{os}):

Maximize,

$$Z_{os} = D(p-c) - \left(\frac{DK}{Q_o} + \frac{h_1Q_o}{2}\right) - \left(D\sum_{t=2}^{n}\frac{KS_t}{QS_t} + \frac{1}{2}\sum_{t=2}^{n}QS_te_t\right), \quad (28)$$

subject to

$$Q_o = N_1QS_2, \quad (29)$$

$$QS_t = N_tQS_{t+1}, \quad t = 2,\ldots,n-1, \quad (30)$$

$$N_1 \ge 1,\ integer, \quad (31)$$

$$N_t \ge 1,\ integer, \quad t = 2,\ldots,n-1, \quad (32)$$

$$p, D, Q_o, QS_t > 0,\ t = 2,3,\ldots,n, \quad (33)$$

and (6).

Now, we develop a heuristic algorithm based on Proposition 2. The basic idea is to first treat the optimal retail price for Model M_o as the initial price to find the order quantity for each stage, then adjust the order quantities using the POT policy, and refine the last stage order quantity (QS_n) by solving Model M_{os} given the multiplicative factors. Once QS_n is found, adjust the order quantities at the stages according to the POT policy. Then, solve Model M_{os} with given $Q_o, QS_t > 0, t = 2,3,\ldots,n$, to get an updated retail price p. The procedure is repeated until the multiplicative factors remain unchanged and the percentage error of the retail price p is smaller than a given value.

The algorithm starts with the determination of the initial price and order quantities. For this purpose, we set the optimal price of Model M_o, which is the root of Eq. (23), as the initial price. Let Q_o^0 and QS_t^0 denote the order quantity submitted at stage 1 and stage $t, t = 2,\ldots,n$. Then, we use the following formula to calculate the initial order quantities at each stage:

$$Q_o^0 = \sqrt{\frac{2Ce^{-(a+bp)}K}{h_1\left[1+e^{-(a+bp)}\right]}},$$

and $QS_t^0 = \sqrt{\frac{2Ce^{-(a+bp)}KS}{e_t\left[1+e^{-(a+bp)}\right]}} > 0,\ t = 2,3,\ldots n$.

Next, we adjust order quantities to ensure that the order quantity at each stage is a multiple of the order quantity at the immediate-following stage, where the multiplicative factors N_1 and N_t are powers of two. Here, we adopted the POT procedure in Subsection 3.1. Set $QS_n^{POT} = QS_n^0$, find QS_{n-1}^{POT} so that $QS_{n-1}^{POT} = 2^{x_{n-1}}QS_n^{POT}$. Continue to find $QS_{n-2}^{POT} = 2^{x_{n-2}}QS_{n-1}^{POT}$, ….until $Q_o^{POT} = 2^{x_1}QS_2^{POT}$, where $x_1, x_1, \ldots, x_{n-1}$ are nonnegative integers. Meanwhile, update $D = \frac{h_1 (Q_o^{POT})^2}{2K}$.

Once we have the values of D, p, N_1, and N_t, in the next step, we can rewrite Model M_{os} as a function of QS_n^{POT}:

Maximize,

$$Z_{os} = Dp - \frac{1}{QS_n m_1}\left(DK + \frac{h_1 QS_n^2 m_1^2}{2} + DQS_n cm_1 \right) - \left(\frac{D}{QS_n}\sum_{t=2}^{n}\frac{KS_t}{m_t} + \frac{QS_n}{2}\sum_{t=2}^{n} m_t e_t \right) \quad (34)$$

where $m_t = N_t N_{t+1} ... N_{n-1}, t = 1, 2, \ldots, n-1$.

Similarly, after taking the first derivative of Z_{os} with respect to QS_n^{POT} and setting it to 0, we have:

$$\frac{D}{QS_n^2}\left(\frac{K}{m_1} + \sum_{t=2}^{n}\frac{KS_t}{m_t}\right) - \frac{1}{2}\left(h_1 m_1 + \sum_{t=2}^{n} m_t e_t\right) = 0, \quad (35)$$

and

$$QS_n^* = \sqrt{\frac{2D\left(\frac{k}{m_1} + \sum_{t=2}^{n}\frac{KS_t}{m_t}\right)}{h_1 m_1 + \sum_{t=2}^{n} m_t e_t}}. \quad (36)$$

Update the initial order quantity at each stage given the updated D and repeat the POT procedure starting with the refined value of QS_n^* from Eq. (36).

Lastly, we need to update the selling price. Based on the order quantities obtained from the POT procedure, Z_{os} becomes a function of the retail price. Let the first derivative of Z_{os} with respect to p be zero. Then, we have

$$\frac{dZ_{os}}{dp} = D\left[1 - \frac{b}{1+e^{-(a+bp)}}\left(p - c - \frac{k}{s} - D\sum_{t=2}^{n}\frac{KS_t}{QS_t}\right)\right] = 0 \quad (37)$$

and

$$1 - \frac{b}{1+e^{-(a+bp)}}\left(p - c - \frac{k}{s} - D\sum_{t=2}^{n}\frac{KS_t}{QS_t}\right) = 0. \quad (38)$$

Since $QS_t, t = 2, ..., n$, are treated as input parameters, we can easily obtain the root of Eq. (38), which is the updated selling price. Then, repeating the foregoing steps until the multiplicative factors remain unchanged and the price deviation is within a relative small number, we can terminate the algorithm and report the corresponding results. The algorithm is summarized below.

Heuristic algorithm 2

Step 1. Set $k = 0$, derive the initial selling price p_0 and D_0 according to Proposition 2.

Step 2. Calculate QS_t^0, $t = 2, \ldots, n$ and Q_o^0.

Step 3. Determine $QS_t^{POT}, t = 2, 3, \ldots n-1$ and Q_o^{POT} using the POT procedure.

Step 4. Calculate and update D.

Step 5. Update QS_n using Eq. (36).

Step 6. Repeat step 2 and 3 using D and QS_n obtained in steps 4 and 5, respectively.

Step 7. Solve Eq. (38) for the updated selling price p_k.

Step 8. If $k = 0$, go to step 2. Otherwise, check:

(1) The multiplicative factors remain unchanged for two consecutive iterations.

(2) $\frac{|p_k - p_{k-1}|}{p_{k-1}} < \varepsilon$.

If both (1) and (2) hold, terminate the algorithm and report the result as p_k, $J_i, Q_o^{POT}, N_1, QS_t^{POT}, N_t, t = 2, 3, \ldots, n-1$.

Else, let $k = k + 1$ and go to step 2.

Without employing any optimization software, we can easily and efficiently solve Model M_{os} using Heuristic algorithm 2.

4. Numerical Examples

We present two numerical examples to demonstrate the application and efficiency of the proposed heuristic algorithms by comparing the obtained approximated solutions with the optimal solutions. These examples are conducted using the global solver in LINGO 17.0 and Matlab 2015a on a PC with INTEL® Core™ 2 Duo Processor at 2.1 GHz and 4.0 gigabytes RAM.

4.1 Example 1

Three potential suppliers with capacity and quality constraints are available to provide raw materials for a company to produce a certain product. The minimum quality level for the product is set to 0.95. Finished products are transported sequentially from the manufacturing facility to the local warehouse, two regional warehouses, and then the DC. The relevant parameters regarding suppliers and stages are shown in Tables 1 and 2. The DC's demand rate is price-sensitive and can be represented by the following logit function of the selling price: $D = 5000 \times \frac{e^{-(-6+0.015p)}}{1+e^{-(-6+0.015p)}}$. We need to select suppliers and determine the optimal decisions on pricing, ordering, and inventory at every stage of the serial system.

We start the heuristic by setting an initial price for the product, which is obtained from Proposition 1 in Subsection 3.1. In this example, we start with $p_0 = \$312.9$/unit. The algorithm terminates after two iterations, as there is no further update with respect to the number of orders placed to each selected supplier as well as the multiplicative factors. Table 3 shows the corresponding results at each iteration. The algorithm stops with an optimal selling price of $335.79/unit. As a result, the overall profit of this serial supply chain system is $821,815.7/month.

These results are compared with the optimal solution when we solve the model directly using the global solver in LINGO 17.0 (see Table 4). From Table 4, one observes that the resulting profit by implementing the heuristic is only 0.05% less than the optimal profit while the CPU time drops from one minute to less than 1 second. Therefore, the proposed heuristic works efficiently with near optimal results.

Table 1. Parameters for suppliers

Supplier (*i*)	F_i (units/month)	K_i ($/order)	q_i	c_i ($/unit)
1	2,000	3,500	0.92	86
2	2,500	2,900	0.95	92
3	2,200	3,300	0.98	103

Table 2. Parameters for stages

Stage (*t*)	h_t ($/unit/month)	e_t ($/unit/month)	KS_t ($/order)
1	5	5	-
2	15	10	2,000
3	28	13	1,300
4	45	17	900
5	70	25	800

Table 3. Detailed solution of Heuristic 1 for Model M_s

Stage (t)		**Initial Values**	**Iteration 1**			**Iteration 2**		
		Q_i / QS_t (units/order)	J_i	N_{1i} / N_t	Q_i / QS_t (units/order)	J_i	N_{1i} / N_t	Q_i / QS_t (units/order)
1	Supplier 1	2347.02	2	2	2007.24	2	2	2103
	Supplier 2	2136.39	6	2	2007.24	6	2	2103
	Supplier 3	2278.97	2	2	2007.24	2	2	2103
2		1254.53		1	1003.62		1	1051.5
3		887.09		2	1003.62		2	1051.5
4		645.45		1	501.81		1	525.75
5		501.81		-	501.81		-	525.75
Retail price ($/unit)		312.9	335.81			335.79		
Revised order quantity at last stage (units/order)		501.81	525.75			525.77		

Table 4. Comparison between Heuristic 1 solution and the optimal solution for Model M_s

Stage (t)		Heuristic 1		Optimal solution	
		N_{1i} / N_t	Q_i / QS_t (units/order)	N_{1i} / N_t	Q_i / QS_t (units/order)
1	Supplier 1	2	2103	2	1986.8
	Supplier 2	2	2103	3	2980.2
	Supplier 3	2	2103	2	1986.8
2		1	1051.5	1	993.4
3		2	1051.5	2	993.4
4		1	525.75	1	496.7
5		-	525.75	-	496.7
Retail price ($/unit)		335.79		337.53	
Total profit ($/month)		821,815.7		822,224.4	
Profit Deviation (%)		0.05		-	

4.2 Example 2

Example 1 from [16] considers a five-stage serial supply chain system including a manufacturer, warehouses and a DC to meet customers' demand. After analysing the historical sales, the company concludes that the form of the price-sensitive demand function is $D(p) = C \times \frac{e^{-(a+bp)}}{1+e^{-(a+bp)}}$, where $C = 3E+04,\quad a = -4,\ b = 0.05$. The manufacturer requires a minimum quality level of 95%. Now, we revisit this problem and determine the company's optimal pricing and replenishment policy. The relevant parameters for suppliers and stages are shown in Tables 5 and 6, respectively.

Table 5. Parameters for suppliers

Supplier (i)	F_i (units/month)	K_i ($/order)	q_i	c_i ($/unit)
1	30,000	5,000	0.94	18
2	25,000	1,500	0.92	15
3	36,000	4,500	0.96	24
4	28,000	3,500	0.98	30

Table 6. Parameters for stages

Stage (t)	h_t ($/unit/month)	e_t ($/unit/month)	KS_t ($/order)
1	5	5	-
2	20	15	200
3	50	30	150
4	95	45	100
5	145	50	50

According to Proposition 1, we set the initial unit selling price of the item p_0 to $64.16. After two iterations, the algorithm terminates as we observe the unchanged values of the selling price and the number of orders from each selected supplier.

Table 7 shows the detailed results with respect to the proposed Heuristic 1 algorithm and the optimal solution. The relative deviations of the retail price and the profit are 0.7% and 0.03%, respectively. While the optimal decisions suggest selecting only suppliers 1 and 4, the heuristic algorithm solutions instead choose suppliers 1, 3, and 4. Furthermore, none of the selected suppliers completely utilizes the capacity in order to meet the minimum quality level.

Next, let us consider Model M_{os} with supplier 1 as the only raw material provider and apply Heuristic algorithm 2 searching for the solutions. By solving Eq. (23) in Matlab 2015a, we obtain p_o^*=$70.68. Then, the order quantities placed at each stage are calculated accordingly and shown in Table 8.

Then the initial order quantities in Table 8 are adjusted so that the order quantity at each stage is an integer multiple of that in the successive stage and the integer multiplicative factors must be powers of two, as shown in Table 9.

Once we have the integer multiplicative factors, we update D and refine QS_n^k according to Eq. (36), which becomes 185.12 units. Then, we repeat the POT procedure to get the updated order quantities at each stage, as shown in Table 10.

Now, using the data in Table 10 as input parameters, we solve Eq. (38) for the selling price, i.e., $p = \$71.17$ /unit. With this retail price, in iteration 2, repeat the same process and the results in Table 11 are achieved.

Table 7. Comparison between Heuristic 1 solution and the optimal solution for Model M_s

Stage (t)		Heuristic solution			Optimal solution		
		J_i	N_{1i} / N_t	Q_i / QS_t (units/order)	J_i	N_{1i} / N_t	Q_i / QS_t (units/order)
1	Supplier 1	7	8	5702.72	7	9	6223.25
	Supplier 2	0	0	0	0	0	0
	Supplier 3	1	8	5702.72	0	0	0
	Supplier 4	2	8	5702.72	3	7	4840.31
2			2	712.84		2	691.47
3			1	356.42		1	345.74
4			2	356.42		2	345.74
5			-	178.21		-	172.87
Retail price ($/unit)		71.86			72.37		
Total profit ($/month)		841,191.71			841,440.6		
Profit Deviation (%)		0.03			-		
CPU time (sec.)		6.01			112.54		

Table 8. Initial order quantities for each stage

Stage	**1**	**2**	**3**	**4**	**5**
Order quantity (units/order)	6071.75	701.11	429.34	286.23	192.01

Table 9. Adjusted POT order quantities for each stage

Stage	**1**	**2**	**3**	**4**	**5**
Order quantity (units/order)	6144.18	768.02	384.01	384.01	192.01
Multiplication factor	8	2	1	2	1

Table 10. Adjusted POT order quantities for each stage using refined QS_n

Stage	**1**	**2**	**3**	**4**	**5**
Order quantity(units/order)	5923.84	740.48	370.24	370.24	185.12
Multiplicative factor	8	2	1	2	1

Table 11. Order quantities for each stage in iteration 2

Stage	**1**	**2**	**3**	**4**	**5**
Order quantity (units/order)	5895.68	736.96	368.48	368.48	184.24
Multiplicative factor	8	2	1	2	1

Meanwhile, the selling price remains at $p = \$71.17$ /unit. Since the multiplicative factors and the selling price remain unchanged, the algorithm stops.

Without using any optimization software, M_{os} can be solved with the profit error of less than 0.001% deviation from the optimal profit obtained by running the model in Lingo17 (see Table 12). This measurement shows the efficiency of algorithm 2 for Model M_{os} .

Table 12. Comparison between Heuristic 2 solution and the optimal solution for Model M_{os}

Stage (t)	Heuristic 2 solution		Optimal solution	
	N_t	QS_t (units/order)	N_t	QS_t (units/order)
1	8	5895.68	9	6255.57
2	2	736.96	2	695.06
3	1	368.48	1	347.53
4	2	368.48	2	347.53
5	-	184.24	-	173.77
Retail price ($/unit)	71.17		71.18	
Total profit ($/month)	894,331.1		894,339.8	
Profit Deviation (%)	< 0.001		-	
CPU time (sec.)	< 1		16.19	

5. Implication and analysis

5.1. Impact of the pricing parameters

In the logit demand function, there are three basic parameters, which are C, a, and b. Parameter C denotes the whole market size the product faces, parameter b denotes the steepness of the demand function while parameter a indicates the wideness of the demand curve.

In order to capture the effect of the demand function parameters on the optimal decisions, we first analyse 10 scenarios by varying parameter b while fixing all other parameters. We know that as b increases, the logit demand function becomes steeper and closer to the y axis. From Figure 3 (1) and (2), it can be observed that both the monthly profit and retail price decrease as b mounts. The increasing value of b indicates a greater demand change as price changes. Therefore, it is intuitive to decrease the retail price for the purpose of improving the sales. Another reasonable explanation is that the lower bound on the retail price also decreases as b goes up.

In addition, the impact of parameter b on the number of selected suppliers and the corresponding order quantities from the selected suppliers is obvious while there is no influence of b on the multiplicative factors for the stages, which always stay as 2, 1, and 2 for all the instances. In sum, the value of parameter b has a significant influence on the raw material sourcing decisions as well as the order quantities at stage 1 while less impact has been noticed on the inventory replenishment policy at the following stages (see Table 13).

Table 13. Procurement policies with respect to parameter b

b	**Price** ($/unit)		**Profit** ($/month)	**Selected suppliers**	**Supplier multipl. factors**	**Stage multipl. factors**	**Order quantity at last stage** (units/order)
	Lower bound	**Optimal**					
0.01	320.79	328.13	6,115,628	1,4	9,7	2,1,2	183.88
0.02	160.40	168.57	2,783,468	2,4	2,8	2,1,2	160.48
0.03	106.93	114.92	1,700,605	2,3	4,8	2,1,2	183.55
0.04	80.20	88.27	1,161,207	1,2,3	12,3,7	2,1,2	177.42
0.05	64.16	72.37	841,441	1,4	9,7	2,1,2	172.87
0.06	53.47	61.93	627,770	1,4	9,7	2,1,2	169.39
0.07	45.83	54.57	477,590	1,4	9,7	2,1,2	165.54
0.08	40.10	49.13	367,266	1,4	9,7	2,1,2	161.3
0.09	35.64	44.99	283,673	1,4	9,7	2,1,2	156.61
0.1	32.08	43.52	180,114	3	8	2,1,2	146.85

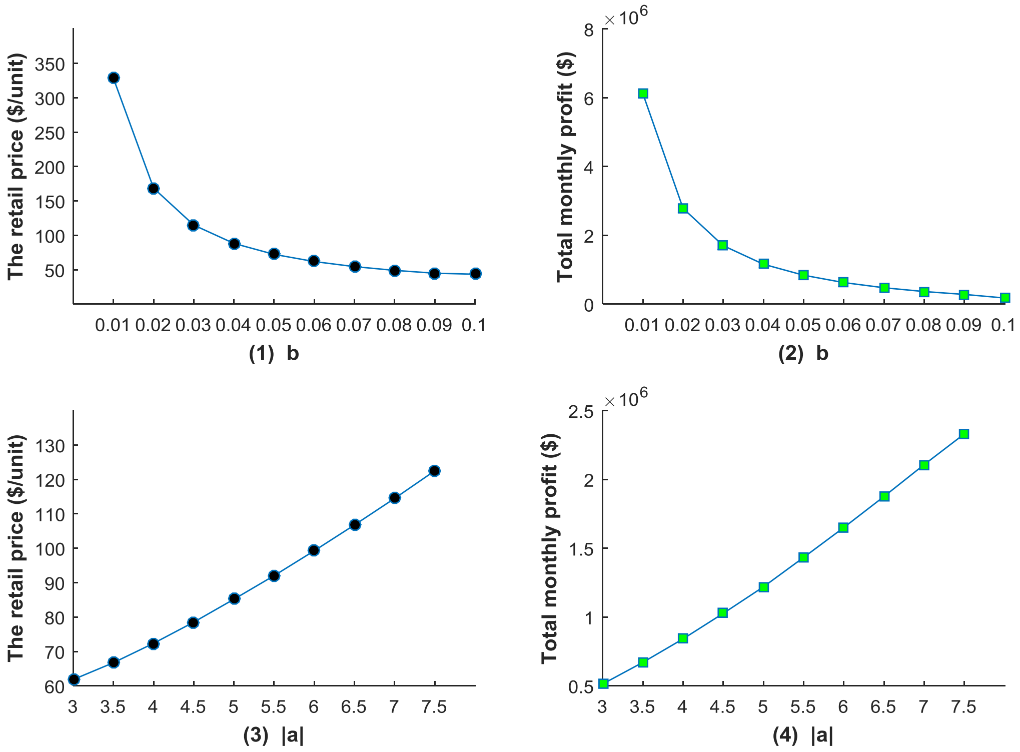

Figure 3. Effect of the price parameters b and a :

(1) retail price vs. b , (2) profit vs. b , (3) retail price vs. a , (4) profit vs. a

Next, let us study the effect of parameter a on the optimal decisions for the serial supply chain system. We consider 10 scenarios by increasing $|a|$ by 0.5 at a time while fixing all other parameters. Figure 3 (3) and (4) show the effect of $|a|$ on the selling price and monthly profit. As $|a|$ goes up, both the selling price and monthly profit show an increasing trend. However, compared to b, both the selling price and monthly profit are less sensitive to $|a|$. Meanwhile, variation of $|a|$ leads to distinct purchasing and inventory replenishment policies at each stage, however, the multiplicative factors from stage 2 to the last stage do not vary as we keep increasing parameter $|a|$ (Table 14).

Table 14. Procurement policies with respect to parameter a

$\lvert a \rvert$	**Price** ($/unit)		**Profit** ($/month)	**Supplier selection**	**Supplier multipl. factors**	**Stage multipl. factors**	**Order Quantity at last stage** (units/order)
	Lower Bound	**Optimal**					
3	51.14	61.84	513,785	1,2,3	12,6,6	2,1,2	159.71
3.5	57.45	66.73	670,989	1,4	9,7	2,1,2	164.91
4	64.16	72.37	841,441	1,4	9,7	2,1,2	172.87
4.5	71.20	78.52	1,026,716	1,4	9,7	2,1,2	179.36
5	78.53	85.18	1,219,217	1,2,3	11,2,8	2,1,2	184.96
5.5	86.09	92.03	1,433,015	1,4	9,7	2,1,2	189.05
6	93.87	99.27	1,650,148	1,3,4	9,9,6	2,1,2	192.75
6.5	101.82	106.76	1,875,532	1,4	9,7	2,1,2	195.72
7	109.93	114.48	2,107,129	1,4	9,7	2,1,2	198.29
7.5	118.18	122.51	2,330,181	2,3	4,8	2,1,2	205.83

It is noticeable that multiple suppliers are not fully utilized in some instances, which has been proved in [15]. Thus, [25]'s claim that there exists an optimal solution where at most one of the selected suppliers does not fully utilize its capacity for the optimization problem with multiple capacitated suppliers and constant demand does not apply to our problem. In our case, considering the quality constraint and price-sensitive demand, it is possible to select multiple suppliers without fully utilizing their capacities.

5.2 Impact of the number of preferred suppliers

To study the effect of the number of suppliers on the raw material procurement decisions, we next perform sensitivity analysis with respect to the number of the preferred suppliers in the supplier pool, that is to say, by varying the number of candidates available.

Now, we consider an even larger pool of suppliers, based on the data provided in [26], [27], with 20 possible vendors. Since we are using new data with a large pool of suppliers, we intend on ranking the suppliers and then proceed with solving the supplier selection problem considering a different number of preselected suppliers. We have not selected AHP for ranking as suggested in [27], because we do not have an experienced decision maker; instead, we use three alternative ranking mechanisms that have been widely recommended in the multi-criteria literature [28]. These are metrics L_1 and L_∞, and the Borda count method. Below we briefly describe each of them as detailed in [28]:

L_p

This metric is a measure of the distance between two vectors x and y and is given by the equation below:

$$x - y_p = \left[\sum_{j=1}^{n} \left| x_j - y_j \right|^p \right]^{\frac{1}{p}}.$$

We consider two cases for the value of p as two separated ranking methods to rank the suppliers. These two cases are $p = 1 \; and \; p = \infty$. In the second case, when p tends to infinity, the value of the distance is calculated as shown below:

$$L_\infty = \max_{j=1,\dots,n} \left| x_j - y_j \right|.$$

Borda count

This method considers n criteria to rank different alternatives. The first step is to rank the criteria from 1 to n, being 1 the least important and n the most important. Next, for each criterion, its corresponding weight is calculated as follows:

$$\text{Criterion ranked } 1 = \frac{n}{s},$$

$$\text{Criterion } 2 = \frac{(n-1)}{s}, \dots ,$$

$$\text{Last criterion } = \frac{1}{s} \text{ , where } s = \frac{n(n-1)}{2}.$$

Then each alternative is evaluated by summing each criterion multiplied by its corresponding weight value. The higher the sum, the higher the ranking for that alternative.

Table 15. Ranking of suppliers according to each method

Supplier	L_1	L_∞	Borda
1	15	20	6
2	2	7	5
3	9	2	13
4	13	12	15
5	1	3	4
6	8	18	2
7	5	13	11
8	6	14	10
9	18	19	16
10	19	15	19
11	10	16	7
12	14	8	17
13	11	4	12
14	16	10	3
15	7	5	14
16	20	17	20
17	17	11	18
18	12	6	1
19	3	9	9
20	4	1	8

The criteria used for each ranking method are: setup cost, capacity, unit price, quality level and service level. The last one is described as the percentage of the units that are expected to be received at the desired lead time. The only criterion not used in the actual mathematical model is the service metric, but we assume it is related to the setup cost of the supplier. Tables 16 and 17 in the Appendix show the data utilized in this example and the scaled data used for the ranking methods, respectively. To use Borda count, we consider the following order for the selection criteria: unit price, quality level, setup cost, service level, and capacity. After using the methods described above, the resulting ranking is shown above in Table 15.

Figure 4 shows how the profit changes when a different pool of suppliers is considered. We would expect the profit to increase until a certain set of suppliers becomes available. Indeed, in the three cases, the maximum profit achievable is \$2,950,253 with a price of \$166.53/unit and a demand of 19,834 units/month. The curve described by the changes in profit follows a step function-type shape, where the total profit increases, and remains the same until a supplier with potential to increase the objective function comes in, which is in line with the intuition behind this experiment. Clearly, Borda count achieves this maximum profit earlier than the other two methods. The latter can be explained given that this method assigns a relative importance to each criterion, and within this pre-emptive approach the cost parameters were the criteria with the higher rank. As these cost parameters are the ones included in Model M_s, we would expect that a set of suppliers ranked using Borda count generates a more cost-conscious pool of vendors.

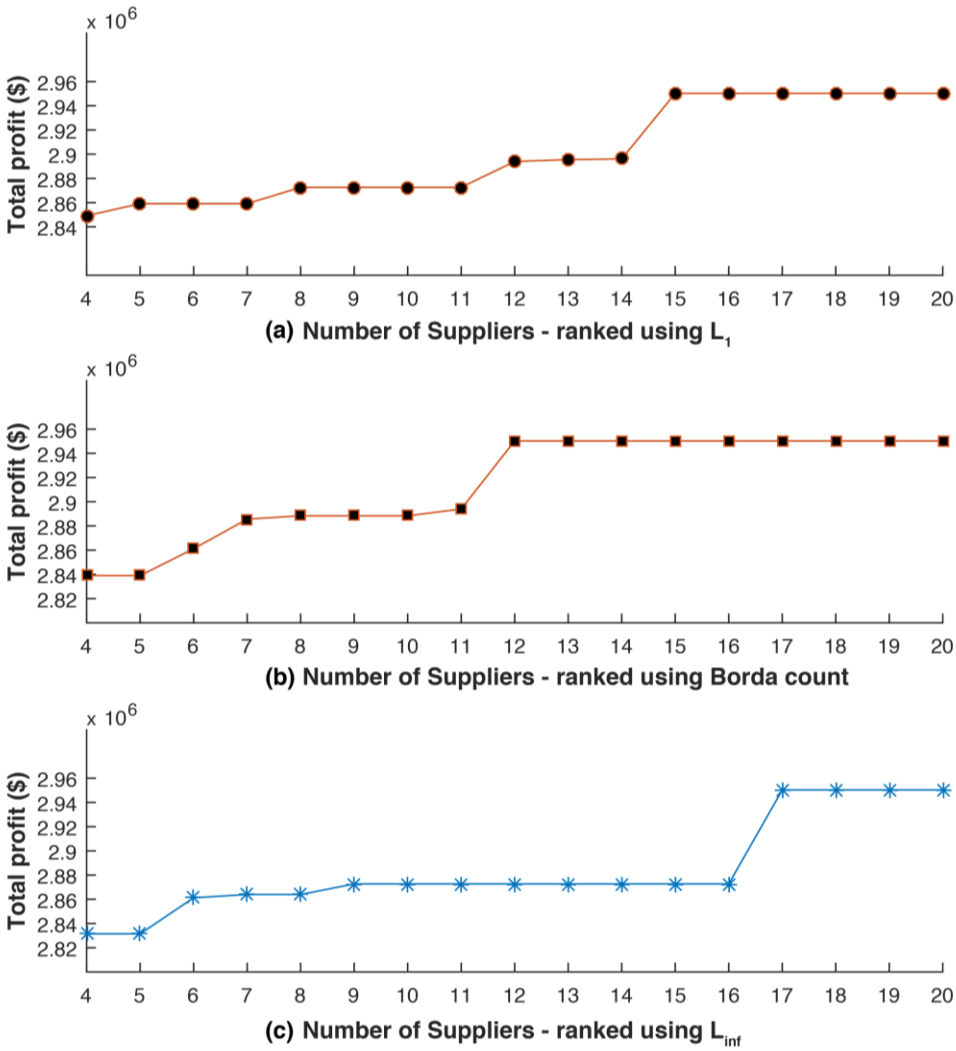

Figure 4. Impact of the number of preferred suppliers: (a) total profit with suppliers ranked with L_1; (b) total profit with suppliers ranked using Borda count; (c) total profit with suppliers ranked using L_∞

6. Conclusions and Future Research

In this paper, we investigate an integrated supplier selection, inventory replenishment, and pricing problem for a serial supply chain facing price-sensitive demand, which can be represented by a logit demand function. Two MINLP models and the corresponding solution algorithms are discussed and developed in this research. First, an MINLP formulation is developed to determine the strategic decisions in the selection of suppliers, order frequency and order quantity from each selected vendor, inventory lot sizes between consecutive stages, and the selling price. Then, a heuristic algorithm is proposed to obtain a near optimal solution in a timely manner. Second, we develop an MINLP formulation and propose a heuristic algorithm when considering a single uncapacitated supplier in a serial supply chain. These two algorithms are then implemented, and their performance is illustrated by solving two examples. Furthermore, we conduct a computational analysis with respect to the pricing parameters in the logit demand function and show their significant influence on the decisions of supplier selection, order allocation, and retail price. Lastly, after ranking the candidate suppliers with various ranking methods, we analyse the impact of the number of suppliers in the supplier pool on the final selection of suppliers and the corresponding profit per time unit. It is concluded that, for the different ranking methods, the optimal number of suppliers that leads to the maximum profit per time unit varies.

Considering that the current research is for a single product, it is natural to extend the proposed models to multiple products with various price-sensitive demand functions. Meanwhile, as stated previously, the proposed algorithms can be generalized to obtain near optimal solutions on pricing, lot sizing, supplier selection, and inventory replenishment when the retailer faces other demand functions, such as the linear function or exponential function. Therefore, it would be interesting to implement these algorithms with various demand functions and compare the corresponding performance efficiency. In addition, it could be important to include supply contracts and supplier quantity discount offers into our proposed models.

Appendix

Table 16. Data for 20 suppliers

Supplier (i)	F_i (units/month)	K_i ($/order)	q_i	c_i ($/unit)	**Service**
1	11000	5000	0.94	15	0.98
2	23300	3000	0.92	18	0.9
3	30200	4500	0.96	24	0.98
4	25700	3500	0.98	30	0.95
5	24800	3000	0.92	18	0.92
6	12800	3200	0.9	15	0.9
7	39700	6000	0.96	20	0.9
8	27100	3000	0.97	30	0.95
9	11400	3400	0.98	28	0.96
10	19200	5400	0.98	26	0.91
11	18900	4000	0.95	17	0.92
12	30900	4200	0.95	28	0.95
13	31100	4900	0.98	22	0.91
14	20800	2300	0.97	23	0.95
15	35800	4700	0.97	25	0.97
16	14900	5700	0.93	26	0.96
17	20200	4100	0.97	28	0.99
18	23500	3700	0.90	12	0.91
19	29600	3000	0.97	28	0.95
20	31400	4500	0.91	19	0.97

Table 17. Scaled data for 20 suppliers

Supplier (i)	F_i (units/month)	K_i ($/order)	q_i	c_i ($/unit)	**Service**
1	0.28	0.60	0.96	1.00	1.00
2	0.59	1.00	0.94	0.83	0.92
3	0.76	0.67	0.98	0.63	1.00
4	0.65	0.86	1.00	0.50	0.97
5	0.62	1.00	0.94	0.83	0.94
6	0.32	0.94	0.92	1.00	0.92
7	1.00	0.50	0.98	0.75	0.92
8	0.68	1.00	0.99	0.50	0.97
9	0.29	0.88	1.00	0.54	0.98
10	0.48	0.56	1.00	0.58	0.93
11	0.48	0.75	0.97	0.88	0.94
12	0.78	0.71	0.97	0.54	0.97
13	0.78	0.61	1.00	0.68	0.93
14	0.52	1.30	0.99	0.65	0.97
15	0.90	0.64	0.99	0.60	0.99
16	0.38	0.53	0.95	0.58	0.98
17	0.51	0.73	0.99	0.54	1.01
18	0.59	0.81	0.92	1.25	0.93
19	0.75	1.00	0.99	0.54	0.97
20	0.79	0.67	0.93	0.79	0.99

References

[1] Basnet C, Leung JMY (2005) Inventory lot-sizing with supplier selection. Comput Oper Res 32(1), 1–14.

[2] Rezaei J, Davoodi M (2008) A deterministic, multi-item inventory model with supplier selection and imperfect quality. Appl Math Model 32(10), 2106–2116.

[3] Mendoza A, Ventura JA (2010) A serial inventory system with supplier selection and order quantity allocation. Eur J Oper Res 207(3), 1304–1315.

[4] Ventura JA, Valdebenito VA, Golany B (2013) A dynamic inventory model with supplier selection in a serial supply chain structure. Eur J Oper Res 230(2), 258–271.

[5] Pazhani S, Ventura JA, Mendoza A (2016) A serial inventory system with supplier selection and order quantity allocation considering transportation costs. Appl Math Model 40(1), 612–634.

[6] Whitin TM (1955) Inventory Control and Price Theory. Manage Sci 2(1), 61–68.

[7] Ghasemy Yaghin R, Fatemi Ghomi SMT, Torabi SA (2014) Enhanced joint pricing and lotsizing problem in a two-echelon supply chain with logit demand function. Int J Prod Res 52(17), 4967–4983.

[8] Wang C, Huang R, Wei Q (2015) Integrated pricing and lot-sizing decision in a two-echelon supply chain with a finite production rate. Int J Prod Econ 161:44–53.

[9] Bhattacharjee S, Ramesh R (2000) Multi-period profit maximizing model for retail supply chain management: An integration of demand and supply-side mechanisms. Eur J Oper Res 122(3), 584–601.

[10] Biller S, Chan LMA, Simchi-Levi D, Swann J (2005) Dynamic pricing and the direct-to-customer model in the automotive industry. Electron Commer Res 5(2), 309–334.

[11] Deng S, Yano CA (2006) Joint production and pricing decisions with setup costs and capacity constraints. Manage Sci 52(5), 741–756.

[12] Ho W, Xu X, Dey PK (2010) Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. Eur J Oper Res 202(1), 16–24.

[13] Qi X (2007) Order splitting with multiple capacitated suppliers. Eur J Oper Res 178(2), 421–432.

[14] Rezaei J, Davoodi M (2012) A joint pricing, lot-sizing, and supplier selection model. Int J Prod Res 50(16), 4524–4542.

[15] Adeinat H, Ventura JA (2015) Determining the retailer's replenishment policy

considering multiple capacitated suppliers and price-sensitive demand. Eur J Oper Res 247(1), 83–92.
[16] Adeinat H, Ventura JA (2018) Integrated pricing and lot-sizing decisions in a serial supply chain. Appl Math Model 54:429–445.
[17] Huang J, Leng M, Parlar M (2013) Demand functions in decision modeling: A comprehensive survey and research directions. Decis Sci 44(3), 557–609.
[18] Love SF (1972) A facilities in series inventory model with nested schedules. Manage Sci 18(5-part-1), 327–338.
[19] Muckstadt J, Roundy R (1993) Analysis of Multistage Production Systems. In: Handbooks in Operations Research and Management Science, Volume 4-Chapter 2. Elsevier, pp 59–131.
[20] Schwarz LB, Schrage L (1975) Optimal and system myopic policies for multi-echelon production/inventory assembly systems. Manage Sci 21(11), 1285–1294.
[21] Hahm J, Yano CA (1995) The economic lot and delivery scheduling problem: powers of two policies. Transp Sci 29(3), 222–241.
[22] Ouenniche J, Boctor FF (2001) The two-group heuristic to solve the multi-product, economic lot sizing and scheduling problem in flow shops. Eur J Oper Res 129:539–554.
[23] Roundy R (1986) A 98 % -effective lot-sizing rule for a multi-product, multi- stage production / inventory system. Mathematics of Operations Research 11 (4), 699-727.
[24] Duan L, Ventura JA (2018) A joint pricing, supplier selection, and inventory replenishment model using the logit demand function. IME Working Paper, Pennsylvania State University, University Park, PA.
[25] Rosenblatt MJ, Herer YT, Hefter I (1998) Note. An acquisition policy for a single Item multi-supplier system. Manage Sci 44(11-part-2), S96–S100.
[26] Saaty T (2008) Decision making with the analytic hierarchy process. Int J Serv Sci 1(1), 83–98.
[27] Mendoza A, Ventura JA (2008) An effective method to supplier selection and order quantity allocation. Int J Bus Syst Res 2(1), 1–15.
[28] Ravindran A, Warsing D (2013) Supply Chain Engineering: Models and Applications. Taylor & Francis, Boca Raton, FL.