Constructing Two-Sided Group Chain Acceptance Sampling Plans for Non-Symmetrical Data

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***Abstract*— In developing acceptance sampling plan, the mean is often the choice due to the fact that it is a good estimator. However, it is only a good estimator when dealing with symmetrical data. If the data is not symmetrical, then the mean is no longer a good estimator, but the median is. The recent two-sided group chain acceptance sampling plans (TSGChSP) has only been applied to Pareto distribution of the 2nd kind by using the mean to cater for symmetrical data. In reality, some data are actually non-symmetrical. Hence, motivated by such scenario, the aim of this study is to produce the TSGChSP for Pareto distribution of the 2nd kind by using the median. The performances of TSGChSP are measured based on minimum number of groups, g and probability of lot acceptance, $L(p)$. The g is calculated by satisfying the consumer's risk while the $L(p)$ is obtained at several values of median ratio. Finally, the number of million revolutions for 23 ball bearings is shown in order to show the application of the TSGChSP in the industry.**

***Keywords*—** *Two-sided group chain acceptance sampling plans, Pareto distribution of the 2nd kind, Consumer's risk, Non-symmetrical data.*

1. Introduction

Group chain acceptance sampling plan (GChSP) is essentially a combination of chain acceptance sampling plan (ChSP) and group acceptance sampling plan (GSP), as initiated by Mughal, Zain and Aziz in 2015 [1]. The purpose of the GChSP was to rectify the issues in the previous sampling plans, particularly the time consuming inspection (ChSP) and the unstable probability of lot acceptance (GSP). By introducing the GChSP, they managed to rectify the problems in sampling plans, whereby the GChSP requires less time for the inspection, and manages to stabilize the probability of lot acceptance. To-date, the GChSP has been developed by using two different lifetime distributions [2]-[3].

The problem in the GChSP is that it only considers the preceding lots, i and overlooks the succeeding lots, j. In order to solve this problem, Mughal [4] proposed the two-sided group chain acceptance sampling plans (TSGChSP), where the suggested sampling plans consider the i and j lots. The TSGChSP provides better protection to the consumer and producer as it emphasizes that good product must be produced in continuation (chain element).

For the TSGChSP, Mughal [4] has developed the plan for Pareto distribution of the 2nd kind using the mean as quality parameter. The mean works well for symmetrical data, but it has problem when the data is skewed (non-symmetrical). For non-symmetrical data, the median overpowers the mean as quality parameter; the median is a good estimator instead of the mean. Therefore, this article develops the TSGChSP for Pareto distribution of the 2nd kind by taking the median as quality parameter.

The median has been used consistently by some researchers in acceptance sampling. Ramaswamy and Jayasri [5] applied the median when they developed the ChSP for Marshall Olkin extended exponential distribution. Apart from them, Al-Nasser and Gogah [6] used the median ranked for developing the reliability test plans, while Gogah and Al-Nasser [7] also used the median ranked technique when developing the single acceptance sampling plans (SSP) for exponential distribution.

2. Glossary of Symbols

g : Minimum number of groups
$L(p)$: Probability of lot acceptance
β : Consumer's risk
r : Number of products

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)
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t_0 : Test termination time
E : Total number of defectives
d : Number of defectives in the current lot
i : Number of preceding lots
d_i : Number of defective in the i lots
j : Number of succeeding lots
d_j : Number of defectives in the j lots
p : Fraction defective
a : Specified constant
λ : Shape parameter
$\frac{\mu_{median}}{\mu_{median\_0}}$: Median ratio

3. Operating Procedure

The operating procedure for the TSGChSP, as documented by Mughal [4] is as follows:

i. For each lot, the minimum number of groups, g is found satisfying $L(p) \leq \beta$.
ii. The number of products, r is allocated to the g groups.
iii. The test termination time, t_0 for the inspection activity is specified. During the inspection activity, the total number of defectives, $E(E = d_i + d + d_j)$ is counted.
iv. The current lot is accepted when $E = 0$.
v. The current lot is also accepted when $E = 1$ given that d_i or d_j only have one defective.
vi. The current lot is rejected when $E > 1$.

4. Probability of lot acceptance

Mughal [4] has shown that the $L(p)$ for the TSGChSP is given by:

$$L(p) = (1-p)^{gr(2i+1)}\left[\frac{2grip}{1-p} + 1\right]. \quad (1)$$

5. Fraction Defective

Fraction defective, p is defined as the time taken before a product stops to function [8] and, it is usually given by its cumulative distribution function (CDF). In this article, the CDF of Pareto distribution of the 2nd kind is chosen in order to develop the TSGChSP.

The Pareto distribution of the 2nd kind has been used by numerous researchers when developing the sampling plans. Among them are Aslam et al. [9] for GSP, Aslam et al. [10] for economic reliability of GSP and Aslam et al. [11] for reliability SSP based on progressive censoring.

In this article, the p is written as:

$$p = 1 - \left[1 + \frac{a(\sqrt[\lambda]{2} - 1)}{(\mu_{median}/\mu_{median\_0})}\right]^{-\lambda}. \quad (2)$$

The performances of the TSGChSP are measured based on (i) g and, (ii) $L(p)$. The g is obtained by solving:

$$L(p) \leq \beta. \quad (3)$$

The $L(p)$ is calculated by using equation (1) at different values of median ratio, ranging from 1 to 12.

In order to measure the performances of TSGChSP, different values of design parameters are used. The values are $\lambda = \{1, 2, 3\}$, $\beta = \{0.01, 0.05, 0.10, 0.25\}$, $a = \{0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\}$ and $(i, r) = \{(1,2), (2,3), (3,4), (4,5)\}$.

6. Results, Discussion and Application

The first aspect in measuring the performances of sampling plans in the g. Plans with low g are always better compared to those with higher g since the former reduce the inspection time. Tables 1 to 3 show the g for the TSGChSP based on Pareto distribution of the 2nd kind with different shape parameters.

Table 1. The g for Pareto distribution of the 2nd kind when λ=1

β	i	r	α							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	5	3	2	2	2	2	1	1
	2	3	2	2	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.05	1	2	4	2	2	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.10	1	2	3	2	1	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.25	1	2	2	1	1	1	1	1	1	1
	2	3	1	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1

Table 2. The g for Pareto distribution of the 2nd kind when λ=2

β	i	r	α							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	6	3	2	2	2	1	1	1
	2	3	3	2	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.05	1	2	4	2	2	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.10	1	2	3	2	1	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.25	1	2	2	1	1	1	1	1	1	1
	2	3	1	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1

Table 3. The g for Pareto distribution of the 2nd kind when λ=3

β	i	r	α							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	6	3	2	2	2	1	1	1
	2	3	3	2	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.05	1	2	4	2	2	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.10	1	2	3	2	1	1	1	1	1	1
	2	3	2	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1
0.25	1	2	2	1	1	1	1	1	1	1
	2	3	1	1	1	1	1	1	1	1
	3	4	1	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1

Tables 1 to 3 provide a guideline for the industrial practitioners in choosing the suitable g when designing the TSGChSP. If a product has Pareto distribution of the 2nd kind $(\lambda = 1)$ as its lifetime distribution and the industrial practitioners tend to design the inspection at the 1% consumer's risk with the inspection platform only allows two products to be placed on the platform, then the g required is 6. The 6 groups is selected if the industrial practitioners decides to truncate the inspection time at 25%. If they decide to stop the inspection time at the actual lifetime of the product (100%), then the g needed is 2.

The second aspect of the performances measurement is the $L(p)$. In this article, the $L(p)$is calculated at different values of median ratio. The median ratio represents the quality of products, which means higher median ratio indicates the products have higher lifetime.

Based on Tables 4 to 6, it tells the industrial practitioners that the products with higher median ratio have higher $L(p)$ compared to the products with lower median ratio. For instance, the $L(p)$ has range from 0.00594 to 0.74760 if the industrial practitioners produce a product with different median ratio (1 to 12), conditions that the product has Pareto distribution of the 2nd kind $(\lambda = 1)$, consumer's risk is at 1% and the inspection time is truncated at 25%. This is not surprising as if the industrial practitioners produce better quality products (higher median ratio), then the consumer (market) is willing to buy the products.

Mughal [4] proved that the number of million revolutions of 23 ball bearings exhibited Pareto distribution of the 2nd kind with 2 as the shape parameter. If the same ball bearings is inspected using the TSGChSP for skewed data, then the number of groups required is 2 when the other design parameters are $(\beta, a, i, r) = (0.01, 1, 1, 2)$. Based on the information, the industrial practitioners need to take a sample of 4 and place 2 products into 2 groups. The lot is accepted if the total defective recorded is zero or one provided that there is at most one defective in the previous or succeeding 1 lot. Otherwise, reject the lot.

Table 4. The $\boldsymbol{L(p)}$ for TSGChSP when $\boldsymbol{\lambda = 1}$

β	a	g	$\frac{\mu}{\mu_0}$						
			1	2	4	6	8	10	12
0.01	0.25	5	0.00594	0.09085	0.34355	0.51719	0.62599	0.69767	0.74760
	0.5	3	0.00316	0.05765	0.26671	0.43707	0.55308	0.63317	0.69063
	0.75	2	0.00485	0.06370	0.26773	0.43256	0.54589	0.62490	0.68205
	1	2	0.00110	0.02569	0.16493	0.31453	0.43256	0.52140	0.58876
	1.25	2	0.00029	0.01089	0.10204	0.22779	0.34081	0.43256	0.50558
	1.5	2	0.00009	0.00485	0.06370	0.16493	0.26773	0.35756	0.43256
	1.75	1	0.00673	0.05523	0.21681	0.36119	0.46946	0.54977	0.61055
	2	1	0.00412	0.03906	0.17558	0.31146	0.41943	0.50235	0.56653
0.05	0.25	4	0.01889	0.15787	0.43935	0.60066	0.69501	0.75515	0.79628
	0.5	2	0.02569	0.16493	0.43256	0.58876	0.68205	0.74245	0.78427
	0.75	2	0.00485	0.06370	0.26773	0.43256	0.54589	0.62490	0.68205
	1	1	0.03906	0.17558	0.41943	0.56653	0.65769	0.71842	0.76138
	1.25	1	0.02055	0.11698	0.33534	0.48745	0.58815	0.65769	0.70799
	1.5	1	0.01147	0.07958	0.26904	0.41943	0.52554	0.60150	0.65769
	1.75	1	0.00673	0.05523	0.21681	0.36119	0.46946	0.54977	0.61055
	2	1	0.00412	0.03906	0.17558	0.31146	0.41943	0.50235	0.56653
0.10	0.25	3	0.05765	0.26671	0.55308	0.69063	0.76628	0.81319	0.84483
	0.5	2	0.02569	0.16493	0.43256	0.58876	0.68205	0.74245	0.78427
	0.75	1	0.07958	0.26904	0.52554	0.65769	0.73430	0.78358	0.81773
	1	1	0.03906	0.17558	0.41943	0.56653	0.65769	0.71842	0.76138
	1.25	1	0.02055	0.11698	0.33534	0.48745	0.58815	0.65769	0.70799
	1.5	1	0.01147	0.07958	0.26904	0.41943	0.52554	0.60150	0.65769
	1.75	1	0.00673	0.05523	0.21681	0.36119	0.46946	0.54977	0.61055
	2	1	0.00412	0.03906	0.17558	0.31146	0.41943	0.50235	0.56653
0.25	0.25	2	0.16493	0.43256	0.68205	0.78427	0.83787	0.87050	0.89235
	0.5	1	0.17558	0.41943	0.65769	0.76138	0.81773	0.85282	0.87669
	0.75	1	0.07958	0.26904	0.52554	0.65769	0.73430	0.78358	0.81773
	1	1	0.03906	0.17558	0.41943	0.56653	0.65769	0.71842	0.76138
	1.25	1	0.02055	0.11698	0.33534	0.48745	0.58815	0.65769	0.70799
	1.5	1	0.01147	0.07958	0.26904	0.41943	0.52554	0.60150	0.65769
	1.75	1	0.00673	0.05523	0.21681	0.36119	0.46946	0.54977	0.61055
	2	1	0.00412	0.03906	0.17558	0.31146	0.41943	0.50235	0.56653

7. Conclusion

The TSGChSP is developed for Pareto distribution of the 2nd kind using the median as quality parameter with the objective to provide a guideline for the industrial practitioners when dealing with the non-symmetrical (skewed) data. The performances of the plan are measured from two aspects: (i) the g and, (ii) the $L(p)$. The g will assist the industrial practitioners in selecting the appropriate g when conducting the inspection with some specific parameters in order to minimize the inspection time while the $L(p)$ will motivate producers to produce higher quality products, and eventually the products will be accepted (bought) by the consumers.

In the future, the TSGChSP can be further developed by using (i) different quality parameter (for example, percentile), (ii) using different underlying distributions (for example, Poisson distribution, weighted Poisson distribution) and, (iii) another lifetime distribution (for example, Rayleigh distribution, inverse Rayleigh distribution).

Table 5. The $\boldsymbol{L(p)}$ for TSGChSP when λ=2

β	a	g	$\frac{\mu}{\mu_0}$						
			1	2	4	6	8	10	12
0.01	0.25	6	0.00424	0.08469	0.34077	0.51742	0.62750	0.69968	0.74976
	0.5	3	0.00508	0.08547	0.33408	0.50748	0.61690	0.68933	0.73997
	0.75	2	0.00594	0.08616	0.32766	0.49789	0.60660	0.67924	0.73039
	1	2	0.00110	0.03489	0.21161	0.37772	0.49789	0.58359	0.64631
	1.25	2	0.00022	0.01428	0.13533	0.28366	0.40517	0.49789	0.56859
	1.5	1	0.00866	0.08770	0.30987	0.47102	0.57745	0.65042	0.70286
	1.75	1	0.00431	0.05827	0.25072	0.41015	0.52189	0.60101	0.65895
	2	1	0.00222	0.03906	0.20282	0.35666	0.47102	0.55465	0.61714
0.05	0.25	4	0.03252	0.21634	0.51240	0.66180	0.74484	0.79644	0.83123
	0.5	2	0.03489	0.21161	0.49789	0.64631	0.73039	0.78334	0.81939
	0.75	2	0.00594	0.08616	0.32766	0.49789	0.60660	0.67924	0.73039
	1	1	0.03906	0.20282	0.47102	0.61714	0.70286	0.75816	0.79650
	1.25	1	0.01805	0.13300	0.38252	0.53988	0.63778	0.70286	0.74875
	1.5	1	0.00866	0.08770	0.30987	0.47102	0.57745	0.65042	0.70286
	1.75	1	0.00431	0.05827	0.25072	0.41015	0.52189	0.60101	0.65895
	2	1	0.00222	0.03906	0.20282	0.35666	0.47102	0.55465	0.61714
0.10	0.25	3	0.08547	0.33408	0.61690	0.73997	0.80520	0.84494	0.87149
	0.5	2	0.03489	0.21161	0.49789	0.64631	0.73039	0.78334	0.81939
	0.75	1	0.08770	0.30987	0.57745	0.70286	0.77241	0.81607	0.84588
	1	1	0.03906	0.20282	0.47102	0.61714	0.70286	0.75816	0.79650
	1.25	1	0.01805	0.13300	0.38252	0.53988	0.63778	0.70286	0.74875
	1.5	1	0.00866	0.08770	0.30987	0.47102	0.57745	0.65042	0.70286
	1.75	1	0.00431	0.05827	0.25072	0.41015	0.52189	0.60101	0.65895
	2	1	0.00222	0.03906	0.20282	0.35666	0.47102	0.55465	0.61714
0.25	0.25	2	0.21161	0.49789	0.73039	0.81939	0.86498	0.89243	0.91070
	0.5	1	0.20282	0.47102	0.70286	0.79650	0.84588	0.87616	0.89657
	0.75	1	0.08770	0.30987	0.57745	0.70286	0.77241	0.81607	0.84588
	1	1	0.03906	0.20282	0.47102	0.61714	0.70286	0.75816	0.79650
	1.25	1	0.01805	0.13300	0.38252	0.53988	0.63778	0.70286	0.74875
	1.5	1	0.00866	0.08770	0.30987	0.47102	0.57745	0.65042	0.70286
	1.75	1	0.00431	0.05827	0.25072	0.41015	0.52189	0.60101	0.65895
	2	1	0.00222	0.03906	0.20282	0.35666	0.47102	0.55465	0.61714

Table 6. The $L(p)$ for TSGChSP when λ=3

β	a	g	$\frac{\mu}{\mu_0}$						
			1	2	4	6	8	10	12
0.01	0.25	6	0.00553	0.09822	0.36590	0.54094	0.64760	0.71672	0.76437
	0.5	3	0.00597	0.09659	0.35687	0.52970	0.63632	0.70607	0.75448
	0.75	2	0.00640	0.09504	0.34823	0.51882	0.62535	0.69565	0.74477
	1	2	0.00110	0.03871	0.22859	0.39893	0.51882	0.60294	0.66386
	1.25	2	0.00020	0.01570	0.14802	0.30324	0.42654	0.51882	0.58829
	1.5	1	0.00769	0.09086	0.32440	0.48825	0.59411	0.66572	0.71673
	1.75	1	0.00355	0.05946	0.26309	0.42688	0.53906	0.61731	0.67405
	2	1	0.00168	0.03906	0.21296	0.37244	0.48825	0.57158	0.63315
0.05	0.25	4	0.03850	0.23731	0.53527	0.68006	0.75940	0.80836	0.84125
	0.5	2	0.03871	0.22859	0.51882	0.66386	0.74477	0.79533	0.82960
	0.75	2	0.00640	0.09504	0.34823	0.51882	0.62535	0.69565	0.74477
	1	1	0.03906	0.21296	0.48825	0.63315	0.71673	0.77015	0.80697
	1.25	1	0.01713	0.13914	0.39883	0.55694	0.65337	0.71673	0.76109
	1.5	1	0.00769	0.09086	0.32440	0.48825	0.59411	0.66572	0.71673
	1.75	1	0.00355	0.05946	0.26309	0.42688	0.53906	0.61731	0.67405
	2	1	0.00168	0.03906	0.21296	0.37244	0.48825	0.57158	0.63315
0.10	0.25	3	0.09659	0.35687	0.63632	0.75448	0.81647	0.85406	0.87911
	0.5	2	0.03871	0.22859	0.51882	0.66386	0.74477	0.79533	0.82960
	0.75	1	0.09086	0.32440	0.59411	0.71673	0.78385	0.82570	0.85415
	1	1	0.03906	0.21296	0.48825	0.63315	0.71673	0.77015	0.80697
	1.25	1	0.01713	0.13914	0.39883	0.55694	0.65337	0.71673	0.76109
	1.5	1	0.00769	0.09086	0.32440	0.48825	0.59411	0.66572	0.71673
	1.75	1	0.00355	0.05946	0.26309	0.42688	0.53906	0.61731	0.67405
	2	1	0.00168	0.03906	0.21296	0.37244	0.48825	0.57158	0.63315
0.25	0.25	2	0.22859	0.51882	0.74477	0.82960	0.87278	0.89871	0.91594
	0.5	1	0.21296	0.48825	0.71673	0.80697	0.85415	0.88296	0.90233
	0.75	1	0.09086	0.32440	0.59411	0.71673	0.78385	0.82570	0.85415
	1	1	0.03906	0.21296	0.48825	0.63315	0.71673	0.77015	0.80697
	1.25	1	0.01713	0.13914	0.39883	0.55694	0.65337	0.71673	0.76109
	1.5	1	0.00769	0.09086	0.32440	0.48825	0.59411	0.66572	0.71673
	1.75	1	0.00355	0.05946	0.26309	0.42688	0.53906	0.61731	0.67405
	2	1	0.00168	0.03906	0.21296	0.37244	0.48825	0.57158	0.63315

Acknowledgement

This research was supported by School Centres of Excellence (SCoe) Grant (S/O Code: 13729).

References

[1] Mughal, A. R., Zain, Z., & Aziz, N. (2015). Time truncated group chain sampling strategy for Pareto distribution of the 2nd kind. *Research Journal of Applied Sciences, Engineering and Technology, 10*(4), 471-474.

[2] Teh, M. A. P., Aziz, N., & Zain, Z. (2016). Time truncated group chain sampling plans for Rayleigh distribution. *Global Journal of Pure and Applied Mathematics, 12*(4), 3693-3699.

[3] Teh, M. A. P., Aziz, N., & Zain, Z. (2019). Time truncated group chain sampling plans for gamma Distribution. *International Journal of Innovative Technology and Exploring Engineering, 8*(5S), 62-65.

[4] Mughal, A. R. (2018). A family of group chain acceptance sampling plans based on truncated life test. (Unpublished Doctoral dissertation.) Universiti Utara Malaysia, Sintok, Kedah, Malaysia.

[5] Ramaswamy, A. R. S., & Jayasri, S. (2013). Time truncated chain sampling plans for Marshall-Olkin extended exponential distributions. *IOSR Journal of Mathematics (IOSR-JM), 5*(1), 1-5.

[6] Al-Nasser, A. D., & Gogah, F. S., (2017). On using the median ranked set sampling for developing reliability test plans under generalized exponential distribution. *Pakistan Journal of Statistics and Operation Research, 13*(4), 757–774.

[7] Gogah, F. S., & Al-Nasser, A. D., (2018). Median ranked acceptance sampling plans for exponential distribution. *Afrika Matematika, 29*(3-4), 477-497.

[8] Pham, H. (2006). *System software reliability: springer series in reliability engineering*. London: Springer.

[9] Aslam, M., Mughal, A. R., Ahmad, M., & Zab, Y. (2010). Group acceptance sampling plans for Pareto distribution of the second kind. *Journal of Testing and Evaluation, 38*(2), 143-150.

[10] Aslam, M., Mughal, A. R., Hanif, M., & Ahmad, M. (2010). Economic reliability group acceptance sampling based on truncated life tests using Pareto distribution of the second kind. *Communications for Statistical Applications and Methods, 17*(5), 725-731.

[11] Aslam, M., Huang, S. Y., Jun, C. H., Ahmad, M. & Rasool, M. (2011). A reliability sampling plan based on progressive interval censoring under Pareto distribution of second kind. *IEMS, 10*(2), 154-160.