Modified Group Chain Acceptance Sampling Plans (MGChSP) based on the Minimum Angle Method

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***Abstract*— The established modified group chain acceptance sampling plans (MGChSP) only considered the consumer's risk when developing the plans and turned down the producer's risk, which means the established MGChSP satisfied the consumer but not the producer. Based on the above problem, this article proposes the minimum angle method for the MGChSP as the method considers both risks (consumer's and producer's) simultaneously. In order to develop the proposed MGChSP, the generalized exponential distribution is chosen as its lifetime distribution, and the time truncated simulation is done. The simulation produces the optimal number of groups, g at different values of specified design parameters. It turns out that the g satisfies both parties, consumer and producer, while the number of groups for the established MGChSP only satisfied the consumer. For application purpose, failure time (in hours) for computer software is used in order to demonstrate the usage of MGChSP in the industry.**

***Keywords*—** *Modified group chain acceptance sampling plans (MGChSP), Minimum angle method, Generalized exponential distribution, Consumer's risk, Producer's risk.*

1. Introduction

Modified group chain acceptance sampling plans (MGChSP) was initiated by Mughal [1] in order to impose tighter acceptance criteria on the previous established sampling plan, namely group chain acceptance sampling plan (GChSP). Acceptance criteria is the conditions imposed on the sampling plans in order to decide whether to accept or to reject a lot. The tighter acceptance criteria in the MGChSP will put the pressure on the producer to manufacture higher quality products. If the producer does not produce higher quality products, then the product will not be accepted and eventually, the product cannot be sold to the consumer.

For the MGChSP, Mughal [1] proposed the plan for Pareto distribution of the 2nd kind using the mean as quality parameter. For the Pareto distribution of the 2nd kind, it has been used by several researchers including Aslam et al. [2] for progressive censoring on the single acceptance sampling plans (SSP), Aslam et al. [3] for economic reliability based on multiple inspection and Aslam et al. [4] for multiple inspection without the economic reliability element.

Apart from the mean as quality parameter [5]-[6], researchers have the options to use median or percentile when proposing a sampling plan. For instance, the median has been used by Gogah and Al-Nasser [7] when proposing the SSP for a product that has exponential distribution as its lifetime. For the percentile, it has been demonstrated by Kaviyarasu and Fawaz [8] when they studied the SSP for modified Weibull distribution.

All the above sampling plans [1] – [8] have been developed by minimizing the consumer's risk, β and overlooked at the other risk related to the acceptance sampling, which is producer's risk, α. The β is defined as the probability of accepting a bad lot while the α is the probability of rejecting a good lot [9]. The two risks emerge in acceptance sampling as acceptance sampling only inspects the sample, not the whole lot.

There is a method in acceptance sampling where both risks are considered and it is called minimum angle method. This article proposes to develop the MGChSP considering both risks, consumer and producer, or known as the minimum angle method as we noted that the previous MGChPSP only catered the consumer and ignored the producer.

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2. Minimum Angle Method

The minimum angle is a method, where both risks associated with acceptance sampling (β and α) are considered. The method considers the tangent angle between the lines joining the points A $(p_1, 1-\alpha)$ and point B (p_2, β) as shown in Figure 1.

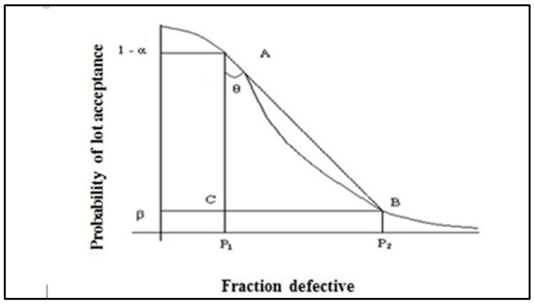

Figure 1. The minimum angle method by Ramaswamy and Sutharani [10]

In order to calculate the angle, the trigonometric function (tangent) is applied and the angle is calculated by using the following formula

$$\tan\theta = \frac{BC}{AC} = \frac{(p_2 - p_1)}{L(p_1) - L(p_2)}. \qquad (1)$$

For the minimum angle method, it has been applied to several sampling plans such as Bayesian double acceptance sampling plans (BDSP) by Suresh and Usha [11] and double acceptance sampling plans (DSP) by Ramaswamy Sutharani [10] [12].

3. Operating Procedure

For MGChSP, the operating procedure is

i. For each lot, the optimal number of groups, g is found satisfying two conditions, which are (i) the β and α are below 0.10, and (ii) the g has the smallest angle.
ii. The number of products, r is allocated to the g groups
iii. The test termination time, t_0 for the inspection activity is specified.
iv. During the inspection activity, the number of defectives, d is counted.
v. Accept the current lot if $d = 0$ provided that the preceding i samples have at most one defective.
vi. Reject the current lot if $d > 0$.

4. Probability of Lot Acceptance

The probability of lot acceptance, $L(p)$ for the MGChSP is given by

$$L(p) = (1-p)^{gr(i+1)}\left[\frac{grip}{1-p} + 1\right]. \qquad (2)$$

5. Fraction Defective

In this article, the fraction defective, p is derived by using the cumulative distribution function (CDF) of generalized exponential distribution. The generalized exponential distribution has been used by Aslam, Kundu and Ahmad [13], Aslam et al. [14] and Rasmaswamy and Jayasri [15].

In this article, the p for generalized exponential distribution is given by

$$p = \left[1 - exp\left[-a\left(\frac{1}{\mu/\mu_0}\right)\right]\right]^{\lambda}. \qquad (2)$$

The article deals with the β and α, therefore there is difference between the p at the consumer and producer levels. At the consumer level, the p is calculated when the mean ratio is 1 while at the producer level, the p is obtained when mean ratio ranges from 2 to 12.

The performance of the MGChSP is measured based on the g. The g is only obtained when (i) $\alpha \leq 0.10$, (ii) $\beta \leq 0.10$ and, (iii) the theta has the smallest value. Apart from that, the pattern of the g is observed at various values of design parameters such as $\lambda = \{2, 3\}$, $a = \{0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\}$ and $(i, r) = \{(1,2), (2,3), (3,4), (4,5)\}$.

6. Discussion and Application

Tables 1 and 2 show the g for generalized exponential distribution at different values of shape and design parameters.

Based on Tables 1 and 2, the g decreases as the specified constant, a increases. The finding is the evident that when a product is tested for a longer period of time (a is higher), then the likelihood to find one defective product is higher compared to the shorter testing time. For instance, the g decreases from 165 to 2 when the a increases from 0.25 to 2 while the other design parameters are set at $\left(\lambda, \frac{\mu}{\mu_0}, i, r\right) = (3, 6, 1, 2)$.

Besides the a, the g also decreases as the number of preceding lots, i and the number of products, r increase. The pattern explains that if a company does the inspection with more information from the previous lot and the inspection platform allows more products to be placed on it, therefore less g is required. This scenario eventually leads to the less inspection time since there is less groups to be inspected.

For the θ, it is getting larger when the a increases. For example, the θ is 0.63260^o when the a is 0.25 while the other design parameters are $\left(\lambda, \frac{\mu}{\mu_0}, i, r\right) = (3, 6, 1, 2)$. The θ increases to 34.50869^o when the a increases to 2.00.

The above finding is significant for the industrial practitioners when they want to design the MGChSP. They can design the MGChSP with low testing time as it shows that as the testing is lower, the θ created is smaller, and closed to 0^o. The 0^o is very important in this study as it resembles the ideal operating characteristic (OC) curve. The ideal OC curve is actually depicted by AC line in Figure 1.

Wood [16] listed a total of nine failure time (in hours) for a computer software: 5218, 4422, 3625, 3058, 2490, 1893, 1430, 968, 519. Aslam et al. [13] has shown that the above failure time follows generalized exponential distribution with 3 as the shape parameter. If the computer software is scheduled to be inspected using MGChSP using the minimum angle method with $\left(\frac{\mu}{\mu_0}, i, r, a\right) = (4, 2, 3, 1)$ as the design parameters, then the g is 2. Take a sample of 6 and allocate 3 computer software into 2 groups. Accept the lot if there is no defective in the current lot given that there is at most one defective in the previous 2 lots. Otherwise, the current lot is rejected.

7. Conclusion

The MGChSP is developed by using the minimum angle method, where the method caters both risks, β and α. The plans improve the previous MGChSP as the previous MGChSP only catered the β. Apart from improving the previous MGChSP, the θ presented is actually the smallest theta and, it should resemble the ideal OC curve.

The generated tables acts as a tool for the industrial practitioners when designing the MGChSP. The table are generated at various values of design parameters and the industrial practitioners have the edge to select what design parameters they want to use for the inspection.

Finally, the MGChSP can be further studied by using (i) different quality parameter (median or percentile), (ii) using different underlying distributions (Poisson distribution, weighted binomial distribution) and, (iii) another lifetime distributions (Weibull distribution).

Glossary of Symbols

g	: Optimal number of groups
β	: Consumer's risk
α	: Producer's risk
p_1	: Fraction defective at the producer level
p_2	: Fraction defective at the consumer level
$L(p_1)$	: Probability of lot acceptance at the producer level
$L(p_2)$	: Probability of lot acceptance at the consumer level
r	: Number of products
t_0	: Test termination time
d	: Number of defectives in the current lot
i	: Number of preceding lots
d_i	: Number of defective in the i lots
a	: Specified constant
λ	: Shape parameter
$\frac{\mu}{\mu_0}$	: Mean ratio

Table 1. The g for generalized exponential distribution ($\lambda = 2$)

			Generalized exponential distribution, $\lambda = 2$							
			Specified constant, a							
mean ratio	i	r	0.25	0.5	0.75	1	1.25	1.5	1.75	2
2	1	2	-	-	-	-	-	-	-	-
	2	3	-	-	-	-	-	-	-	-
	3	4	-	-	-	-	-	-	-	-
	4	5	-	-	-	-	-	-	-	-
4	1	2	-	-	-	-	-	-	-	-
	2	3	8 (3.20036^{o})	-	-	-	-	-	-	-
	3	4	5 (3.09885^{o})	-	-	-	-	-	-	-
	4	5	3 (3.11437^{o})	1 (9.58979^{o})	-	-	-	-	-	-
6	1	2	26 (3.02676^{o})	7 (9.58665^{o})	3 (17.26457^{o})	2 (24.00782^{o})	-	1 (34.65002^{o})	-	-
	2	3	13 (2.95460^{o})	4 (9.32650^{o})	2 (16.49435^{o})	1 (23.30124^{o})	-	-	-	-
	3	4	8 (2.91688^{o})	2 (9.20969^{o})	1 (16.30348^{o})	-	-	-	-	-
	4	5	5 (2.89149^{o})	2 (9.25798^{o})	1 (16.35366^{o})	-	-	-	-	-

Table 1 continued.

8	1	2	30 (2.94280^o)	9 (9.28999^o)	4 (16.48008^o)	3 (23.05332^o)	2 (28.49694^o)	1 (34.30741^o)	1 (36.53942^o)	1 (38.63136^o)
	2	3	14 (2.89574^o)	4 (9.12704^o)	2 (16.15804^o)	2 (23.00753^o)	1 (27.93128^o)	1 (32.49748^o)	-	-
	3	4	9 (2.87025^o)	3 (9.06842^o)	1 (16.14488^o)	1 (22.47487^o)	-	-	-	-
	4	5	6 (2.85440^o)	2 (9.01396^o)	1 (15.92784^o)	1 (23.10628^o)	-	-	-	-
10	1	2	32 (2.89939^o)	9 (9.14306^o)	5 (16.19233^o)	3 (22.67353^o)	2 (28.10205^o)	2 (32.46136^o)	1 (36.28535^o)	1 (38.26339^o)
	2	3	16 (2.86580^o)	5 (9.02935^o)	2 (16.01795^o)	2 (22.53592^o)	1 (27.67012^o)	1 (31.98313^o)	1 (35.65662^o)	-
	3	4	9 (2.84739^o)	3 (8.96597^o)	2 (16.01076^o)	1 (22.20321^o)	1 (27.91273^o)	1 (32.85646^o)	-	-
	4	5	6 (2.83566^o)	2 (8.92619^o)	1 (15.77603^o)	1 (22.51438^o)	1 (28.71164^o)	-	-	-
12	1	2	34 (2.87385^o)	10 (9.05375^o)	5 (16.02991^o)	3 (22.46844^o)	2 (27.88694^o)	2 (32.08844^o)	2 (35.66650^o)	1 (38.05923^o)
	2	3	17 (2.84867^o)	5 (8.96493^o)	3 (15.90147^o)	2 (22.29668^o)	1 (27.53719^o)	1 (31.72109^o)	1 (35.22518^o)	1 (38.09578^o)
	3	4	10 (2.83473^o)	3 (8.91644^o)	2 (15.85548^o)	1 (22.07332^o)	1 (27.58701^o)	1 (32.23911^o)	1 (36.12753^o)	-
	4	5	7 (2.82685^o)	2 (8.88602^o)	1 (15.70755^o)	1 (22.24236^o)	1 (28.06132^o)	1 (33.11269^o)	-	-

Table 2. The g for generalized exponential distribution (λ=3)

			Generalized exponential distribution, $\lambda = 3$							
			Specified constant, a							
mean ratio	i	r	0.25	0.5	0.75	1	1.25	1.5	1.75	2
2	1	2	-	-	-	-	-	-	-	-
	2	3	-	-	-	-	-	-	-	-
	3	4	-	-	-	-	-	-	-	-
	4	5	-	-	-	-	-	-	-	-
4	1	2	134 (0.65376^{o})	22 (3.71352^{o})	8 (8.98840^{o})	4 (15.38456^{o})	2 (22.40460^{o})	-	-	-
	2	3	66 (0.64262^{o})	11 (3.63888^{o})	4 (8.77934^{o})	2 (14.98385^{o})	1 (21.75001^{o})	-	-	-
	3	4	39 (0.63654^{o})	6 (3.60098^{o})	3 (8.72205^{o})	1 (14.99493^{o})	1 (20.93111^{o})	-	-	-
	4	5	26 (0.63277^{o})	4 (3.57480^{o})	2 (8.65668^{o})	1 (14.67499^{o})	-	-	-	-
6	1	2	165 (0.63260^{o})	27 (3.57318^{o})	11 (8.60549^{o})	6 (14.65610^{o})	4 (20.71682^{o})	3 (26.17167^{o})	2 (30.57407^{o})	2 (34.50869^{o})
	2	3	80 (0.62824^{o})	13 (3.54289^{o})	5 (8.51734^{o})	3 (14.49075^{o})	2 (20.45973^{o})	1 (25.85822^{o})	1 (30.13745^{o})	1 (33.98005^{o})
	3	4	47 (0.62584^{o})	8 (3.52598^{o})	3 (8.46904^{o})	2 (14.43386^{o})	1 (20.27551^{o})	1 (25.73959^{o})	1 (30.71202^{o})	-
	4	5	31 (0.62433^{o})	5 (3.51580^{o})	2 (8.43894^{o})	1 (14.33341^{o})	1 (20.38582^{o})	1 (26.25711^{o})	-	-

Table 2 continued.

8	1	2	187 (0.62617°)	31 (3.52906°)	12 (8.48103°)	6 (14.42193°)	4 (20.33854°)	3 (25.63014°)	2 (30.05585°)	2 (33.65531°)
	2	3	90 (0.62403°)	15 (3.51378°)	6 (8.43683°)	3 (14.33038°)	2 (20.19839°)	2 (25.58735°)	1 (29.79976°)	1 (33.36730°)
	3	4	53 (0.62285°)	9 (3.50539°)	4 (8.41858°)	2 (14.28571°)	1 (20.14996°)	1 (25.37195°)	1 (29.93074°)	1 (33.77183°)
	4	5	35 (0.62211°)	6 (3.50013°)	2 (8.40191°)	1 (14.27881°)	1 (20.12133°)	1 (25.53697°)	1 (30.27882°)	1 (34.39439°)
10	1	2	204 (0.62353°)	34 (3.51061°)	13 (8.42860°)	7 (14.31735°)	5 (20.19675°)	3 (25.42526°)	2 (29.85630°)	2 (33.32497°)
	2	3	98 (0.62232°)	16 (3.50188°)	6 (8.40337°)	3 (14.27373°)	2 (20.10579°)	2 (25.36459°)	1 (29.67947°)	1 (33.14926°)
	3	4	57 (0.62165°)	10 (3.49711°)	4 (8.38938°)	2 (14.23676°)	1 (20.10931°)	1 (25.25221°)	1 (29.67844°)	1 (33.32398°)
	4	5	38 (0.62123°)	6 (3.49422°)	3 (8.38467°)	2 (14.25411°)	1 (20.04118°)	1 (25.32101°)	1 (29.82948°)	1 (33.58794°)
12	1	2	218 (0.62224°)	37 (3.50147°)	14 (8.40221°)	8 (14.26765°)	5 (20.10736°)	3 (25.33049°)	3 (29.69651°)	2 (33.16980°)
	2	3	104 (0.62148°)	17 (3.49597°)	7 (8.38597°)	4 (14.23581°)	2 (20.06434°)	2 (25.26530°)	1 (29.62535°)	1 (33.05119°)
	3	4	61 (0.62107°)	10 (3.49294°)	4 (8.37677°)	2 (14.21562°)	2 (20.06061°)	1 (25.20097°)	1 (29.57135°)	1 (33.13586°)
	4	5	40 (0.62081°)	7 (3.49107°)	3 (8.37287°)	2 (14.22020°)	1 (20.00831°)	1 (25.23358°)	1 (29.65060°)	1 (33.27143°)

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References

[1] Mughal, A. R. (2018). A family of group chain acceptance sampling plans based on truncated life test. (Unpublished Doctoral dissertation.) Universiti Utara Malaysia, Sintok, Kedah, Malaysia.

[2] Aslam, M., Huang, S. Y., Jun, C. H., Ahmad, M. & Rasool, M. (2011). A reliability sampling plan based on progressive interval censoring under Pareto distribution of second kind. *IEMS, 10*(2), 154-160.

[3] Aslam, M., Mughal, A. R., Hanif, M., & Ahmad, M. (2010). Economic reliability group acceptance sampling based on truncated life tests using Pareto distribution of the second kind. *Communications for Statistical Applications and Methods, 17*(5), 725-731.

[4] Aslam, M., Mughal, A. R., Ahmad, M., & Zab, Y. (2010). Group acceptance sampling plans for Pareto distribution of the second kind. *Journal of Testing and Evaluation, 38*(2), 143-150.

[5] Teh, M. A. P., Aziz, N., & Zain, Z. (2016). Time truncated group chain sampling plans for Rayleigh distribution. *Global Journal of Pure and Applied Mathematics, 12*(4), 3693-3699.

[6] Teh, M. A. P., Aziz, N., & Zain, Z. (2019). Time truncated group chain sampling plans for gamma Distribution. *International Journal of Innovative Technology and Exploring Engineering, 8*(5S), 62-65.

[7] Gogah, F. S., & Al-Nasser, A. D., (2018). Median ranked acceptance sampling plans for exponential distribution. *Afrika Matematika, 29*(3-4), 477-497.

[8] Kaviyarasu, V., & Fawaz, P. (2017). Certain studies on acceptance sampling plans for percentiles based on the modified Weibull distribution. *International Journal of Statistics and Systems, 12*(2), 343-354.

[9] Montgomery, D. C. (2009). *Statistical quality control: a modern introduction* (6th ed.). Arizona: Wiley.

[10] Ramaswamy, A. R. S., & Sutharani, R. (2013). Designing double acceptance sampling plans based on truncated life tests in Rayleigh distribution using minimum angle method. *American Journal of Mathematics and Statistics, 3*(4), 227-236.

[11] Suresh, K. K., & Usha, K. (2016). Construction of Bayesian double sampling plan using minimum angle method. *Journal of Statistics and Management Systems, 19*(3), 473-489.

[12] Ramaswamy, A. R. S., & Sutharani, R. (2014). Designing double acceptance sampling plans based on truncated life tests under various distributions using minimum angle method. *International Journal of Scientific & Engineering Research, 5*(1), 626-640.

[13] Aslam, M., Kundu, D., & Ahmad, M. (2010). Time truncated acceptance sampling plans for generalized exponential distribution. *Journal of Applied Statistics, 37*(4), 555-566.

[14] Aslam, M., Kundu, D., Jun, C. H., & Ahmad, M. (2010). Time truncated group acceptance sampling plans for generalized exponential distribution. *Journal of Testing and Evaluation, 39*(4), 1-7.

[15] Ramaswamy, A. R. S., & Jayasri, S. (2012). Time truncated chain sampling plans for generalized exponential distribution. *International Journal of Computational Engineering Research, 2*(5), 1402-1407.

[16] Wood, A. (1996). Predicting software reliability. *IEEE Transactions Software Engineering, 29*(11), 69-77.