A Meta-Heuristic Hybrid Algorithm for the Rich Vehicle Routing Problem

Evangelos Moschos[#1], Stavros T. Ponis[#2], Elena Rokou[*3]
[#1-2]School of Mechanical Engineering, National Technical University Athens, Heroon Polytechniou 9, Zografos 157 80, Athens, Greece
[*3]Creative Decisions Foundation, 4922 Ellsworth Ave., 15212 Pittsburgh, Pennsylvania, USA
[1]moschos.vangelis@gmail.com
[2]staponis@central.ntua.gr
[3]erokou@icloud.com

***Abstract*—This paper presents an enriched variation of the original Vehicle Routing Problem (VRP), in which traffic zones, CO2 emissions and heterogeneity of vehicles are all taken into consideration. Despite the extensive research already focused at VRP, its applicability in Supply Chain Management, has pushed researchers in investigating richer, and more realistic versions of VRP with various business constraints. The objective of the model is to minimize the total travelled distance, while assigning specific customers to specific type of vehicles and maintaining the emissions of the entire fleet under a specified limit. The proposed hybrid algorithm (TDHF-GVRP) is composed of two nested genetic algorithms, interacting with each other as well as with other local search optimization methods. The external genetic algorithm (parent) is responsible for the assignment of customers to vehicle types, whereas the internal (child) algorithm is responsible for solving the vehicle routing problem for each category of vehicles separately.**

***Keywords*—*Rich VRP, Vehicle Routing Problem, Hybrid Algorithm, Meta-heuristic, Green Transportation, Sustainable Transportation, Heterogeneous Fleet*

1. Introduction

Increased demand variability between the discrete tiers of a supply chain is perhaps the most significant criterion for strategic supply chain decisions, for example facility location and supply chain network design, as it directly affects the efficient operation of the whole supply chain, especially transportation [1]. Some authors record the first instance of the VRP problem in literature back in 1954, in the efforts of to provide a solution of a large-scale Travelling Salesman Problem (TSP), since the TSP is considered a specific case of the generic vehicle routing problem [2]. Others move this date five years forward to when the seminal paper by [3] was published, calculating the optimum routing of a fleet of identical gasoline delivery trucks between a bulk terminal and a large number of service stations supplied by the terminal [4]. Since then and after the introduction of the term 'vehicle routing' by [5], VRP has been a very popular, amongst researchers, scientific area something which is translated in an abundance of publications that can roughly be divided into theoretical papers providing mathematical formulations and exact or approximate solution methods for academic problems and case-oriented papers [6].

VRP is an NP-Hard combinatorial optimization problem. For decades authors proposed solutions of manageable scale problems using exact algorithms [7], [8], [9], [10], [11] and heuristic algorithms [12], [13], [14], [15]. Still, the tight connection of the VRP with real life applications and the inherent complexity of the actual business cases created, early enough, the need for the introduction of a large number of the problem's variants, driving research away from the simplistic nature of the generic vehicle routing problem towards more realistic models and problem constraints.

It was the 90's that marked a significant increase in VRP research. The advent of the microcomputers and the vast spread of computing power availability in research institutions and the industry, created a new wave of scientific publications focusing in more complex, realistic and tractable algorithmic approaches of real-life business problems. During this era the term meta-heuristics was introduced to define a number of search algorithms for solving these VRPs as well as other combinatorial optimization problems [16]. The introduction of meta-heuristics in the research agenda of the vehicle routing problem created a new wave of algorithmic approaches addressing more complex and dynamic business situations utilizing field data and information. At the same time, a number of conveniently overlooked real life constraints started to gain meaning and interest, thus enhancing the proposed model's and mathematical formulations' validity leading to the development of a wide array of VRP variants. Several literature review and taxonomical efforts exist in the literature identifying, providing definitions and exploring these VRP variations. The underlying logic and structure of these taxonomies vary. For example, the taxonomy proposed by [4] is based on the nature of the problem and application of VRP, while the taxonomy proposed [6] utilizes central routing concepts present in industrial applications in order to produce a meaningful VRP variant categorization. One has to note, the most influential work by the authors of [16] who classify contributions based on five different aspects, i.e. type of study, scenario characteristics, problem physical characteristics, information characteristics and data characteristics.

This paper presents a new variation of the traditional CVRP, in which traffic congestion is taken into account (TDVRP), customers are served by a heterogeneous fleet of vehicles with various capacities (HFVRP) and environmental pollution (GVRP) is included in the model's mathematical formulation. The objective of the proposed TDHF-GVRP algorithm is to minimize the total travelled distance, while assigning specific customers to specific type of vehicles and maintaining the emissions of the entire fleet under a specified limit. In that sense, the proposed problem can be considered a Rich Vehicle Routing Problem as argumentation in the next section will show.

The remainder of this paper is organized as follows. Section 2 briefly explores current definitions of the 'Rich VRP' problem in order to provide adequate justification for including this paper under the 'Rich VRP' category. Section 3 details the proposed hybrid meta-heuristic algorithm. Finally, Section 4 provides the conclusions to this work and outlines further research orientation and goal setting.

2. Literature Review

The proposed problem in this paper, utilizes elements from three major VRP variants as these can be found in the taxonomic efforts in literature, i.e. the Time Dependent VRP (TDVRP), the Heterogeneous Fleet VRP (HFVRP) and the Green VRP (GVRP). TDVRP assumes that the travel times between depots and customers are deterministic but not constant. Instead they are a function of current time and as such, the effects of congestion on the total route duration can be determined [17], while making these problems harder to model and solve. The first instance of TDVRP in literature can be found in the initial PhD [18] and the subsequent paper in [19], which addresses both the TDVRP and the TDTSP. Since then, many researchers have proposed metaheuristics to address the TDVRP problem most of the times in combination with setting soft or hard time windows in servicing the nodes of the network [20], [21], [22], [23], [24]. For pure TDVRP heuristics research, the reader can refer to the work in [25], which proposes a parallel tabu search heuristic for improving results in comparison with a fixed travel time model. The HFVRP is NP-hard as a generalization of the classical Vehicle Routing Problem in which customers are served by a heterogeneous fleet of vehicles with various capacities, fixed costs, and variable costs [26]. The HFVRP is a very important problem, since fleets are likely to be heterogeneous in most practical situations, even when at the time of the fleet acquisition the initial fleet vehicles were identical. Moreover, insurance, maintenance and operating costs may have different values based on the level of depreciation or usage time of the fleet [27]. A relatively recent review of HFVRP can be found in [28]. Finally, the GVRP refers to vehicle routing problems where externalities of using vehicles, such as carbon dioxide-equivalents emissions, are explicitly taken into account so that they are reduced through better planning [29]. According to the authors, existing GVRP studies only cover vehicle capacity and time windows constraints, while at the same time heterogeneous vehicles are still not explored in the existing literature.

Table 1, provides brief information (name and acronym, definition, description, indicative references) on VRP variants relative to the research efforts described in this paper, i.e. CVRP, TDVRP, HFVRP and GVRP. The PRP, VRPTW and OVRP problems are also included in this Table since they all constitute possible extensions of our proposed algorithm and items of the authors' future research agenda.

Table 1. Variants of the VRP

VRP Variant	Description	Indicative References
Capacitated Vehicle Routing Problem **(CVRP)**	It is the simplest form of a VRP problem where a homogeneous fleet of vehicles supplies customers from a depot. Each vehicle has the same capacity (homogeneous fleet) and each customer has a certain demand that must be satisfied. Additionally, there is a cost matrix that measures the costs associated with moving a vehicle from one node to another [9].	Dantzig & Ramser, 1959 [16]; Osman, M.F.S [30]; Sahroni et al., 2018 [31]; Chen et al., 2010 [32].
Time Dependent Vehicle Routing Problem **(TDVRP)**	The TDVRP assumes that the travel times between depots and customers are deterministic but not constant. Instead they are a function of current time and as such, the effects of congestion on the total route duration can be determined [7].	Malandraki & Daskin, 1992 [19]; Ichoua et al., 2003 [25]; Jabali et al., 2012 [33]; Kuo & Wang, 2012 [34]; Lorini et al., 2011 [35].
Heterogeneous Fleet Vehicle Routing Problem **(HFVRP)**	It is a variant of the classical Vehicle Routing Problem in which customers are served by a heterogeneous fleet of vehicles with various capacities, fixed costs, and variable costs [29].	Baldacci et al., 2008 [36]; Jiang et al., 2014 [37]; Subramanian et al., 2012 [38]; Penna et al., 2013 [39].
Green Vehicle Routing Problem **(GVRP)**	Green Vehicle Routing problems are characterized by the objective of harmonizing the environmental and economic costs by implementing effective routes to meet the environmental concerns and financial indexes of the problem at hand [54].	Erdoğan & Miller-Hooks, 2012 [40]; Demir et al., 2014 [41]; Park & Chae, 2014 [42]; Bektas et al., 2016 [29].
Pollution Routing Problem **(PRP)**	It is in essence an extension of the VRPTW problem. The PRP routes a number of vehicles to serve a set of customers within preset time windows, and determining their speed on each route segment, so as to minimize a function comprising emissions and driver costs [28].	Demir et al., 2012 [43]; Bektaş & Laporte, 2011 [44]; Kramer et al., 2015 [45]; Kumar et al., 2016 [46].
Vehicle Routing Problem with Time Windows **(VRPTW)**	A generalization of the VRP where the service at any customer starts within a given time interval, called a time window. Time windows are called soft when they can be considered non-binding for a penalty cost. They are hard when they cannot be violated [44].	Chiang & Russell, 1996 [47]; Gayialis et al., 2018 [48]; Bräysy & Gendreau, 2005 [49]; Ponis et al., 2015 [50]; Baldacci et al., 2012 [51].
Open VRP **(OVRP)**	The open vehicle routing problem (OVRP) differs from the classic vehicle routing problem (VRP) because the vehicles either are not required to return to the depot, or they have to return by revisiting the customers assigned to them in the reverse order [66].	Li et al., 2005 [52]; Li et al., 2007 [53]; Li et al., 2012 [54]; Repoussis et al., 2007 [55];

VRP research has often been criticized for being too focused on idealized models with little or no practical application, because of the use of non-realistic assumptions [56]. In recent years, methodological progress and the development and vast availability of computer power and infrastructures, has led to the emergence of numerous research studies addressing more complex VRP variants introducing new, and closer to real life applications, constraints and objectives.

These problems can be found in literature under the general definition of the so called 'Rich Vehicle Routing Problems', or in brief 'RVRP'.

There are numerous research papers addressing the Rich VRP, still to our knowledge, there is no single and unified definition of what this family of problems entails. According to [57], RVRPs are the ones that capture the high complexities, large data sizes, uncertainties and dynamisms that exist in real life. Ref. [58] define RVRPs as problems, which deal with realistic optimization functions along with a wide variety of real-life constraints related to time and distance factors. What seems to be the most recent and elaborated definition of the RVRPs can be found in [6]. The authors create a taxonomy of RVRPs based on publications from 2006 to 2015 and according to their findings, propose a more precise and strict definition of the RVRP, based on the specific characteristics and categorization criteria of their taxonomy. Still, the same authors agree that in literature the "RVRP is defined as a problem which simultaneously includes several types of challenging and complicated features associated with the complexity of real-life routing problems".

As stated earlier, recent literature presents a substantial number of publications in the RVRP area, especially those utilizing metaheuristics, drifting away from specialized and traditional heuristic algorithms, which were used for decades in solving complex combinatorial optimization problems [59]. Today, thirty years after their initial introduction, metaheuristics have proved to be remarkably effective, and for that metaheuristics are now widely considered the most appropriate methods to address the combinatorial nature, the complexity and variety of rich vehicle routing problems [60]. According to [61], metaheuristics are "solution methods that orchestrate an interaction between local improvement procedures and higher-level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space". While metaheuristics are not able to certify the optimality of the solutions they find, they have proved themselves capable of providing better results than exact solution procedures, especially in real world problems of high complexity [62]. For a survey of metaheuristics for RVRPs, the reader could refer to [63].

3. Methodology

3.1 Problem Definition and Formulation

The TDHF-GVRP problem is defined in a complete direct network G (N, A), where N is the set of nodes and A= $\{(i,j): i \neq j, i,j \text{ E } N\}$ is the set of arcs. Node 0 represents the central depot and N'=N \ {0} represents the customers. For every arc *i-j*, which represents the path between nodes *i* and *j*, D_{ij} stands for the distance of the arc. Each customer *i* places a demand R_i, which has to be serviced by exactly one vehicle. The scheduling period is set to one day and is divided in *m* time periods, so that the traffic speed in a specific arc during a time period is fixed, but may be different between two time periods. Every time period *k* belongs in the set of K periods and is characterized by its beginning time b_k and its end time e_k.

The average speed for a specific route depends both on the period *k* that the route is scheduled for and the different traffic zones, which form the final route. Every *i-j* route is divided in *B* smaller routes, defined by B+1 points (*incl. i,j*), in a way that every small route (i= p_0,p_1),(p_1,p_2),(p_2, p_3)…(p_{B-1},j=p_B) belongs in exactly one traffic zone, and subsequently has the same average speed for a specific time period *k* (see Figure 1). The traffic zones affect the average speed of vehicles in the zone ($V^{(zik)}$, i=1……B and k=1…..m) in the sense that the average speed is fixed within a sub route for a given time period.

The TDHF-GVRP problem considers a set of H heterogeneous vehicles, consisting of H_1 low capacity vehicles and H_2 high capacity vehicles. The vehicles are visiting a set of n customers, each with a known and non-variable demand, randomly located in a region. All vehicles start their route from a central depot and end it at the same central depot. Each vehicle type has its own capacity C_h, load time t_{load}, and unload time t_{unload}. The load and unload times correspond to full truck loads (FTL). In the case of less than truckload (LTL) assignments, load and unload times are linearly reduced.

For calculating the emissions of the vehicles, the model described in [64] is utilized. Since the model mandates a steady vehicle speed, the total emissions of the route are calculated as the sum of the emissions for every steady-speed smaller route that the original route was divided into. The company, except the proprietary fleet of vehicles (set H) has the potential to assign routing tasks to an external 3PL service provider. The additional cost of subcontracting is C_s per measure of distance, when compared to the cost of using its own vehicles. The purpose of the problem is to minimize the total weighted travelled distance, while the emissions remain below a specified limit.

The distance travelled by the company's own vehicles has been assigned with a weight value of one (1), whereas the subcontracted vehicles have been assigned a weight value of C_s, which represents the additional cost per unit of measure. The C_s may receive any positive value, but is typically above one.

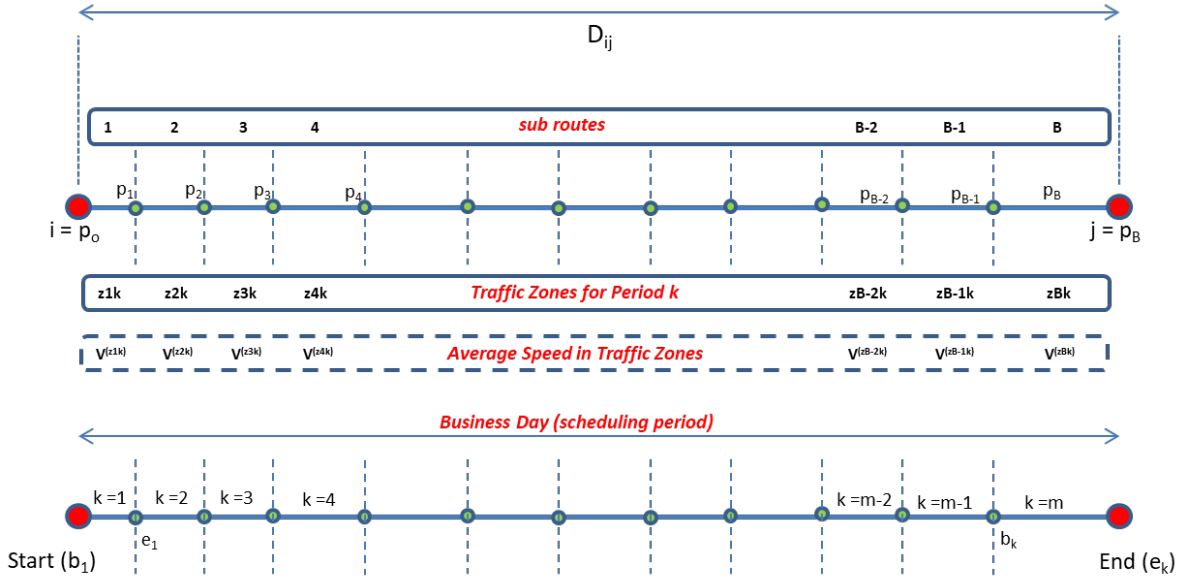


Figure 1. Route (i,j) Segmentation in sub routes

The mathematical formulation of the problem is based on the recent works of [65]. The decision variables of the problem are the following:

Primary Decision Variables

X_{ij}: Binary variable = {1, if ij route is travelled, otherwise 0}

Y_{ijh}: Binary variable = {1 ij route is travelled by vehicle h, otherwise 0}

X_{ijkh}: Binary variable = {1 ij route is travelled by vehicle h in period k, otherwise 0}

Secondary Decision Variables (bound to the primary decision variables)

d_{ijkh} : Continuous variable that depicts the travelled distance in the ij route, travelled by vehicle h at period k.

τ_{ijkh} : Continuous variable that depicts the travel time of the d_{ijkh} route.

l_i : Continuous variable that depicts the departure time from node i.

a_i : Continuous variable that depicts the arrival time to node i.

$$\min(F) = \sum_{h=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} d_{ijhk} + C_s \cdot \sum_{h=1}^{o}\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} d_{ijhk} \qquad (1)$$

$$\sum_{j=0}^{n} X_{ij} = 1 \qquad \forall\, i \in N' \qquad (2)$$

$$\sum_{i=0}^{n} X_{ij} = 1 \qquad \forall\, j \in N' \qquad (3)$$

$$Xij = \sum_{h=1}^{q} Y_{ijh} \qquad \forall\, i,j \in A \qquad (4)$$

$$Y_{ijh} \geq X_{ijhk} \qquad \forall (i,j) \in A, k \in K, h \in H \qquad (5)$$

$$Y_{ijh} \leq \sum_{k=1}^{m} X_{ijhk} \qquad \forall\, i,j \in A, h \in H \qquad (6)$$

$$\sum_{i=0:i\neq j}^{n} Y_{ijh} = \sum_{i=0:i\neq j}^{n} Y_{jih} \qquad \forall\, j \in N, h \in H \qquad (7)$$

$$\sum_{j=1}^{n} Y_{0jh} \leq 1 \qquad \forall\, h \in H \qquad (8)$$

$$d_{ijkh} \leq D_{ij} \cdot Z_{ijkh} \qquad \forall (\, i,j) \in A, k \in K, h \in H \qquad (9)$$

$$\sum_{k=1}^{m}\sum_{h=1}^{q} d_{ijkh} = X_{ij} \cdot D_{ij} \qquad \forall\, i,j \in A \qquad (10)$$

$$\sum_{(i,j)\in A}^{n} \tau_{ijkh} \leq e_k - b_k \qquad \forall\, k \in K, h \in H \qquad (11)$$

$$l_i \leq e_k - \tau_{ijkh} + M \cdot (1 - Z_{ijkh}) \qquad \forall (\, i,j) \in A, k \in K, h \in H \qquad (12)$$

$$a_j \geq b_k + \tau_{ijkh} - M \cdot \left(1 - Z_{ijkh}\right) \quad \forall (i,j) \in A, k \in K, h \in H \tag{13}$$

$$a_j \geq l_i + \sum_{k=1}^{m} \sum_{h=1}^{q} \tau_{ijkh} - M \cdot \left(1 - X_{ij}\right) \quad \forall (i,j) \in A \tag{14}$$

$$g_i = t_h^{unload} \cdot \frac{\mathrm{R}_i}{C_h} \quad \forall h \in H \tag{15}$$

$$a_i + g_i \leq l_i \quad \forall i \in N' \tag{16}$$

$$a_0 \leq e_m \tag{17}$$

$$\sum_{j=1}^{n} \sum_{i=0}^{n} R_j \cdot Y_{ijh} \leq C_h \quad \forall h \in H \tag{18}$$

$$d_{ijkh} = \sum_{n=0}^{B-1} d_{p_{(n)}p_{(n+1)}kh} \quad \forall h \in H, k \in K \tag{19}$$

$$\tau_{ijkh} = \sum_{n=0}^{B-1} \tau^{(z)}_{p_{(n)}p_{(n+1)}kh} \quad \forall h \in H, k \in K, z \in Z \tag{20}$$

$$\tau^{(z)}_{p_{(n)}p_{(n+1)}kh} = \frac{d_{p_{(n)}p_{(n+1)}kh}}{V^{(z)}_{p_{(n)}p_{(n+1)}kh}} \quad \forall (p_n, p_{n+1}) \in B, k \in K, h \in H \tag{21}$$

$$V^{(z)}_{p_{(n)}p_{(n+1)}kh} = \begin{cases} VP^{(z)}_{p_{(n)}p_{(n+1)}kh} & if\ off\ peak\ hours \\ VR^{(z)}_{p_{(n)}p_{(n+1)}kh} & if\ k\ in\ peak\ hours \end{cases} \quad \forall \left(p_{(n)}, p_{(n+1)}\right) \in B, k \in K, h \in H \tag{22}$$

$$Vol_{co2} = \left(a_0 + a_1 V_{ij} + a_2 V_{ij}^3 + \frac{a_3}{V_{ij}^2}\right) d_{ij} \tag{23}$$

$$Volijkh = \sum_{n=0}^{B-1} (a_0 + a_1 V^{(z)}_{p_{(n)}p_{(n+1)}kh} + a_2 \left(V^{(z)}_{p_{(n)}p_{(n+1)}kh}\right)^3 + a_3 \frac{1}{(V^{(z)}_{p_{(n)}p_{(n+1)}kh})^2}) d_{p_{(n)}p_{(n+1)}kh} \tag{24}$$

$$\sum_{j=0}^{n} \sum_{i=0}^{n} \sum_{k=1}^{m} \sum_{h=1}^{q} Vol_{ijkh} \cdot Z_{ijkh} \leq \mathrm{Co2}_{lim} \tag{25}$$

$$l_0 \geq b_0 + \mathrm{t_{load}} \cdot (\sum_{j=1}^{n} \sum_{i=0}^{n} R_j \cdot Y_{ijh}) / C_h \quad \forall h \in H \tag{26}$$

The constraint in eq.2 ensures that only one vehicle departs from each customer node, while eq.3 ensures that only one vehicle reaches each customer node. Eq.4 states that each arc, if chosen, has to be travelled by only one vehicle. Eq.5 forces the sum of routings, for every vehicle, to be greater than the sum of routings of that vehicle for one time period. Eq.6 forces the sum of routings, for every vehicle, to be less than the sum of routings for all time periods of that vehicle. Eq.5 and Eq.6 collectively, ensure that Y_{ijh} is consistent with X_{ijkh}. Eq.7 forces every vehicle to exit a node the same number of times it entered it. Eq.8 ensures that every vehicle leaves the central depot one time at most, and cannot be reused after it returns there. Eq.9 forces the continuous variable d_{ijkh} to be less than the total distance of route ij. If X_{ijkh} is 0 then d_{ijkh} also needs to be 0, but if X_{ijkh} is 1, then d_{ijk} has to be equal or less than D_{ij}. Eq.10 states that if an arc is chosen (X_{ij}=1) then the entire distance D_{ij} has to be travelled, which means that the continuous variable d_{ijkh} for every period and every vehicle has to be equal to the distance of the arc ij. If the arc is not chosen, then d_{ijkh} for every period and vehicle has to be equal to 0. Eq.11 ensures that the travel time of an arc ij, travelled by any vehicle is less than the difference between its start and end period, which in turn forces the travel time to be less than the duration of the time period. Eq.12 ensures that if an arc is chosen in the period k by the vehicle h, the departure time from the node i has to be before the end time of the period minus the time it takes to travel the arc ij. Eq.13 states that if an arc is travelled in period k by the vehicle h, then the arrival time in node j has to be more than the sum of the arrival time and the time it takes to travel the arc ij. Eq.14 ensures that if an arc is chosen, the arrival time at a node j has to exceed the sum of the departure time and the travel time of the arc. For eq.12, eq.13 and eq.14, if the arc is not chosen, the constraint is automatically satisfied due to the large number M used in the equation. Eq.15 calculates the service time of each customer as a function of the customer's demand and the vehicle's unload time and capacity. Eq.16 ensures that the departure time of any vehicle from node i has to be after its arrival and service time from the same node. Eq.17 states that the arrival period of any vehicle at the central depot doesn't exceed the end time of the last time period, since every vehicle has to return to the depot to be ready for use the next day. Eq.18 forces the sum of demands for the customers served by a specific vehicle to be less than the capacity of that vehicle for every vehicle h. Eq.19 calculates the travelled distance d_{ijkh} as the sum of each of the B-1 smaller arcs, in which the average speed is steady. Eq.20 calculates the travel time of the d_{ijkh} distance. Eq.21 calculates the travel time of a smaller arc, as the quotient of the distance of that arc and the average speed in that arc (which is steady by the definition of the smaller arcs). For eq.20 and eq.21 all the units used for the calculations are in the appropriate system of units. Eq.22 states that the average speed of a vehicle can take two possible values, one for peak and one for non-peak hours. Eq.23 calculates the emissions for an arc ij, using the model developed in [64]. The constants a1, a2, a3, a4 depend on the vehicle type. Eq.24 calculates the emissions for an arc ij, from a vehicle h, during the period k, as the sum of emissions for all the smaller arcs that constitute the original arc. Eq.25 forces the sum of emissions of the entire fleet of vehicles to be below the specified limit. Eq.26 forces all vehicles to depart from the central depot after the start of the first period and their respective load times.

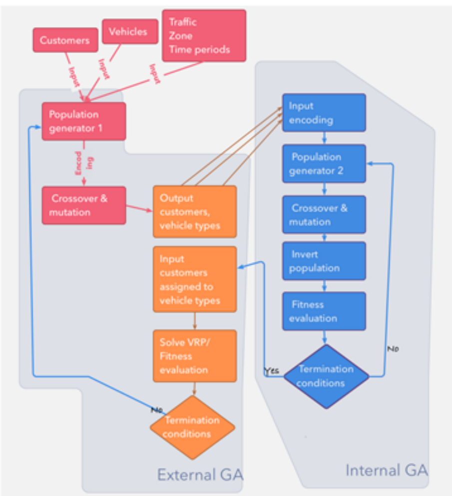
3.2 Problem Solution

The proposed solution method is a hybrid algorithm, consisting of two nested genetic algorithms and on top of that we use local search methods to improve solutions. The external (parent) genetic algorithm, which is responsible for assigning customers to vehicle types, forwards each customer-set, corresponding to a vehicle type, separately to the internal (child) genetic algorithm. The interior genetic algorithm solves the VRP of the given customer-set, and uses local search heuristics to improve its solutions. The results of the internal genetic algorithm are subsequently forwarded into the exterior genetic algorithm, in which the objective function of the final solution is calculated, the new population is chosen and the iterations continue. The problem's solution process is schematically presented in Figure 2.

3.2.1 Initialization & Solutions Encoding

The external genetic algorithm receives all the related information about the customers, vehicles, traffic-zones and time periods. The algorithm creates an initial population with a method that inserts randomly customers into customer-sets. These customer-sets represent the pairing of customers to specific vehicle types and are encoded as chromosomes for the external genetic algorithm. The chromosomes are first imposed to cross-over and mutation and then decoded back into customer sets. Each customer set, group of customers that will be serviced by the same vehicle type, is sent to the internal genetic algorithm, and the VRP problem for that group of customers now can be solved using standard VRP techniques embedded in the genetic algorithm. The results of all customer-sets are sent as feedback to the exterior genetic algorithm, in order to evaluate the quality of the solution of the problem, as a whole. Based on the results, a new population is generated and the process continues.

For the exterior genetic algorithm, each chromosome is a vector of length N where N is the total number of customers for the problem at hand. Each customer receives an identification number (unique for each customer). This number indicates the position of the customer in the chromosome. Each chromosome position can get an integer value between 1 and the number of vehicle types, indicating which vehicle type has to service each customer. For instance, if a customer has id 34, position 34 in the chromosome will contain the information about the vehicle type that should be used to make the delivery to that customer. Usually ids correspond to the reading order from the input data. To generate the initial population the roulette wheel methodology is used [66].



set populationSizeExt= $POP1$, populationSizeInt= $POP2$;
set genNumExt= $GEN1$, genNumInt= $GEN2$;
set probCrossoverInt, probCrossoverExt, probMutationInt, probMutationExt;
set customers, vehicles, trafficZones, timePeriods;
set generation counter $g1 = 0$;
// generate population $P1_0$ of external GA
for $i = 0 \ldots POP1$ **do**
  $P1_0 = P1_0 \cup$ PopulationGenerator () ;
end
$P1_g = P1_0$;
while *stopping criteria not met* **do**
  EncodingExt ($P1_g$);
  // generate offspring population $P1_{g_{children}}$
  $P1_{g_{children}}$ = Crossover $P1_g$;
  $P1_{g_{children}}$ = Mutate $P1_{g_{children}}$;
  OutputCustVTypes = DecodingExt ($P1_g$);
  // begin Internal GA
  EncodingInt (OutputCustVTypes);
  for $i = 0 \ldots POP2$ **do**
    $P2_0 = P2_0 \cup$ PopulationGenerator () ;
  end
  $P2_g = P2_0$;
  while *stopping criteria not met* **do**
    // generate offspring population $P2_{g_{children}}$
    $P2_{g_{children}}$ = Crossover $P2_g$;
    $P2_{g_{children}}$ = Mutate $P2_{g_{children}}$;
    InvertPop ($P2_g$);
    $R2_g = P2_g \cup P2_{g_{children}}$;
    $g = g + 1$;
    $P2_g$ =Fitness ($R2_g$);
  end
  // end Internal GA
  $P1_g$ =EncodingExt (SolveVRP (DecodingInt ($P2_g$));
  // combine current and offspring population
  $R1_g = P1_g \cup P1_{g_{children}}$;
  $g = g + 1$;
  $P1_g$=Fitness (R_g);
end

Algorithm 1: Internal and External GA

Figure 2. Solution Process

3.2.2 Genetic operators

The main operators we used in the exterior genetic algorithm are Crossover and Swap. After conducting an adequate number of tests, we observed that a two-point Crossover produced the best results. Both operators are shown in Figure 3. In the two-point Crossover, chromosomes are randomly selected two at a time, and two random positions are chosen. The first offspring, which is produced, receives the first (before the first position) and third part (after the second position) from the first chromosome and the middle part (between the two positions) from the second chromosome, whereas the second offspring receives the middle part from the first chromosome and the other parts from the second chromosome. The Swap operator changes the positions of two random chromosome locations, but the rest of the chromosome remains the same. Swap acts as a mutating operator to offer diversity in the genetic population, in order to explore the solution space, in search for the optimal solution.

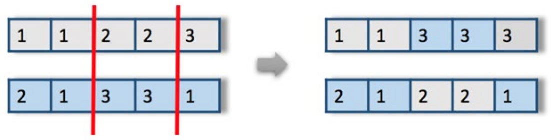


Figure 3. Two-point crossover for the external GA

3.2.3 Fitness Function

The fitness value of a given chromosome represents the quality of the produced solution. For most optimization problems, the fitness function is the same as the objective function. In VRP, fitness function represents the total travelled distance of the entire fleet in the solution. Since the genetic algorithm cannot directly satisfy the constraints of the problem, we decided to add a penalty function to the fitness function. Therefore, each time a constraint is violated the solution receives a penalty value, depending on the iteration of the algorithm and the extent of the violation. Typically, we allowed constraint violations in the early stages of the algorithm, so that the algorithm can search the space of infeasible solutions, as well as offer diversity in the population. As the iterations came to an end the penalty function increased in magnitude, making infeasible solutions less likely to participate in the new population for the next iteration.

To calculate the fitness of each chromosome we needed to get the results of the interior algorithm for that chromosome or in other words for every given specific pairing of customers to vehicle types (one chromosome from the exterior algorithm) to find the optimal routes (final population of the interior algorithm for the given input/chromosome) and then calculate the total distance and any penalties due to violations of the CO_2 limit.

3.2.4 Internal Genetic Algorithm

The internal Genetic Algorithm receives a specific customer set (customers grouped by the vehicle type to be used) as input from the exterior genetic algorithm, as well as the number of available vehicles and their characteristics and solves the vehicle routing problem for those customers. The algorithm generates an initial set of chromosomes with both heuristics, i.e. Sweep [67], Savings [68], Moles & Jameson [69] and random generating methods. The ratio of heuristic to random generated chromosomes is 1 to 5, since diversity of the population generally produced better results, while experimenting with the algorithm. The main operators we used on the chromosomes were crossover, mutation and inversion. At the early stages of the algorithm, no local search methods were used, due to the added computational time needed to implement those methods. Moreover, in those stages the algorithm did not produce solutions close to optimal and it is more important for the population to keep its diversity, in order for the algorithm not to converge too soon. When the algorithm execution was terminated, at least 90% of its iterations, local search methods (2-opt) were implemented to the existing solutions. Then, the final best solution was forwarded to the exterior genetic algorithm, and the process continued with the next customer set.

For the interior genetic algorithm, each chromosome has as many positions as the number of customers in the customer set. Each customer has its own unique id number and each chromosome consists of a sequence of these identification numbers. Since each customer can only be visited once, its id number can only appear once in the chromosome. Starting from the beginning of the chromosome, customers are assigned to the first vehicle, according to the sequence, as long as the remaining capacity of the vehicle can support the addition of the next customer. When the remaining capacity of the vehicle is not enough for the demand of the next customer, the route of that vehicle is complete and the process continues for the next vehicle until no customers are left. In every vehicle besides the customers that have to be serviced in the given order, the central depot is added as the first and last node of the route. It is important to note that this encoding produces only feasible solutions. The whole encoding and decoding process is illustrated in Figure 4.

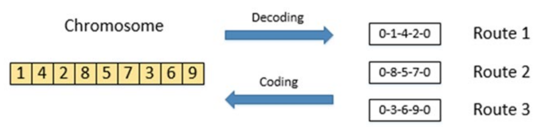


Figure 4. The Encoding – Decoding Process

For the interior genetic algorithm, we used three genetic operators: PMX crossover, swap and inversion. PMX crossover is a special type of crossover, in which parent 1 donates a swath of genetic material and the corresponding swath from the other parent is sprinkled about in the child. Once that is done, the remaining alleles are copied direct from parent 2. This type of crossover offers better performance in some optimization problems, and was picked due to the solution encoding. The mutation operator (swap) was used to offer diversity in the population, in the same way it was used in the exterior genetic algorithm. Inversion is another genetic operator that is used only when the newly produced solution is better than the previous. More specifically, a part of a given chromosome is chosen and the order of the genes in it is reversed. The inversion is illustrated in Figure 5:

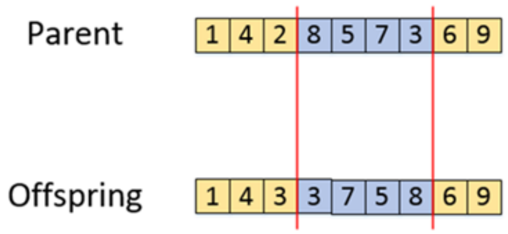


Figure 5. Gene Inversion

4. Conclusions

In this paper, we introduced a metaheuristic algorithm for a rich VRP problem, addressing the issues of fleet heterogeneity and CO_2 emissions, while at the same time taking into account in its solution the often met -in real life cases- option to assign parts of the transportation workload to third party contractors. The proposed algorithm is using a nested set of GAs to effectively decompose this complex problem to two simpler ones quickly leading to really good quality of results. The first one (external) tries to pair customers to vehicle types in a pretty much random way letting the genetic algorithm to lead the way to relatively good solutions. The second one (internal) for each pairing goes and solves X (where X the number of available vehicle types) simple instances of VRP (vehicle type does not affect the internal problem) making the most of the known heuristic techniques and the characteristics of the evolutionary methods, i.e. genetic algorithms.

The results of applying the proposed algorithm in selected VRP benchmark problems provide quite competitive results and the application of the proposed algorithm in our demonstrative case study shows that the proposed algorithm exhibits great potential for a wide range of complex practical problems. It is within the immediate research plans of the authors to further enrich the proposed algorithm by introducing the concept of time windows, thus expanding its applicability to specific PRP and green VRPTW problems. Finally, the business case of assigning a number of vehicles to a 3PL contractor is partially addressed in this paper, since the algorithm works under the assignment that all vehicles return to the depot. Still, this is not the case in the majority of real life cases, which roughly follow the Open VRP problem description, where vehicles are not required to return to the depot, or they have to return by revisiting the customers assigned to them in the reverse order.

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