

Investigating the Vehicle Routing Problem with Simultaneous Pickup and Delivery for Multi-Product Distribution: An Optimization Approach

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Abstract — This study addresses the Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD), which carries multi-products in multiple compartments within a single-vehicle. The unique characteristics of the study lies on the determination of the vehicle route from the depot to customers because not only does it consider the vehicle's capacity but also the compartment capacity which also serves as a limitation. We calculate the set of instances using two customer grouping methods namely smallest maximum load (SML) and largest maximum load (LML). The solution obtained by the cheapest insertion method can be improved by the Tabu Search algorithm. Finally, the computational result is reported from the test instance.

Keywords—vehicle routing problem, simultaneous pickup and delivery, multiple compartment, tabu search

1. Introduction

Vehicle Routing Problem (VRP) are the classical optimization problem which examines various alternative routes from a depot to several delivery locations and back to the originating point with a vehicle capacity as the prime constraint. By ensuring the carrying demand on a route does not exceed the capacity of the vehicle, the VRP model aims to minimize the total distance and the number of operating vehicles. Among its development, the model has run several variations such as namely VRP with Backhauls (VRPB), VRP with Mixed Pickup and Delivery (VRPMPD), and VRP with Simultaneous Pickup and Delivery (VRPSPD) [1].

The transportation and distribution of product are the most commonly used cases in the VRPSPD branch, particularly for improving distribution handling of both raw materials and finished goods, from manufacturing companies to their vendors and vice versa [2]. If the delivery of raw materials and pick-up of finished goods are conducted separately,

it will double the amount of transportation costs and time due to longer routes taken and time consumed. Handling the delivery and pickup of products separately not only requires handling efforts to both activities [1], but also results in increased area requirements or space for vendors to store both types of products. With the VRPSPD system, the companies and their manufacturing counterparts able to generate benefits from the simultaneous delivery and goods collection process as the number of vehicles used, total distance travelled, handling time, and space for warehousing reduced [3].

While most of VRPSPD studies discuss a single product routing problem, the Vehicle Routing Problem with Multiple Products (VRPDMC) holds a specific application for transporting multiple products. The primary VRPDMC characteristics are the usage of vehicle with compartment as the capacity limitation for storing various types of product [4]. This practice is similar in the distribution of food and fuel where the use of vehicles with compartments allow to transport heterogenous goods altogether on the same vehicle, but in different placement [5], [6], [7]. In some applications, the compartment layout of the vehicle may be reconfigurable as the separators between the compartments can be adjusted to fit with the total volume of the vehicle. In contrast to this flexible arrangement, other scenarios would consider fixed compartment for an unchangeable setup [5].

The paper aims to propose a new method combining Vehicle Routing Problem with Simultaneous Pickup and Delivery with Multiple Compartment (VRPSPDMC) to optimise the number and the total transportation cost of vehicles with compartments. The organization of the paper

commence with brief introduction of two distinct branches and their research potential, followed by section 2 in which reviews the development of VRPSPD literature, multiple compartments and methodological approaches (SML, LML and Tabu search), then section which present the data used, mathematical formulations and assumptions, continued by section 4 in which describes the results obtained, and then concluded with section 5 which summarizes the research, findings, and potential direction for future research.

2. Literature Review

2.1 VRPSPD

In VRPSPD system, the vehicle will deliver and collect goods simultaneously so that it will reduce the total distance travelled by the vehicle and the time that has resulted in a reduction in transportation costs. The VRPSPD was first proposed by [8] whose study presents a heuristic method to solve a real-life problem concerning the distribution with tree-stage procedure, namely (1) clients clustering, (2) assigning every vehicle to every cluster, and (3) solving the Traveling Salesman Problem. These stages are also called “the multi-vehicle Hamiltonian one-to-many-to-one pickup and delivery problem with combined demands” by [9] on their work on a classification of static pickup and delivery problems. Apart from that, [10] and [11] implemented four steps heuristic insertion with multi-depots to solve the VRPSPD. [12] proposed an exact procedure of Set Partitioning (SP) combined with Iterated Local Search (ILS) heuristic-based algorithm to solve Heterogenous Fleet VRP case. [7] and [13] also outlines examples of VRPSPD cases using exact method while [14] propose the same cases with particle swarm optimization.

2.2 Multi-Products and Multiple Compartments

A vehicle with multiple compartments carries a number of compartments where the same type of product is designated to a specific compartment and is not mixed with a different type of product. [5] analyse the demand for compartments with the methods and software used in two industries: the distribution of petrol and food. The authors conclude that the distribution of petrol typically involves different fuel types and vehicles with five compartments thus a single petrol station

demanding several different fuel types can be served by one vehicle with several compartments. Other studies have been developed discussing the application of the multi-compartment VRP by [15] and mathematical model for VRP for multiple product types, compartments and trips with soft time window called VRPMPMTSTW [16].

2.3 Tabu Search

In this study, the heuristic method that will be used to find a solution from VRPSPDMC is the Tabu Search algorithm. Tabu search is a mathematical optimization method that guides the procedure for searching local search to explore the solution area outside the local optimal point by giving taboo status to the solution that has been found [17]. The solution to the solution with the Tabu Search method on VRP can produce the best optimization solution value and require faster calculation time compared to other methods [1]. The Tabu Search algorithm can be used to obtain robust solutions, showing a route with minimum total distance considering the vehicle capacity [3], [18], [19]. Among these studies, [20] presents a four combination of local search with neighborhood algorithm, tabu search, and constructive algorithms in solving VRPSPD and show better computational outcomes than [19].

The initial step of using the Tabu Search algorithm is that it starts with determining the Initial Solution where the research uses the Nearest Neighbour and client grouping (smallest maximum load or largest maximum load) [2]. After getting the initial solution value, the next step is to evaluate the moves where each move will produce a neighbourhood solution. The moves are evaluated in three ways, namely, relocation, exchange, and crossover. If the dismissal criteria are met, the Tabu Search algorithm process stops, but if not, then the move evaluation will be done again.

3. Method

In this study, determining the initial Tabu Search solution uses a combination approach of client grouping with route determination procedures [2]. Client grouping used is grouping clients based on the net weight of the carrying load of each client or what is called net demand. Net demand (nd) is the difference from the number of goods transported (pi) to the number of items sent (to) to the client (i),

so the net demand equation from the client (i), namely:

$$nd(i) = p_i - d_i$$

If $nd(i) < 0$, the vehicle load will decrease by $nd(i)$ after visiting the client i . If $nd(i) > 0$ then the vehicle load will increase by $nd(i)$ after visiting the client i . Client grouping is divided into two types, Smallest Maximum Load (SML) and Largest Maximum Load (LML). The smallest maximum load is the grouping of clients based on the smallest net demand to the largest. In the SML the vehicle visits a client that has $nd(i) \leq 0$ and then visits a client that has $nd(i) > 0$. The maximum load margin is the grouping of clients based on the largest net demand to the smallest. In LML vehicles visit all clients with $nd(i) > 0$ first then visit the client with $nd(i) \leq 0$. Insertion of new clients into a group can be done if additional pick-up and subsequent client delivery does not violate the maximum capacity of the vehicle and does not violate the compartment limits of each product type.

After grouping clients, route determination procedures are carried out based on Independent Grouping and Routing (IGR), which is determining the route by determining the group for each client first, then determining the route of the vehicle. This procedure is used on problems without maximum distance constraints. The heuristic procedure that can be used to determine the initial Tabu Search solution is:

- a. IGR1: client grouping is based on smallest maximum load by ordering client visits based on the smallest to largest net demand load, then repairing the initial route obtained by the 2-opt heuristic method.
- b. IGR2: client grouping is based on Largest Maximum Load by ordering client visits based on the largest to the smallest net demand, then repairing the initial route obtained by the 2-opt heuristic method.

3.1 Assumptions and Notation

3.1.1 Assumptions

The following are the assumption considered:

1. Every route carried out by a vehicle always starts and ends at the depot.
2. Each client must and should only be visited by exactly one vehicle.

3. The vehicle used has a limited carrying capacity.
4. Every vehicle visit to the client is carried out by the delivery and pickup of goods in accordance with the number of requests.
5. Total items sent to clients and taken from clients in one route may not exceed the capacity of the vehicle.
6. Each type of product has each specific compartment that is different from the other types of products
7. Each type of product transported must not exceed the compartment capacity of each type of product

3.1.2 Notation

The mathematical model for the VRPSPDMP problem is as follows:

- N : Client location node $\{0,1,2,3,\dots,n\}$
- $N(i,j)$: Client and warehouse location set, $i,j \in N, i \neq j$
- V : Set vehicle, $\{1,2,3,\dots,k\}$
- P : Set product type $\{1,2,3,\dots,p\}$
- T : Set Compartment for each type of product $\{1,2,3,\dots,p\}$
- R : Set trips $\{1,2,3,\dots,r\}$
- n : Number of clients = $|N|$
- c_{ij} : Distance between location of client i and client j
- $d_{mi,p}$: The number of goods needed by the client i on the product p
- d_{ij} : The number of items requested is sent to locations j
- p_{ij} : The number of items requested is taken from the location j
- Q : Vehicle Capacity
- $C_{pk,t}$: Compartment capacity for each type of product p in vehicle v
- TC_k : Fixed costs for vehicle trips t

3.2 Decision Variable

$$X_{ij}^k = \begin{cases} 1, & \text{if arc } (i,j) \text{ belongs to the route operated by vehicle } k \\ 0, & \text{Otherwise} \end{cases}$$

y_{ij} : the number of items taken from the initial collection location to location i and transported on the (i, j) side

z_{ij} : the number of items sent from the initial collection location to location i and transported on the (i, j) side

XTk, r, i, p : The quantity of product type p sent from node i to node j using vehicle k on trip r

XLk, r, i, p : The quantity of product type p available to be taken from node i to node j using vehicle k on trip r .

3.3 Objective Function

The objective function is to minimize the total distance of the vehicle

$$\sum_{k=1}^k \sum_{i=0}^n \sum_{j=0}^n c_{ij} \cdot x_{ij}^k$$

3.4 Constraints

The objective function is to minimize the total distance of the vehicle

1. Every client visited can only be one time and only by one vehicle

$$\sum_{i=0}^n \sum_{k=1}^n x_{ij=1}^k, \quad j=0,1,2,\dots,n$$

2. After visiting a client, the vehicle must leave the client

$$\sum_{i=0}^n x_{ij}^k - \sum_{i=0}^n x_{ji=0, j=0,1,\dots,n; k=1,2,\dots,k}^k$$

3. The number of items sent to location j must fulfil delivery requests at the location j

$$\sum_{i=0}^n z_{ij} - \sum_{i=0}^n z_{ji} = dj, \quad \forall \neq 0$$

4. The number of items taken from location j must fulfil pickup requests at the location j

$$\sum_{i=0}^n y_{ji} - \sum_{i=0}^n y_{ij} = dj, \quad \forall \neq 0$$

5. The total number of items taken and shipped must not exceed the maximum capacity of the vehicle

$$y_{ij} + z_{ij} \leq Q \sum_{k=1}^k x_{ij}^k, \quad i, j = 0,1,2, \dots, n$$

6. The number of items sent that are transported on the (i, j) side is not a negative number

$$y_{ij} \geq 0, \quad i, j = 0,1,2, \dots, n$$

7. The number of items taken that are transported on the (i, j) side is not a negative number

$$z_{ij} \geq 0, \quad i, j = 0,1,2, \dots, n$$

8. The number of product quantity type p sent from node i to node j using vehicle k on trip r must not exceed the capacity of the compartment

$$XTk, r, i, p \leq C_{pk, t}, \quad \forall r = 1,2, \dots, NR; \\ i = 1,2, \dots, N;$$

9. The number of product quantity type p are taken from node i to node j using vehicle k on trip r must not exceed the capacity of the compartment

$$XLk, r, i, p \leq C_{pk, t}, \quad \forall r = 1,2, \dots, NR; \\ i = 1,2, \dots, N;$$

10. Total number of product quantity type p are sent from depot to the customer using vehicle k on trip r should in total be greater than the total sum of the demands at all customers.

$$\sum_k \sum_r \sum_j XTk, r, i, p = \sum_{j \in N} dm_{j,p}; \quad \forall i \in N, \forall p \in P$$

4. Result and Discussion

This study uses small scale instances based on the data set collected by [21] who took data sets from fashion retails in Singapore, each of which serves 10-26 customers.

4.1 Problem Instances

In order to get VRPSPD with multiple products, commodity m is used in each agency and is categorized into several different m quantities. m which is used to generate data sets is 5 commodities with different quantities in each commodity. To get classic VRPSPD, calculate the number of shipping requests using a probability of $2/3$ of the quantity, while for the number of requests for taking goods with a probability of $1/3$. After, the distribution of shipping needs and the

need for collection is done randomly to each commodity. Vehicle capacity is set to $Q * m / 20$, where Q is the number of original capacities of the agency. The data set is then calculated using the SML and LML approaches [2] with a limit to the number of compartments of each commodity m . The data set is then executed using an algorithm encoded through the NetBeans IDE 8.2 application. All instances are executed on an Intel Pentium® Dual-Core T4400 @ 2.20GHz Windows 7 Ultimate computer, with 1GB of RAM

Table 1. Result Comparison between [21] with the Current Methodological Approach

Inst.	n	m	Cap	Literature		SML		LML		TS	
				V	Travel Cost	V	Travel Cost	V	Travel Cost	V	Travel Cost
1	10	5	100	1	57.79	2	94.07	3	90.51	3	83.34
2	12	5	100	2	47.59	5	134.7	5	105.16	5	107.3
3	14	5	100	2	73.88	4	113.8	5	142.74	5	124.8
4	16	5	100	2	85.74	5	208.9	6	199.66	5	128.7
5	18	5	150	1	76.9	3	190	3	194.48	3	151.1
6	20	5	150	2	91.12	4	228.7	5	263.8	4	141.9
7	22	5	150	1	81.53	3	249.4	4	233.03	4	128.8
8	24	5	150	2	99.49	5	295.6	6	306.82	7	186.6
9	26	5	100	3	101.3	6	305.7	9	333.83	7	188.8

Table 1 presents the results obtained from [20] instances, showing the cost resulted from the calculation of the proposed mathematical formulation on the literature instances with SML, LML, and TS approach methods are higher. Such increase occurs due to the limitation of compartment capacity of each type of product. Further, the Table also shows that Tabu Search has the smallest gap amount from the literature hence considered as the best solution among the three approach methods.

5. Conclusions

Based on the explanation above, we found the mathematical formulation as proposed in this paper could be useful to determine the optimal value of VRPSPDMC despite the results obtained from the literature's total cost are higher. Moving forward from these results, we suggest a deeper analysis of another approach to solve this problem and applying a more parameters heterogenous vehicle or flexible compartments provide promising pathways to explore as future research directions. Comparisons among them with the current results will shed new insights in the field.

References

- [1] Yousefikhoshbakht, M., Didehvar, F., & Rahmati, F., "A combination of modified tabu search and elite ant system to solve the vehicle routing problem with simultaneous pickup and delivery", *Journal of Industrial and Production Engineering* 31(2), pp: 65-75, 2014.
- [2] Osman, S., & Mojahid, F., "Capacitated transport vehicle routing for joint distribution in supply chain networks" *International Journal of Supply Chain Management* 5(1), pp: 25-32, 2016.
- [3] Avci, M., & Topaloglu, S., "A hybrid metaheuristic algorithm for heterogeneous vehicle routing problem with simultaneous pickup and delivery", *Expert Systems with Applications* 53, pp: 160-171, 2016.
- [4] Kritikos, Manolis N., and George Ioannou, "The heterogeneous fleet vehicle routing problem with overloads and time windows", *International Journal of Production Economics*, 144 (1), pp: 68-75, 2013.
- [5] Derigs, Ulrich, Jens Gottlieb, Jochen Kalkoff, Michael Piesche, Franz Rothlauf, and Ulrich Vogel. "Vehicle routing with compartments:

- applications, modelling and heuristics*", OR spectrum 33(4), pp: 885-914, 2011.
- [6] Silvestrin, Paulo Vitor, and Marcus Ritt. "An iterated tabu search for the multi-compartment vehicle routing problem", Computers and Operations Research Vol. 81, pp: 192-202, 2017.
- [7] Jamali, M. "Presenting a Location-Routing Problem for Multi-vehicle Hazardous Materials Transport, Considering the Cost Dependent to the Amount of Materials Loaded", International Journal of Supply Chain Management 8 (3), pp 1079-1100, 2019.
- [8] Min. H, "The Multiple Vehicle Routing Problem with Simultaneous Delivery and Pick-up Points", Transportation Research 23(5), pp: 377-86, 1989.
- [9] Berbeglia, Gerardo, Jean-François Cordeau, Irina Gribkovskaia, and Gilbert Laporte. "Static pickup and delivery problems: a classification scheme and survey." *Top* 15(1), pp: 1-31, 2007.
- [10] S. Salhi and G. Nagy, "A Cluster Insertion Heuristic for Single and Multiple Depot Vehicle Routing Problems with Backhauling," Journal of the Operational Research society, Vol. 50, No. 10, pp. 1034-1042, 1999.
- [11] Nagy, Gabor, and Saïd Salhi. "Heuristic algorithms for single and multiple depot vehicle routing problems with pickups and deliveries." *European Journal of Operational Research* Vol. 162, no. 1 pp: 126-141, 2005.
- [12] Subramanian, A., Uchoa, E., & Ochi, L. S., "A hybrid algorithm for a class of vehicle routing problems", *Computers & Operations Research*, 40(10), pp. 2519-2531, 2013.
- [13] Dell'Amico, Mauro, Giovanni Righini, and Matteo Salani. "A branch-and-price approach to the vehicle routing problem with simultaneous distribution and collection." *Transportation science* Vol. 40(2), pp: 235-247, 2006.
- [14] Ai, The Jin, and Voratas Kachitvichyanukul. "A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery." *Computers & Operations Research* 36(5) pp: 1693-1702, 2009.
- [15] Silvestrin, Paulo Vitor, and Marcus Ritt. "An iterated tabu search for the multi-compartment vehicle routing problem." *Computers & Operations Research* Vol. 81, pp: 192-202, 2017.
- [16] Kabcome and T. Mouktonglang, "Vehicle Routing Problem for Multiple Product Types, Compartments, and Trips with Soft Time Windows," Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Sciences, 2015
- [17] Koç, Çağrı, and Gilbert Laporte. "Vehicle routing with backhauls: Review and research perspectives." *Computers and Operations Research* Vol. 91, pp: 79-91, 2018.
- [18] Yin, Chuanzhong, Lei Bu, and Haitao Gong, "Mathematical model and algorithm of split load vehicle routing problem with simultaneous delivery and pickup" *International Journal of Innovative Computing, Information and Control* 9(11), pp: 4497-4508, 2013.
- [19] Dethloff, Jan, "Vehicle routing and reverse logistics: the vehicle routing problem with simultaneous delivery and pick-up" *OR-Spektrum*, Vol 23, No. 1, pp 79-96, 2001.
- [20] Bianchessi, Nicola, and Giovanni Righini. "Heuristic algorithms for the vehicle routing problem with simultaneous pick-up and delivery." *Computers & Operations Research* 34(2), pp: 578-594, 2007.
- [21] Z. Zhang, "Multi-commodity demand fulfillment via simultaneous pick up and delivery for a fast fashion retailer," *Computer and Operation Research* Vol 103, pp 81-96, 2019.