A Judgement Model for Direct-Indirect Delivery

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Abstract—We study the selection of direct delivery / indirect delivery for LTL shipment in a two-echelon supply chain composed of a central distribution center (DC), an intermediate cross-docking center (XD) and customers, where the direct delivery is a shipment directly from the DC to customers, and the indirect delivery is a shipment to customers via the XD. We develop an optimization model that selects direct delivery / indirect delivery, taking into account the feature of volume discount. The volume discount is modelled as a nonlinear concave function and reformulated as MILP by a piecewise linearization technique. We propose a new analysis framework to account for the choice of direct / indirect delivery based on two indices, Distance-Ratio (DR), which is the ratio of the distance between direct and indirect delivery, and Demand Quantity (DQ). Based on this analysis, a heuristic solution using a support-vector-machine-based discriminant analysis model using the indices DR and DQ was proposed. In numerical experiments, the proposed heuristics were compared to the optimal solution for 100 randomly generated problem cases, and the error was shown to be within 0.8%.

Keywords—Include at least 5 keywords or phrases

1. Introduction

Many companies give high attention to minimize their transportation cost of goods because it is considered as a major share of the total logistics cost. In many companies, the transportation of their goods is relied on third-party distributors. Due to the deregulation, the transportation costs charged by third party distributors has been decreased.

One important mode of transportation used is the LTL (Less-than-Truck-Load) mode, which is attractive when shipment sizes are less than truck capacity [1][2][3][4]. Typically, LTL carriers offer volume, or quantity, discounts to their clients to encourage demand for larger, more profitable shipments (Figure 1). These economies of scales motivate the shipper to set an intermediate cross-docking center (XD). In that system, freight is delivered from the depot to the XD, from where it is delivered to customers. This approach is strongly connected to the design of City Logistics systems for large cities, where it provides the means to efficiently keep large trucks out of the city centre, with small and environment-friendly vehicles providing the last leg of distribution activities [5][6].

In this study, delivery directly performed from the depot to customers is called direct delivery, and delivery via the XD is called indirect delivery as illustrated in figure 2. As mentioned earlier, indirect delivery can enjoy the economy of scale by consolidating freight, while it makes a detour compared to direct delivery. Therefore, it is very important to analyse the advantages and disadvantages of each option, and to properly judge the choice between direct and indirect delivery. To our knowledge, however, there is no study that analyse advantages and disadvantages of these options. Especially, the relationship between the location of customers, the XD, and the depot and the associated costs has yet been studied.

The intend of the research is centred around the question of which direct/indirect delivery is selected under which condition. We build an optimization model to select the direct/indirect shipping in LTL shipper problem, that results in nonlinear concave minimization problem, which is hard to solve. We applied a piecewise linearization technique to approximate the problem as a mixed-integer linear program, which can be solved very efficiently by the off-the-shelf solver.

We proposed the framework of judging the direct/indirect delivery with distance-ratio (DR) and demand quantity (DQ), where DR is the distance...
ratio of distance of direct and indirect distance. We analyse that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa. The hint of this idea comes the distance-intensity (DI) analysis, in the facility planning area, in which the efficiency of transportation is measured by the transport work distance-intensity (DI) analysis.

Further, we proposed a heuristics method for obtaining near optimal judgement of direct/indirect delivery, using the supervised machine learning classification technique. We applied support vector machine (SVM) to classify the direct/indirect delivery judgment using the data pair of obtained optimal judgement and values of DR and DQ as supervised data. By doing so, we can judge the near optimal judgement of direct/indirect delivery for very large-scale problem very quickly. Our heuristics does not require the mathematical solver, the only background required is knowledge of simple linear classification and the usage of spreadsheet. This convenience is especially beneficial in practice, especially when the mathematical programming cannot be used on a daily basis. For daily operations, the demand requests from customers vary from day to day, while the location of the depot and the XD is fixed.

In the remainder of the paper is as follows. In the section 2, we review the related research. In the section 3, we present the mathematical formulation of the problem. In the section 4, we demonstrate the numerical experiments. In the section 5, we present the result of discriminant model. In the section 6, we make concluding remarks.

2. Literature Review

We proposed the framework of judging the direct/indirect delivery with distance-ratio (DR) and demand quantity (DQ), where DR is the distance ratio of distance of direct and indirect distance. We analyse that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa. The hint of this idea comes the distance-intensity (DI) analysis, in the facility planning area, in which the efficiency of transportation is measured by the transport work distance-intensity (DI) analysis.


Another related research is multi-commodity flow problem. See [18] for recent review. There are several studies that study the exact concave minimization problem [19-22]. An exhaustive search of all extreme points would provide an optimal flow, since a concave function achieves its minimum at an extreme point of the convex feasible region. However, such an approach is impractical for all but the simplest of problems. The fixed-charge network design problem is also extensively studied that arise in various applications in
telecommunications, logistics, and transportation [23-27]. Piecewise Linearization is a popular technique to approximately minimize concave cost function [28-32].

The contribution of this paper is as follows. First, we model the choice of direct/indirect delivery mode and present the mixed-integer linear programming formulation using the piecewise-linearization. Second, we place a great deal of emphasizes on the analysis of the optimal solution. Especially, we proposed the new framework to analyze and account for the reason of the choice of direct/indirect delivery with distance-ratio and demand quantity. Third, we proposed a new heuristics method utilizing DR-DQ framework.

3. Methodology

We consider the situation where transportation requests are fully outsourced to the third-party distributors, so the forwarding companies are no responsible for the vehicle scheduling. The shipment cost charged to the third-party distributors is calculated according to the predetermined tariff table. The tariff table provides a list of agreed fixed tariff rates under non-linear consideration of loads and lengths of the tours that the subcontracting company charges for its delivery services. An example illustrating the typical tariff table is shown in table 1. In the tariff table, the cost \( f(d, q) \) is specified according to the distance \( d \) and delivery quantity \( q \). This can be modelled by concave function conceptually as illustrated in figure 2.

The modelling framework is illustrated as in figure 3. Let \( k = 1, \ldots, K \) denote a set of customers. Let \( d_k \) denote distance from XD to Customer \( k \), \( d_0 \) denote distance from DC to XD, \( D_k \) denote distance from DC to Customer \( k \), \( q_k \) denote demand of customer \( k \). We let \( x_k \) denote the binary variable to indicate if the customer \( k \) is fulfilled with direct delivery or indirect delivery as follows.

\[
x_k = \begin{cases} 
1, & \text{if customer } k \text{ is fulfilled with indirect delivery} \\
0, & \text{otherwise} 
\end{cases}
\]

We also use the notation \( q_0 = \sum_{k=1}^{K} x_k q_k \) to denote the shipment volume from the depot to XD.

Using these notations, the direct/indirect delivery selection problem can be modelled

\[
\begin{align*}
\text{Min.} & \quad C_D + C_I \\
\text{S.t.} & \quad C_D = \sum_{k=1}^{K} (1 - x_k) f(D_k, q_k) \quad (2) \\
& \quad C_I = f(d_0, q_0) + \sum_{k=1}^{K} x_k f(d_k, q_k) \quad (3) \\
& \quad q_0 = \sum_{k=1}^{K} x_k q_k \quad (4) \\
& \quad x_k \in \{0,1\} \quad (5)
\end{align*}
\]

The objective function is to minimize the sum of the direct cost \( C_D \) and indirect delivery cost \( C_I \). The constraints (2) presents the calculation of the direct cost, in which the cost \( f(D_k, q_k) \) is added only when \( x_k = 0 \). The constraint (3) presents the calculation of the indirect cost, which composed of the consolidated shipment cost from the depot to the XD and the delivery cost from the XD to each customer. The constraint (4) presents the calculation of \( q_0 \), and the constraint (5) presents binary constraints for \( x_k \).

![Figure 3. Direct and indirect delivery (Left: Direct, Right: Indirect)](image)

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Since \( D_k, d_k, q_k \) are fixed constants, \( f(D_k, q_k) \) and \( f(d_k, q_k) \) are independent on the choice of \( x_k \). We let \( C_{Dk} = f(D_k, q_k) \) and \( C_{dk} = f(d_k, q_k) \) as constants. Further, given \( d_0 \) is constant, it is sufficient to see how much quantity \( q_0 \) becomes for the choice of \( x_k \).

We applied the piecewise linearization technique to model the problem via the mixed-integer linear
programming. We let \( q_0^n \) denote a binary indicator to take \( q_0 \) if \( q_0 \) belongs to quantity segment \( n \), and take zero otherwise as:

\[
q_0^n = \begin{cases} 
q_0, & N_{n-1} \leq q_0 \leq N_n \\
0, & \text{otherwise}
\end{cases}
\]

We let \( z_n \) denote a binary indicator to take 1 if \( q_0 \) belongs to quantity segment \( n \), and take zero otherwise as:

\[
z_n = \begin{cases} 
1, & N_{n-1} \leq q_0 \leq N_n \\
0, & \text{otherwise}
\end{cases}
\]

Using these auxiliary variables, we have the following reformulation.

\[
\text{Min.} \quad C_D + C_I \quad (6)
\]

\[
\text{S.t.} \quad C_D = \sum_{k=1}^{K} (1-x_k) C_{Dk} \quad (7)
\]

\[
C_I = \sum_{n=1}^{S} z_n c_n + \sum_{k=1}^{K} x_k C_{Ik} \quad (8)
\]

\[
q_0 = \sum_{k=1}^{K} x_k q_k \quad (9)
\]

\[
q_0 = \sum_{n=1}^{N} q_0^n \quad (10)
\]

\[
N_{n-1} z_n \leq q_0^n \leq N_n z_n \quad (11)
\]

\[
\sum_{n=1}^{N} z_n = 1 \quad (12)
\]

\[
z_n \in \{0,1\} \quad (13)
\]

\[
x_k \in \{0,1\} \quad (14)
\]

Equation (6) represents the objective function. Equations (7)(8) represent the direct and indirect delivery costs. Equation (9) represents the delivery quantity from the factory to the XD is equals to the sum of demand quantity for indirect delivery. Equation (10) indicates that the delivery quantity from the factory to the XD equals to the sum of the flow section. Equation (11) indicates that the flow section is equal to or less than the upper and lower limits of the category. Equation (12) indicates that one section is selected. The constraint (13)(14) presents binary constraints for \( x_k \) and \( z_n \), respectively.

4. Discussions

This section presents numerical examples. In section 3.2.1, we describe the data set. In section 3.2.2, the results are analysed with respect to the distance-ratio and demand quantity.

4.1 Data Set

We build a virtual case based on the link to a real-world company. In this case, we consider a company delivers to the entire west of Japan with a DC in Kansai and a XD in Kyushu. Figure 5 shows a conceptual diagram of this case. We would like to decide whether to deliver directly from the DC or to perform indirect delivery via XD in Kyushu. The input information and experimental environment were set as follows.

- Number of customers: 10000
- Demand \( q_k \): drawn from uniform distribution \( U(0,4) \)
- Distance from the DC to the XD \( D_0 \): 600km
- Distance from the DC to customers \( d_k \): drawn from uniform distribution \( U(0,1000) \)
- Distance from the XD to customers \( d_0 \): drawn from uniform distribution \( U(0, D_0 + d_k) \)
- Experimental conditions: Intel Core i7 3.5GHz×2, 16GB memory
- Code: MATLAB
- Solver: Gurobi Optimizer

4.2 Distance-ratio and Demand quantity analysis

To analyze the result, we proposed a framework with distance-ratio (DR) and demand quantity (DQ), named DR-DQ analysis. We propose the index distance-ratio (DR), the distance ratio of distance of direct and indirect distance, denoted by the following equation (15).
\[ R_k = \frac{d_k}{d_0 + d_k} \]  

(15)

We plot the customer in scatter diagrams with horizontal axis being distance ratio (DR) \( R_k \) and vertical axis being demand quantity (DQ) \( q_k \) as in figure 6, 7, and 8. Customers with \( x_k = 1 \) are plotted in figure 6, customers with \( x_k = 0 \) are plotted in figure 7, and all customers are plotted in figure 8.

We see that customers with \( x_k = 1 \) are scattered on the left-upper corner and customers with \( x_k = 0 \) are scattered on the right-bottom corner. The results indicate that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa.

The results indicate that the relationship among the location of DC, the XD, and the customer may affect the choice of the direct/indirect delivery. More specifically, the choice depend on how much a detour is made. For example, if the XD is located in the middle of the DC and a customer, then it is reasonable to use the XD. In this case \( R_k = 1 \). On the other hand, if the XD is located in the opposite direction from the DC to customers, it is not reasonable to use the XD. In this case \( R_k \) takes much smaller value than one. Therefore, the rate of detour can be evaluated by the distance ratio presented in the equation (15).

The results also indicate the effect of the shipment volume. The higher the shipment volume, the more likely the direct delivery is selected. This comes from the utilization of volume discount presented in as in the figure 1. This analysis help get better understanding of what depends on the direct/indirect delivery choice.

4.3 Heuristics with Discrimination Model

As a result of the previous section, using the index distance ratio, we can get a hint to judge direct/indirect delivery. In this section, we use this indicator to determine the direct / indirect delivery heuristically. We considered a linear discrimination model that can perform motion. From the graph shape presented in the figure 8, we consider it is possible to judge the direct/indirect delivery from the value of \( R_k \) and \( q_k \) by the linear function. We use the SVM (Support Vector Machine) as discriminator.

SVM is one of the most widely used pattern recognition learning algorithms and is a two-class problem linear discriminant function construction method that achieves the maximum margin. In this research, we use a linear function described as the equation (16), and the direct delivery is selected for the customer \( k \) if the value of \( g (R_k, q_k) \) is positive, and the indirect delivery is selected if \( g (R_k, q_k) \) is negative.
\[ g(R_k, q_k) = \theta_1 R_k + \theta_2 + \theta_0 \]  

(16)

Let \( K_D \) denote the number of customers that are selected direct delivery, \( K_I \) denote the number of customers for indirect delivery, and the subscripts \( i = 1, \ldots, K_D, j = 1, \ldots, K_I \) denote the subscripts for customers of direct delivery and indirect delivery, respectively. The problem of maximizing the margin for obtaining the above-mentioned discriminant function parameters \( \theta = [\theta_0, \theta_1, \theta_2] \) is as follows.

Max. \[ \sqrt{\theta_1^2 + \theta_2^2} + \sum_{i=1}^{K_D} u_i + \sum_{j=1}^{K_I} v_j \]  

(17)

s.t. \[ \theta_1 D_i + \theta_2 q_i + \theta_0 \geq 1 - u_i, \quad (i = 1, \ldots, K_D) \]  

(18)

\[ \theta_1 D_j + \theta_2 q_j + \theta_0 \leq -1 + v_j, \quad (j = 1, \ldots, K_I) \]  

(19)

\[ u_i, v_j \geq 0 \]  

(20)

The results of parameter fitting for the case problem presented in the previous section are as follows.

\[ g(R_k, q_k) = -7.7219R_k + 0.8195q_k - 3.3307 \]

(21)

Table 2 shows the number of errors, the value of the approximate solution when the obtained discriminant is used, and the error rate between the approximate solution and the optimal solution. Table 2 shows that the discriminant model using DR-DQ performs well in terms of the number of errors and the objective function value. It is interesting to see that the error rate of the objective function (0.74%) is extremely low despite some errors (396/10000=3.96%). The possible reason for this phenomenon is that for customers close to the boundary of the discriminant equation, there is no large cost difference regardless of whether direct delivery or indirect delivery is selected, so the effect of errors may be small. To see this hypothesis, for each customer \( k \), the incremental cost when \( x_k \) obtained by the optimal solution is inverted is defined as “error cost \( E_k \)”, and the relationship with the distance from the boundary line, measured by \( ||g(R_k, q_k)|| \), is analyzed as shown in Figure 9. In a region where the distance from the boundary is in the range of \( \pm 3 \), \( E_k \) has a small value. From this viewpoint, the robustness of the discriminant model using DR-DQ analysis can be asserted.

Finally, as additional verification, 100 different problem examples were generated and the error between the optimal solution and the approximate solution was compared. Figure 10 shows the results of the scatter diagram of the objective function value of the optimal solution and the approximate solution. The results indicate that the optimal solution and the approximate solution take very close values. When the maximum value of the error is within \( \pm 1.8\% \), it is considered that a sufficiently good approximation has been performed. By the way, this technique can also be used as a criterion when opening a new XD. The difference between the cost when assuming that all deliveries from the DC to the customer are direct delivery (i.e., \( \forall x_k = 0 \)) and the cost of the approximate solution obtained from the discriminant proposed in this study is calculated, and the cost difference is calculated. If the sum of these costs is larger than the XD opening cost, it is considered better to open the XD.

**Table 1. Best results**

<table>
<thead>
<tr>
<th></th>
<th>Approximate Solution</th>
<th>Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>1,068,074,805</td>
<td>1,066,033,722</td>
</tr>
<tr>
<td># of Errors</td>
<td>396/10000</td>
<td>-</td>
</tr>
<tr>
<td>Error Rate ( \frac{z - z^<em>}{z^</em>} )</td>
<td>0.74%</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 9. DR-DQ analysis
5. Conclusion

The importance of reducing logistics costs has been increasing ever. An important consideration to reduce logistics costs is to make effective use of Less-than-Truckload (LTL). One of the features of LTL is volume discount cost structure. To exploit this structure, an intermediate cross-dock (XD) is often used, where a large-sized truck is used to deliver from a central distribution center to the cross-dock, and a small-sized car is used to deliver from the XD to customers. However, in such a system, it is difficult to determine which customers will be directly delivered and which customers will be indirectly delivered.

The purpose of this study is to make a judgment on the direct / indirect delivery system. We present an optimization model incorporating a piecewise linear function. The optimal solution was plotted on the distance-ratio (DR), which is the ratio of the distance between direct and indirect delivery, and demand quantity (DQ) axis. As a discovery, we found that direct / indirect delivery can be judged by the DR-DQ indices.

Based on this observation we proposed a support-vector-machine-based heuristic to discriminant of direct / indirect delivery by DR and DQ. It was confirmed that the error between the approximate solution obtained from the heuristic and the optimal solution was sufficiently small. Through the above activities, we were able to get the better understanding the principle of determining the delivery system.

For future research, we can incorporate the routing decision into the model. In the case of FTL, a single truck can deliver to multiple customers, so VRP (Vehicle Routing Problem) should be included in the model. It is thought that more accurate cost calculation can be performed by using the incorporated model. As for the network shape, hub-and-spoke network analysis is also very interesting. In many companies, large transportation companies develop a hub-and-spoke distribution network, and the need for this form is expected to increase further in the future. By applying the same analysis to shipping, it is thought that it can be applied to decision-making of bonded warehouses at each port.

References