A Judgement Model for Direct-Indirect Delivery

Shunichi Ohmori[#1], Kazuho Yoshimoto[*2]

[#]*Department of Business Design and Management, Waseda University*
Address Including Country Name
[1]ohmori0406@aoni.waseda.jp
[2]kazuho@waseda.jp

***Abstract*— We study the selection of direct delivery / indirect delivery for LTL shipment in a two-echelon supply chain composed of a central distribution center (DC), an intermediate cross-docking center (XD) and customers, where the direct delivery is a shipment directly from the DC to customers, and the indirect delivery is a shipment to customers via the XD. We develop an optimization model that selects direct delivery / indirect delivery, taking into account the feature of volume discount. The volume discount is modelled as a nonlinear concave function and reformulated as MILP by a piecewise linearization technique. We propose a new analysis framework to account for the choice of direct / indirect delivery based on two indices, Distance-Ratio (DR), which is the ratio of the distance between direct and indirect delivery, and Demand Quantity (DQ). Based on this analysis, a heuristic solution using a support-vector-machine-based discriminant analysis model using the indices DR and DQ was proposed. In numerical experiments, the proposed heuristics were compared to the optimal solution for 100 randomly generated problem cases, and the error was shown to be within 0.8%.**

***Keywords*—** *Include at least 5 keywords or phrases*

1. Introduction

Many companies give high attention to minimize their transportation cost of goods because it is considered as a major share of the total logistics cost. In many companies, the transportation of their goods is relied on third-party distributers. Due to the deregulation, the transportation costs charged by third party distributers has been decreased.

One important mode of transportation used is the LTL (Less-than-Truck-Load) mode, which is attractive when shipment sizes are less than truck capacity [1] [2] [3] [4]. Typically, LTL carriers offer volume, or quantity, discounts to their clients to encourage demand for larger, more profitable shipments (Figure 1). These economies of scales motivate the shipper to set an intermediate cross-docking center (XD). In that system, freight is delivered from the depot to the XD, from where it is delivered to customers. This approach is strongly connected to the design of City Logistics systems for large cities, where it provides the means to efficiently keep large trucks out of the city centre, with small and environment-friendly vehicles providing the last leg of distribution activities [5] [6].

In this study, delivery directly performed from the depot to customers is called *direct delivery*, and delivery via the XD is called *indirect delivery* as illustrated in figure 2. As mentioned earlier, indirect delivery can enjoy the economy of scale by consolidating freight, while it makes a detour compared to direct delivery. Therefore, it is very important to analyse the advantages and disadvantages of each option, and to properly judge the choice between direct and indirect delivery. To our knowledge, however, there is no study that analyse advantages and disadvantages of these options. Especially, the relationship between the location of customers, the XD, and the depot and the associated costs has yet been studied.

The intend of the research is centred around the question of which direct/indirect delivery is selected under which condition. We build an optimization model to select the direct/indirect shipping in LTL shipper problem, that results in nonlinear concave minimization problem, which is hard to solve. We applied a piecewise linearization technique to approximate the problem as a mixed-integer linear program, which can be solved very efficiently by the off-the-shelf solver.

We proposed the framework of judging the direct/indirect delivery with distance-ratio (DR) and demand quantity (DQ), where DR is the distance

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

ratio of distance of direct and indirect distance. We analyse that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa. The hint of this idea comes the distance-intensity (DI) analysis, in the facility planning area, in which the efficiency of transportation is measured by the transport work distance-intensity (DI) analysis.

Further, we proposed a heuristics method for obtaining near optimal judgement of direct/indirect delivery, using the supervised machine learning classification technique. We applied support vector machine (SVM) to classify the direct/indirect delivery judgment using the data pair of obtained optimal judgement and values of DR and DQ as supervised data. By doing so, we can judge the near optimal judgement of direct/indirect delivery for very large-scale problem very quickly. Our heuristics does not require the mathematical solver, the only background required is knowledge of simple linear classification and the usage of spreadsheet. This convenience is especially beneficial in practice, especially when the mathematical programming cannot be used on a daily basis. For daily operations, the demand requests from customers vary from day to day, while the location of the depot and the XD is fixed.

In the remainder of the paper is as follows. In the section 2, we review the related research. In the section 3, we present the mathematical formulation of the problem. In the section 4, we demonstrate the numerical experiments. In the section 5, we present the result of discriminant model. In the section 6, we make concluding remarks.

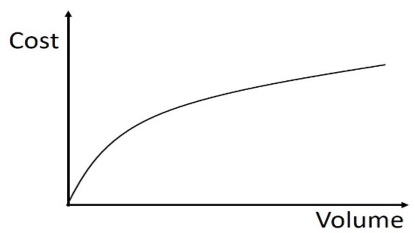


Figure 1. Volume discount for the LTL shipment

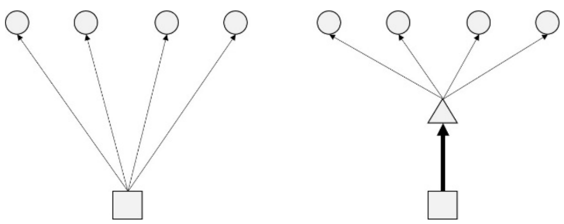


Figure 2. Direct and indirect delivery (Left: Direct, Right: Indirect)

2. Literature Review

We proposed the framework of judging the direct/indirect delivery with distance-ratio (DR) and demand quantity (DQ), where DR is the distance ratio of distance of direct and indirect distance. We analyse that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa. The hint of this idea comes the distance-intensity (DI) analysis, in the facility planning area, in which the efficiency of transportation is measured by the transport work distance-intensity (DI) analysis.

In this section, we review the related literature. There are many studies for the production-distribution problem considering direct and indirect deliveries. For example, survey paper on hierarchical logistics network problem by [7] presented over 50 papers considering direct deliveries. Ref [8] reveals the impact of distribution system characteristics on computational tractability. Ref [9] presented an ant colony optimization algorithm to solve for the transportation problem of cross-docking network. Ref [10] presented an integrated load-planning problem with intermediate consolidated truckload assignments. Ref [11] presented a graph theoretic-based heuristic algorithm for responsive supply chain network design with direct and indirect shipment. Ref [12] considers crossdocking distribution networks with setup cost and time window constraint. Ref [13] studies the transhipment service through cross docks with both soft and hard time windows. Ref [14] presented the bi-objective heuristics for multi-item freights distribution planning problem in crossdocking networks. Ref [15] proposed a hybrid genetic algorithm for the multiple cross docks problem. Ref [16] studied a stochastic programming model for reverse logistics network design under uncertainty. Ref [17] presented an in-transit freight consolidation of indivisible shipments.

Another related research is multi-commodity flow problem. See [18] for recent review. There are several studies that study the exact concave minimization problem [19-22]. An exhaustive search of all extreme points would provide an optimal flow, since a concave function achieves its minimum at an extreme point of the convex feasible region. However, such an approach is impractical for all but the simplest of problems. The fixed-charge network design problem is also extensively studied that arise in various applications in telecommunications, logistics, and transportation [23-27]. Piecewise Linearization is a popular technique to approximately minimize concave cost function [28-32].

The contribution of this paper is as follows. First, we model the choice of direct/indirect delivery mode and present the mixed-integer linear programming formulation using the piecewise-linearization. Second, we place a great deal of emphasizes on the analysis of the optimal solution. Especially, we proposed the new framework to analyse and account for the reason of the choice of direct/indirect delivery with distance-ratio and demand quantity. Third, we proposed a new heuristics method utilizing DR-DQ framework.

3. Methodology

We consider the situation where transportation requests are fully outsourced to the third-party distributers, so the forwarding companies are no responsible for the vehicle scheduling. The shipment cost charged to the third-party distributers is calculated according to the predetermined tariff table. The tariff table provides a list of agreed fixed tariff rates under non-linear consideration of loads and lengths of the tours that the subcontracting company charges for its delivery services. An example illustrating the typical tariff table is shown in table 1. In the tariff table, the cost $f(d, q)$ is specified according to the distance d and delivery quantity q. This can be modelled by concave function conceptually as illustrated in figure 2.

The modelling framework is illustrated as in figure 3. Let $k = 1, \cdots, K$ denote a set of customers. Let d_k denote distance from XD to Customer k, d_0 denote distance from DC to XD, D_k denote distance from DC to Customer k, q_k denote demand of customer k. We let x_k denote the binary variable to indicate if the customer k is fulfilled with direct delivery or indirect delivery as follows.

$$x_k = \begin{cases} 1, & \text{if customer } k \text{ is fulfilled} \\ & \text{with indirect delivery} \\ 0, & \text{otherwise} \end{cases}$$

We also use the notation $q_0 = \sum_{k=1}^{K} x_k q_k$ to denote the shipment volume from the depot to XD.

Using these notations, the direct/indirect delivery selection problem can be modelled

$$\text{Min.} \quad C_D + C_I \tag{1}$$

$$\text{S.t.} \quad C_D = \sum_{k=1}^{K} (1 - x_k) f(D_k, q_k) \tag{2}$$

$$C_I = f(d_0, q_0) + \sum_{k=1}^{K} x_k f(d_k, q_k) \tag{3}$$

$$q_0 = \sum_{k=1}^{K} x_k \, q_k \tag{4}$$

$$x_k \in \{0,1\} \tag{5}$$

The objective function is to minimize the sum of the direct cost C_D and indirect delivery cost C_I. The constraints (2) presents the calculation of the direct cost, in which the cost $f(D_k, q_k)$ is added only when $x_k = 0$. The constraint (3) presents the calculation of the indirect cost, which composed of the consolidated shipment cost from the depot to the XD and the delivery cost from the XD to each customer. The constraint (4) presents the calculation of q_0, and the constraint (5) presents binary constraints for x_k.

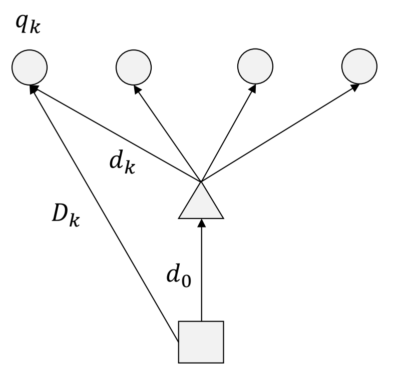


Figure 3. Direct and indirect delivery (Left: Direct, Right: Indirect)

The objective function is to minimize the sum of the direct cost C_D and indirect delivery cost C_I. The constraints (2) presents the calculation of the direct cost, in which the cost $f(D_k, q_k)$ is added only when $x_k = 0$. The constraint (3) presents the calculation of the indirect cost, which composed of the consolidated shipment cost from the depot to the XD and the delivery cost from the XD to each customer. The constraint (4) presents the calculation of q_0, and the constraint (5) presents binary constraints for x_k.

Since D_k, d_k, q_k are fixed constants, $f(D_k, q_k)$ and $f(d_k, q_k)$ are independent on the choice of x_k. We let $C_{Dk} = f(D_k, q_k)$ and $C_{dk} = f(d_k, q_k)$ as constants. Further, given d_0 is constant, it is sufficient to see how much quantity q_0 becomes for the choice of x_k.

We applied the piecewise linearization technique to model the problem via the mixed-integer linear programming. We let q_0^n denote a binary indicator to take q_0 if q_0 belongs to quantity segment n, and take zero otherwise as:

$$q_0^n = \begin{cases} q_0, & N_{n-1} \le q_0 \le N_n \\ 0, & \text{otherwise} \end{cases}$$

We let z_n denote a binary indicator to take 1 if q_0 belongs to quantity segment n, and take zero otherwise as:

$$z_n = \begin{cases} 1, & N_{n-1} \le q_0 \le N_n \\ 0, & \text{otherwise} \end{cases}$$

Using these auxiliary variables, we have the following reformulation.

$$\text{Min.} \quad C_D + C_I \qquad (6)$$

$$\text{S.t.} \quad C_D = \sum_{k=1}^{K} (1 - x_k) C_{Dk} \qquad (7)$$

$$C_I = \sum_{n=1}^{s} z_n c_n + \sum_{k=1}^{K} x_k C_{Ik} \qquad (8)$$

$$q_0 = \sum_{k=1}^{K} x_k\, q_k \qquad (9)$$

$$q_0 = \sum_{n=1}^{N} q_0^n \qquad (10)$$

$$\mathrm{N}_{\mathrm{n-1}} z_n \le q_0^n \le N_n z_n \qquad (11)$$

$$\sum_{n=1}^{N} z_n = 1 \qquad (12)$$

$$z_n \in \{0,1\} \qquad (13)$$

$$x_k \in \{0,1\} \qquad (14)$$

Equation (6) represents the objective function. Equations (7)(8) represent the direct and indirect delivery costs. Equation (9) represents the delivery quantity from the factory to the XD is equals to the sum of demand quantity for indirect delivery. Equation (10) indicates that the delivery quantity from the factory to the XD equals to the sum of the flow section. Equation (11) indicates that the flow section is equal to or less than the upper and lower limits of the category. Equation (12) indicates that one section is selected. The constraint (13)(14) presents binary constraints for x_k and z_n, respectively.

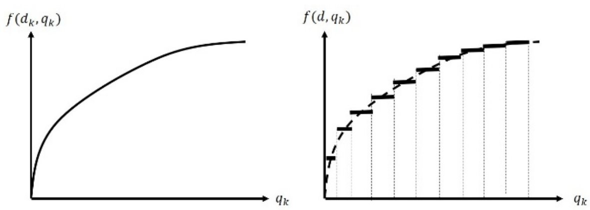


Figure 4. Piecewise linearization technique

4. Discussions

This section presents numerical examples. In section 3.2.1, we describe the data set. In section 3.2.2, the results are analysed with respect to the distance-ratio and demand quantity.

4.1 Data Set

We build a virtual case based on the link to a real-world company. In this case, we consider a company delivers to the entire west of Japan with a DC in Kansai and a XD in Kyushu. Figure 5 shows a conceptual diagram of this case. We would like to decide whether to deliver directly from the DC or to perform indirect delivery via XD in Kyushu. The input information and experimental environment were set as follows.

- Number of customers: 10000
- Demand q_k: drawn from uniform distribution $U(0,4)$
- Distance from the DC to the XD D_0: 600km
- Distance from the DC to customers d_k: drawn from uniform distribution $U(0,1000)$
- Distance from the XD to customers d_0: drawn from uniform distribution $U(0, D_0 + d_k)$
- Experimental conditions: Intel Core i7 3.5GHz×2, 16GB memory
- Code: MATLAB
- Solver: Gurobi Optimizer

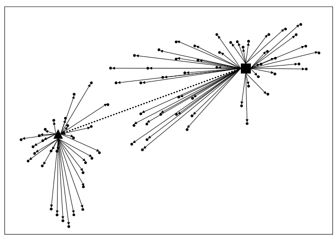


Figure 5. Illustration of case study (square box is the DC, triangle is the XD, and the dots are customers)

4.2 Distance-ratio and Demand quantity analysis

To analyze the result, we proposed a framework with *distance-ratio* (DR) and demand quantity (DQ), named DR-DQ analysis. We propose the index distance-ratio (DR), the distance ratio of distance of direct and indirect distance, denoted by the following equation (15).

$$R_k = \frac{D_k}{d_0 + d_k} \qquad (15)$$

We plot the customer in scatter diagrams with horizontal axis being distance ratio (DR) R_k and vertical axis being demand quantity (DQ) q_k as in figure 6, 7, and 8. Customers with $x_k = 1$ are plotted in figure 6, customers with $x_k = 0$ are plotted in figure 7, and all customers are plotted in figure 8.

We see that customers with $x_k = 1$ are scattered on the left-upper corner and customers with $x_k = 0$ are scattered on the right-bottom corner. The results indicate that when the DR is high and DQ is low, the direct delivery is beneficial, and vice versa.

The results indicate that the relationship among the location of DC, the XD, and the customer may affect the choice of the direct/indirect delivery. More specifically, the choice depend on how much a detour is made. For example, if the XD is located in the middle of the DC and a customer, then it is reasonable to use the XD. In this case $R_k = 1$. On the other hand, if the XD is located in the opposite direction from the DC to customers, it is not reasonable to use the XD. In this case R_k takes much smaller value than one. Therefore, the rate of detour can be evaluated by the distance ratio presented in the equation (15).

The results also indicate the effect of the shipment volume. The higher the shipment volume, the more likely the direct delivery is selected. This comes from the utilization of volume discount presented in as in the figure 1. This analysis help get better understanding of what depends on the direct/indirect delivery choice.

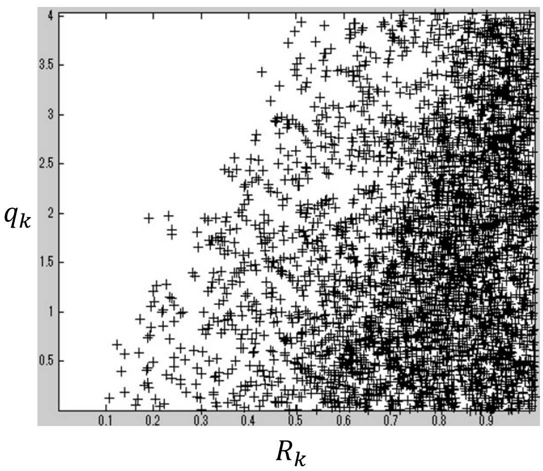


Figure 6. DR-DQ analysis with $x_k = 1$

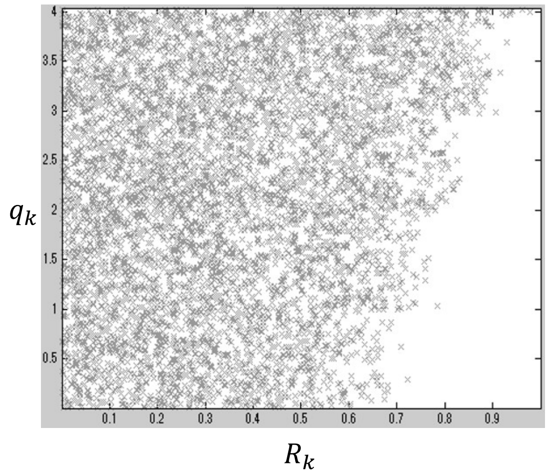


Figure 7. DR-DQ analysis with $x_k = 0$

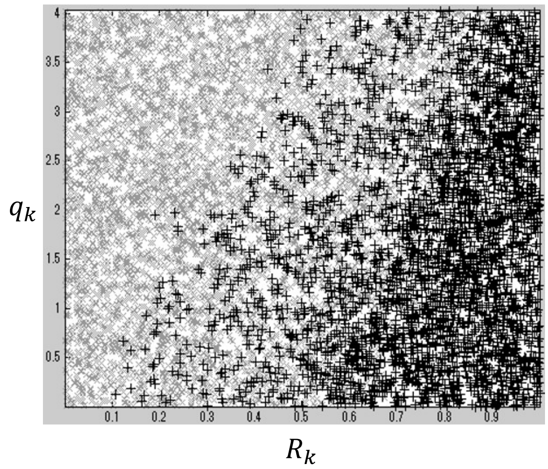


Figure 8. DR-DQ analysis

4.3 Heuristics with Discrimination Model

As a result of the previous section, using the index distance ratio, we can get a hint to judge direct/indirect delivery. In this section, we use this indicator to determine the direct / indirect delivery heuristically. We considered a linear discrimination model that can perform motion. From the graph shape presented in the figure 8, we consider it is possible to judge the direct/indirect delivery from the value of R_k and q_k by the linear function. We use the SVM (Support Vector Machine) as discriminator.

SVM is one of the most widely used pattern recognition learning algorithms and is a two-class problem linear discriminant function construction method that achieves the maximum margin. In this research, we use a linear function described as the equation (16), and the direct delivery is selected for the customer k if the value of $g\ (R_k, q_k)$ is positive, and the indirect delivery is selected if $g\ (R_k, q_k)$ is negative.

$$g(R_k, q_k) = \theta_1 R_k + \theta_2 + \theta_0 \quad (16)$$

Let K_D denote the number of customers that are selected direct delivery, K_I denote the number of customers for indirect delivery, and the subscripts $i = 1, \cdots, K_D, j = 1, \cdots, K_I$ denote the subscripts for customers of direct delivery and indirect delivery, respectively. The problem of maximizing the margin for obtaining the above-mentioned discriminant function parameters $\theta = [\theta_0, \theta_1, \theta_2]$ is as follows.

$$\text{Max.} \quad \sqrt{\theta_1^2 + \theta_2^2} + \sum_{i=1}^{K_D} u_i + \sum_{j=1}^{K_I} v_j \quad (17)$$

$$\text{s.t.} \quad \theta_1 D_i + \theta_2 q_i + \theta_0 \geq 1 - u_i, \quad (i = 1, \cdots, K_D) \quad (18)$$

$$\theta_1 D_j + \theta_2 q_j + \theta_0 \leq -1 + v_j \quad (j = 1, \cdots, K_I) \quad (19)$$

$$u_i, v_j \geq 0 \quad (20)$$

The results of parameter fitting for the case problem presented in the previous section are as follows.

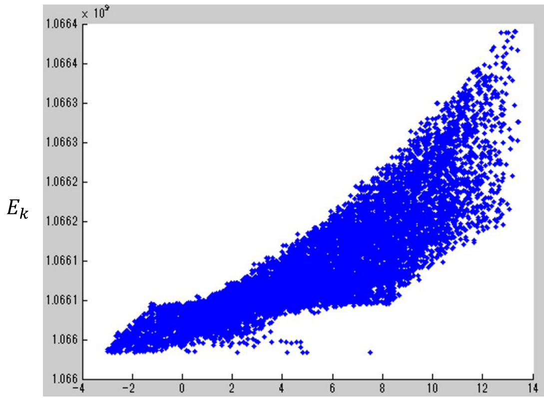
$$g(R_k, q_k) = -7.7219 R_k + 0.8195 q_k - 3.3307$$

Table 2 shows the number of errors, the value of the approximate solution when the obtained discriminant is used, and the error rate between the approximate solution and the optimal solution. Table 2 shows that the discriminant model using DR-DQ perform well in terms of the number of errors and the objective function value. It is interesting to see that the error rate of the objective function (0.74%) is extremely low despite some errors (396/10000=3.96%). The possible reason for this phenomenon is that for customers close to the boundary of the discriminant equation, there is no large cost difference regardless of whether direct delivery or indirect delivery is selected, so the effect of errors may be small. To see this hypothesis, for each customer k, the incremental cost when x_k obtained by the optimal solution is inverted is defined as "error cost E_k", and the relationship with the distance from the boundary line, measured by $\|g(R_k, q_k)\|$, is analyzed as shown in figure 9. In a region where the distance from the boundary is in the range of ± 3, E_k has a small value. From this viewpoint, the robustness of the discriminant model using DR-DQ analysis can be asserted.

Finally, as additional verification, 100 different problem examples were generated and the error between the optimal solution and the approximate solution was compared. Figure 10 shows the results of the scatter diagram of the objective function value of the optimal solution and the approximate solution. The results indicate that the optimal solution and the approximate solution take very close values. When the maximum value of the error is within ± 1.8%, and it is considered that a sufficiently good approximation has been performed. By the way, this technique can also be used as a criterion when opening a new XD. The difference between the cost when assuming that all deliveries from the DC to the customer are direct delivery (i.e., $\forall x_k = 0$) and the cost of the approximate solution obtained from the discriminant proposed in this study is calculated, and the cost difference is calculated. If the sum of these costs is larger than the XD opening cost, it is considered better to open the XD.

Table 1. Best results

	Approximate Solution	Optimal Solution
Objective Function	1,068,074,805	1,066,033,722
# of Errors	396/10000	-
Error Rate $(\frac{\hat{z}-z^*}{z^*})$	0.74%	-



$g(D_k, q_k) = -7.7219Rk + 0.8195qk - 3.3307$

Figure 9. DR-DQ analysis

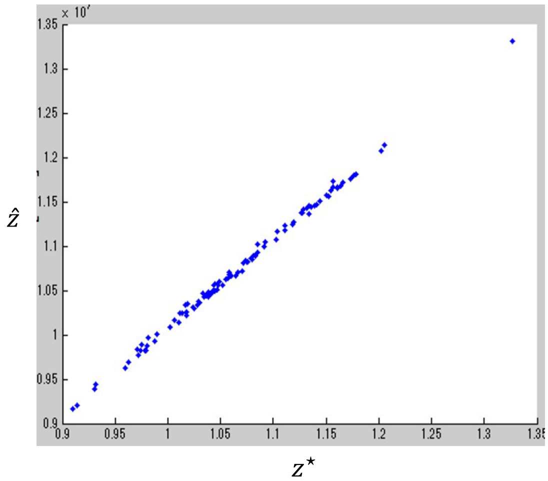


Figure 10. DR-DQ analysis

5. Conclusion

The importance of reducing logistics costs has been increasing ever. An important consideration to reduce logistics costs is to make effective use of Less-than-Truckload (LTL). One of the features of LTL is volume discount cost structure. To exploit this structure, an intermediate cross-dock (XD) is often used, where a large-sized truck is used to deliver from a central distribution center to the cross-dock, and a small-sized car is used to deliver from the XD to customers. However, in such a system, it is difficult to determine which customers will be directly delivered and which customers will be indirectly delivered.

The purpose of this study is to make a judgment on the direct / indirect delivery system. We present an optimization model incorporating a piecewise linear function. The optimal solution was plotted on the distance-ratio (DR), which is the ratio of the distance between direct and indirect delivery, and demand quantity (DQ) axis. As a discovery, we found that direct / indirect delivery can be judged by the DR-DQ indices.

Based on this observation we proposed a support-vector-machine-based heuristic to discriminant of direct / indirect delivery by DR and DQ. It was confirmed that the error between the approximate solution obtained from the heuristic and the optimal solution was sufficiently small. Through the above activities, we were able to get the better understanding the principle of determining the delivery system.

For future research, we can incorporate the routing decision into the model. In the case of FTL, a single truck can deliver to multiple customers, so VRP (Vehicle Routing Problem) should be included in the model. It is thought that more accurate cost calculation can be performed by using the incorporated model. As for the network shape, hub-and-spoke network analysis is also very interesting. In many companies, large transportation companies develop a hub-and-spoke distribution network, and the need for this form is expected to increase further in the future. By applying the same analysis to shipping, it is thought that it can be applied to decision-making of bonded warehouses at each port.

References

[1] Powell, W. B., & Sheffi, Y. (1989). Or practice—design and implementation of an interactive optimization system for network design in the motor carrier industry. Operations Research, 37(1), 12-29.

[2] Cheung, R. K., & Muralidharan, B. (1999). Impact of dynamic decision making on hub-and-spoke freight transportation networks. Annals of Operations Research, 87, 49-71.

[3] Cheung, R. K., & Muralidharan, B. (2000). Dynamic routing for priority shipments in LTL service networks. Transportation science, 34(1), 86-98.

[4] Muriel, A., & Simchi-Levi, D. (2003). Supply chain design and planning–applications of optimization techniques for strategic and tactical models. Handbooks in operations research and management science, 11, 15-93.

[5] Benjelloun, A., & Crainic, T. G. (2008). Trends, challenges, and perspectives in city logistics. Transportation and land use interaction, proceedings TRANSLU, 8, 269-284. Research, 34(8), 2310-2331.

[6] Crainic, T. G., Gendreau, M., & Potvin, J. Y. (2009). Intelligent freight transportation systems: Assessment and the contribution of operations research. Transportation Research Part C: Emerging Technologies, 17(6), 541-557.

[7] Sahin, G., & Sural, H. (2007). A review of hierarchical facility location models. Computers & Operations Research, 34(8), 2310-2331.

[8] Ali, A. I., & O 'Connor, D. J. (2010). The impact of distribution system characteristics on computational tractability. European journal of operational research, 200(2), 323-333.

[9] Musa, R., Arnaout, J. P., & Jung, H. (2010). Ant colony optimization algorithm to solve for the transportation problem of cross-docking

network. Computers & Industrial Engineering, 59(1), 85-92.

[10] Uster, H., & Agrahari, H. (2010). An integrated load-planning problem with intermediate consolidated truckload assignments. IIE Transactions, 42(7), 490-513.

[11] Pishvaee, M. S., & Rabbani, M. (2011). A graph theoretic-based heuristic algorithm for responsive supply chain network design with direct and indirect shipment. Advances in Engineering Software, 42(3), 57-63.

[12] Ma, H., Miao, Z., Lim, A., & Rodrigues, B. (2011). Crossdocking distribution networks with setup cost and time window constraint. Omega, 39(1), 64-72.

[13] Miao, Z., Yang, F., Fu, K., & Xu, D. (2012). Transshipment service through crossdocks with both soft and hard time windows. Annals of Operations Research, 192(1), 21-47.

[14] Marjani, M. R., Husseini, S. M. M., & Karimi, B. (2012). Bi-objective heuristics for multi-item freights distribution planning problem in crossdocking networks. The International Journal of Advanced Manufacturing Technology, 58(9-12), 1201-1216.

[15] Miao, Z., Fu, K., & Yang, F. (2012). A hybrid genetic algorithm for the multiple crossdocks problem. Mathematical Problems in Engineering, 2012.

[16] Ayvaz, B., & Bolat, B. (2014). Proposal of a stochastic programming model for reverse logistics network design under uncertainties. International Journal of Supply Chain Management, 3(3), 33-42.

[17] Hanbazazah, A. S., Castro, L. E., Erkoc, M., & Shaikh, N. I. (2020). Intransit freight consolidation of indivisible shipments. Journal of the Operational
Research Society, 71(1), 37-54.

[18] Mahey, P., & Souza, M. C. D. (2017). Multicommodity netword flows with nonconvex arc costs. Pesquisa Operacional, 37(3), 571-595.

[19] Tuy, H. (1964). Concave programming under linear constraints. In Soviet Mathematics Doklady (Vol. 5, pp. 1437-1440).

[20] Zangwill, W. I. (1968). Minimum concave cost flows in certain networks. Management Science, 14(7), 429-450.

[21] Erickson, R. E., Monma, C. L., & Veinott Jr, A. F. (1987). Send-and-split method for minimum-concave-cost network flows. Mathematics of Operations Research, 12(4), 634-664.

[22] Ward, J. A. (1999). Minimum-aggregate-concave-cost multicommodity flows in strong-series-parallel networks. Mathematics of Operations Research, 24(1), 106-129.

[23] Magnanti, T. L., & Wong, R. T. (1984). Network design and transportation planning: Models and algorithms. Transportation science, 18(1), 1-55.

[24] Balakrishnan, A., Magnanti, T. L., & Mirchandani, P. (1997). Network design. Annotated bibliographies in combinatorial optimization, 311-334.

[25] Teo, C. P., & Shu, J. (2004). Warehouse-retailer network design problem. Operations Research, 52(3), 396-408.

[26] Shu, J. (2010). An efficient greedy heuristic for warehouse-retailer network design optimization. Transportation Science, 44(2), 183-192.

[27] Diabat, A., Batta¨ıa, O., & Nazzal, D. (2015). An improved Lagrangian relaxation-based heuristic for a joint location-inventory problem. Computers & Operations Research, 61, 170-178.

[28] Balakrishnan, A., & Graves, S. C. (1989). A composite algorithm for a concave ‐ cost network flow problem. Networks, 19(2), 175-202.

[29] Amiri, A., & Pirkul, H. (1997). New formulation and relaxation to solve a concave-cost network flow problem. Journal of the Operational Research Society, 48(3), 278-287.

[30] Chan, L. M. A., Muriel, A., Shen, Z. J., & Simchi-Levi, D. (2002). An approximation algorithm for the economic lot sizing model with piece-wise linear cost structures. Operations Research, 50, 1058-1067.

[31] Kim, D., & Pardalos, P. M. (2000). Dynamic slope scaling and trust interval techniques for solving concave piecewise linear network flow problems. Networks: An International Journal, 35(3), 216-222.

[32] Muriel, A., & Munshi, F. N. (2004). Capacitated multicommodity network flow problems with piecewise linear concave costs. IIE Transactions, 36(7), 683-696.