# A Supply Chain Optimization Model for an Enterprise Based on Maximizing the Economic Effect

Mark A. Gorskiy[1], Mikhail A. Khalikov[1], Denis A. Maximov [1]

[1]*Plekhanov Russian University of Economics, Department of Mathematical Methods in Economics, Moscow, Russia*
gorskiy-m@inbox.ru

*Abstract*— **This paper is motivated by the importance of addressing the challenges faced by supply chain systems in the presence of multiple uncertain factors. It focusses on three crucial areas of innovative supply chain optimization models, including supply chain operations processes, competition and interactions in supply chains, and supply chain contracting and coordination. The authors propose to separate into stages the process of determining the elemental composition of a production system (groups of technological equipment) and the sequence of these elements providing the technological process at its individual stages. At the first stage, it is proposed to use the structural-parametric optimization of the production system of the enterprise, and at the second to use the combinatorial optimization. The main result of the paper is the formulation and proof of a theorem on the relation of structurally-parametric and combinatorial supply chain optimization model.**

***Keywords*—** *enterprise, supply chain management, optimization model, economic effect, industrial system.*

## 1. Introduction

The problems of choosing and substantiating an enterprise's investment strategy, managing investment projects at the stages of the restructuring and modernizing of production and technological systems, and analyzing investment efficiency are widely covered in the works of [1-5]. The accumulated theoretical material and practical experience around investment design are summarized in official guidelines (Methodological recommendations for evaluating the effectiveness of investment projects. However, some important issues around the managing of investment projects in terms of the restructuring of production systems at existing enterprises are still practically unresolved in the literature. For example, central to the most widely used approach to designing the production system of a reconfigurable enterprise is the idea of developing design solutions to correspond precisely to stated technical specifications. However, already at the stage of project implementation, situations may arise requiring correction, which will lead to unjustified expenditure of time and effort on the part of the designer, and of money and resources on the part of the customer. The aim of the present study is to introduce a new theoretical approach for the restructuring of the design of industrial enterprise systems, the promotion of changes in the production program, and/or the introduction of new high-tech equipment. If the classical approach involves the development of projects from A to Z without guaranteeing subsequent implementation, the new approach allows us to divide the design process into two large blocks, the implementation of which can be undertaken both sequentially and in parallel. This reduces the cost of designing and adapting the system. The article discusses the mathematical side of this approach and proves the main theorem connecting the above steps. The authors suggest further developing detailed mathematical models and numerical methods for the implementation of the two design stages and adapting them to practical activities in the selected industrial enterprise.

The authors propose to separate the stages of preliminary design of the enterprise's production system for the selected production program and the "final" project, taking into account possible changes and correction of design decisions.

This approach is new and requires appropriate justification and preliminary testing. In this article, the authors suggest that the scientific community consider its prospects. The main emphasis is on the development and adaptation of mathematical tools that allows, at the first stage, to separate the tasks of a technical project, and at the second stage, choose an agreed solution on the structure and elemental composition of the enterprise's production system designed for a new production program.

## 2. Research materials and methods

Industrial system of a factory and its components

The industrial system (IS) of a factory can be divided into two parts based on functional and task-oriented

***International Journal of Supply Chain Management***
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

designation: production and technological structures (PTS) and organizational and technical structures (OTS). In turn, PTS is divided into technological structures (TS) and production structures (PS), and OTS comprises of building structures (BS) and informational and managerial structures (IMS) [6, 7].

TS represents a combination of realized technological routes, including the content and consequences (links) of technological operations. On the other hand, PS denotes the quantitative content and range of main technological equipment (MTE), which depends on the production program, effective working time, and equipment productivity. BS represents the relative positions and interconnections of the main and transport equipment, and IMS comprises of the content and interconnections of control and information reprocessing devices [8].

If the aforementioned structures comprising an IS are presented in the form of oriented graphs, TS can be defined as a combination of technological graphs, PS as production graphs, and PTS as a combination of production and technological graphs.

At the top level of technological and production graphs are technological operations, equipment and work stations, respectively. When these are combined, the resulting production and technological graphs define an MTE intended for carrying out technological operations. On the other hand, the ribs in technological graphs define consequences of implementing technological operations, while the ribs of production graphs pertain to flows of processed items.

The integration level of PTS is increasing in the machinery because of the technological similarity growth of processed items [9]. Conversely, as PTS is utilized for processing items with different geometrical forms, it is characterized by low integration level. Thus, the integration of PTS subgraphs defines the usage intensity of similar equipment.

In turn, BS can be defined as building graphs, IMS as information and managerial graphs, and OTS as organizational and technical graphs.

At the top level of building graphs are the elements of main and supporting equipment, such as machinery, work stations, automatized warehouses, tool and monitoring departments, etc. Thus, in building graphs, ribs define the transport links with IS.

At the top level of information and managerial graphs are elements and devices of management and information reprocessing. Here, ribs define flows of managerial and monitoring information.

The outline of functional and targeted IS structures provides the possibility to present the targeted structures as a managerial combination of the main and supporting equipment, thus representing the production and technological, as well as organizational and technical potential of equipment.

The production and technological capacity defines the efficiency and flexibility of a PTS (i.e., the combined efficiency of the industrial system and the range of its technological opportunities) [10]. The production and technological capacity can be presented by the following pair:

$$\overline{P}(D)=\left\{P;\bigcup_{i=1}^{I}D_i\right\}, \qquad (1)$$

where: $P$ represents the volume of production (in value terms) processed by IS in units,

$I$ denotes the number of items within the technological groups of units, and

$D_i$ denotes a wide range of $i$ - units within the technological group.

If IS efficiency is designated in nits of output, the production and technological capacity is described by I-measured vector line:

$$\overline{P}(D)=\left\{p_1(D_1);p_2(D_2);\ldots;p_i(D_i);\ldots;\right. \qquad (2)$$

where $p_i(D_i)$ denotes the efficiency of IS for the production of $i$ units from the technological group.

The organizational and technological capacity defines the technical and economic level of OTS and, as the integral indicator, outlines the most important characteristics of the IS, including the quantitative measure of detailed specialization (the profile of IS universality), expressed as the number of technological groups of components. Another important characteristic of an OTS is IS flexibility, which is correlated with the time taken to transfer IS from the processing of one unit to another one.

The production and technological capacity is defined by a wide range of $\overline{D}(\underline{D_i})$ components. Thus, a comparison of the production and technological and organizational and technical potentials of two different substations (indices “1” and “2”) in the form of an inclusion operation is possible if the ratio below holds:

$$\overline{D}_1(D_i)\subseteq\overline{D}_1(D_i)\cap\overline{D}_2(D_i)\subseteq\overline{D}_2(D_i)(i=\overline{1,I}) \qquad (3)$$

Comparison is carried out if and only if the following ratio is true:

$$\overline{D}_1(D_i)\cap\overline{D}_2(D_i)\subseteq\overline{D}_1(D_i) \text{ and}$$
$$\overline{D}_1(D_i)\cap\overline{D}_2(D_i)\subseteqq\overline{D}_2(D_i). \qquad (3')$$

However, if $\overline{D}_1(D_i)\cap\overline{D}_2(D_i)=\varnothing$ , the comparison of production and technological, and organizational and technical capacities, makes no sense.

Nonetheless, such comparisons are possible in special technological areas if the following holds:

$$\overline{D}_1(D_i)\cap\overline{D}_2(D_i)\neq\varnothing;$$
$$\text{and } \overline{D}_1(D_i)\subset\overline{D}_2(D_i) \text{ or, conversely,} \qquad (4)$$
$$\overline{D}_2(D_i)\subset\overline{D}_1(D_i).$$

In other words, comparison of different IS based on capacities is possible, if these systems are identical, equivalent. The description of technological conditions given by (4) allows comparison among IS that are targeted for the production of similar units, but are characterized by different output programs.

In additional to the integral (systematic) indicators for the detailed description of PTS and MTE can be defined both general indicators and special ones.

The combination of integral (systematic), general, and special indicators of IS can be divided into three groups.

The first group includes indicators, characterized by the production and technological capacity, such as the range of components, universality of used equipment, annual productivity $N_i$ for separate units and the general one $\sum_{i=1}^{I} N_i$, as well as the type of production (small- and average-scale), characterized by the frequency of readjustments of MTE.

The second group includes indicators that are characterized by organizational and technical capacity: production flexibility, number of workers, the quantity and range of main technological equipment, shift coefficients, equipment overloading, and coefficients of technological and production integration. These indicators are calculated by formulas:

$$\alpha_T = \frac{T_G}{T_{GO}}; \qquad (5)$$

$$\varphi_p = \frac{T_G}{T_{G\Sigma}}, \qquad (6)$$

where $T_G$ is the capital output for operations of technological process of units for this technological group in the volume of annual output program for such IS; $T_{GO}$ is the capital output of the full cycle for the production of units for such technological group in the volume of annual output program and $T_{G\Sigma}$ is the capital output of the full cycle for the production of all units for all technological groups in the volume of the annual output program.

The third group includes the intensity of MTE: coefficients of organizational and technical usage, the duration of the production cycle in separate technological groups of units, the volume of incomplete production, and the average general duration of equipment downtime for the shift, as well as the indicators of production and technological reliability, which characterize risks of failure in the technological process.

Competition and interactions in supply chains

Interaction and competition behaviors among supply chain members are commonly seen in practice, and the issue of multiple uncertainties makes the problem even more complicated. Observe that competition may exist vertically between the upstream supplier and downstream resellers. Characteristics of indicators in the third group are the possibility of its recognition by project and/or normative data, although, they are settled more frequently in the process of usage.

The discrete optimization of supply chain production and technological structure of a company. During the reconstruction and reassignment of current production at the stage of production program change, the most important tasks are the definition of rational content for MTE, and the structure supporting the production process of subsystems. The synthesis process of production system at the company in this case includes two groups of interrelated tasks [11, 12].

The first one includes tasks of PTS synthesis aimed to select options of technological processes for the units handling within the planned production program and related to these processes the nomenclature and quantitative content of MTE.

The second group includes tasks of supporting subsystems synthesis (combined under the terminology OTS) [13]:

Transport and warehousing system (TWS);

System of tool provision and control (STP and SC) and information and diagnosis system (IDS);

Automatization system of operational management by the production (ASOMP);

System of technological sides of production and automatized projection (STSP and AP accordingly).

In regard to the combination of technological processes for units handling in the production system with special indicators of shift coefficients, organizational and technical usage explicitly defines the nomenclature and quantitative content of MTE and parameters of supporting subsystems. At the same time, qualitative and quantitative characters of supporting subsystems influence shift coefficients and organizational usage and consequently affects the characteristics of PTS, indicating the interaction of tasks for first and second groups. The impact assessment of supporting subsystems on the quantitative content of MTE can be carried out by considering the coefficient of organizational usage of equipment $\lambda o_m$ [14]:

$$\lambda r_m = \lambda t_m \cdot \lambda o_m,$$

where $\lambda r_m$, $\lambda t_m$, $\lambda o_m$ represent coefficients of the general, technical and organizational usage of form-technological group equipment, accordingly.

Coefficient $\lambda t_m$ indicates the quality and reliability of main technological equipment, and coefficient $\lambda o_m$ depends on the configuration of technical and economic parameters of supporting subsystems.

Two options for subsystems implementation with different stages of automation are recommended: “1” – mechanized option (with manual system of management), “2” – automated option (with automated system of management). It is essential to take into account how the configuration of supporting subsystems depends on the effective timing of main technological equipment $(f_m^n)$.

The use of analytical solution methods based on a discrete mathematical model—and projected for the new program of main production consolidated analysis tasks and synthesis of PTS and supporting subsystems at the pre-project stage—is suggested. (The original model is presented in papers [15], and an improved, modified option appears below).

At the methodological level, the concept of structural and parameter optimization of the supply chain systems is subordinated to the task of maximum integral economic effect:

$$E = \Delta P - \Delta Q \rightarrow \max, \qquad (8)$$

where $\Delta m$, $\Delta Q$ – changes in the cost estimation of results and costs for the reference period accordingly (to service the operation of the enterprise production system's main technical equipment ).

The targeted function for the optimization of SCM is dependent on unstable technical and economic indicators of a company during its lifecycle, as the detailed expression for a company's economic effect is used in the form of difference between discounting value of results and costs based on the residual value of fixed assets (at the end of reference period). The first case occurs if the discount rate is justified by the cost and risk of the project, as well as if the implementation of the project involves a long (more than 5 years) horizon. In other cases, i.e. at short- and medium - term intervals, the need for discounting cash flows of the project disappears. Thus, it is essential to have reliable information for the task of SCM optimization on the following points: changes in nomenclature and annual plans for the output of units, prices on produced goods, trends in value, further changes to material and labor costs for production, prices on a new equipment, etc.

In conditions of rapidly evolving industrial trends, the issue that arises is whether it is appropriate to consider all factors that influence the changes of the technical and economic indicators of the production system at the pre-project stage, which is during the period of adaptation and function ability of the new output program. To answer, it is impossible to forecast the change of the technical and economic indicators of a PS with a tolerance no higher than 7% in most practical tasks [16]. Similar forecasts should pursue other objectives that include the definition of economic effect for the life cycle period of a modernized production system, the choice of a production strategy within a company, and the determination of a trend in financial and economic indicators [17-24]. For the solution of listed tasks, it is possible to accurately estimate the technical and economic indicators of a modernized production system in accordance with a new output program [25].

However, if there is information on the changes of the technical and economic indicators that show the life cycle stages as the targeted function of a SCM structure optimization, it is preferable to use the economic effect expression by calculating the difference between the discounted value of results and costs [26-30].

The described approach to create targeted functions is not contrary to the mobility feature of production system but rather adds to its importance. Consequently, there may be decomposition of the estimated period for such time lags, during which the technical and economic indicators can be defined as stable. Because the first interval is characterized by the most accurate source data, this should define the following structural frame of a PS. For the structuring task of a company's SCM's next steps, the optimization procedure can be repeated on the basement of the same targeted function. Thus, the targeted function (8) has the most importance.

For the creation of optimality criteria in projecting and restructuring tasks of a PS, there is an essential function that connects the annual effect of equipment usage and its consumption costs with the cost-effective and clear forms usage [31, 32]. This function can be used as the following expression:

$$F = \frac{E}{Q_H}.$$

In such cases, the annual effect $E$ is equal to the annual productivity (PR) of the main technical equipment (MTE). Then the expression (9) has the following mode:

$$F = \frac{PR}{\mathrm{Q}_H}$$

where $PR$ is the productivity of the MTE (in units or rubles). At the stage of evaluating the commercial effectiveness by reconfiguring an enterprise's production system, the calculated cost indicators are used. Then at the stage of evaluating the production capacity of new equipment being built into the enterprise's production system, preference should be given to the equipment's physical performance.

$Q_H$ - the cost of servicing the production system for the period of its operation. The meaning of formulas (9) and (10) is that PS efficiency can be estimated either as a result per unit of cost or productivity per unit of cost. This is not a contradiction. If the PS is built into the general production and technological system of the enterprise, then the first indicator should be used, otherwise, if it is used as a unique link in the general production and technological system of the enterprise, then the second indicator should be used.

The physical sense of expression (9) is the following one: $F$ is a value, which is opposite to specific annual costs for PS usage.

The productivity of a system, working in assembling production, can be defined as it has been suggested in paper [33-37], on the basement of expression:

$$PR = \sum_{i=1}^{I} \sum_{m=1}^{M} \frac{f_m^n \cdot \lambda t_m \cdot b_m}{\tau_{i,m} \cdot t_{i,m}}, \tag{11}$$

where: $\tau_{i,m} = 1/\overline{p}_{i,m}$ – intensity of flows; $t_{i,m}$ – technological labor intensity of processing for i-detail for *m*- group of MTE.

The following definitions can be used:

$\tau_m^{-p}$ – the average period of production process for one unit of MTE for $m$ - group:

$$\tau_m^{-p} = \sum_{i=1}^{I} \tau_{i,m} \cdot t_{i,m}; \tag{12}$$

$\theta_m^y$ – The special time of one unit output of production program based on the equipment of $m$ - group:

$$\theta_m^y = \frac{\tau_m^{-p}}{f_m^n \cdot \lambda t_m \cdot b_m}; \tag{13}$$

$Q_m^G$ – all costs for the equipment of $m$ - technological group.

Based on (12) and (13) we have:

$$F = \frac{1}{\sum_{m=1}^{M} \theta_m^y \cdot Q_m^G} \tag{14}$$

Formula (14) specifies the type of functional F in the event that the production capacity is estimated by the technological group of equipment (in this case, m-group). That is, costs for a group of equipment are calculated as the sum of costs for its individual types. The denominator of expression (14) – special annual costs of PS. The maximum F function complies with minimum denominator in the expression (14).

Consequently, the targeted function of optimization in this case has the following mode:

$$Q = \sum_{m=1}^{M} \theta_m^y \cdot Q_m^G \rightarrow \min. \tag{15}$$

Mentioned in the paper [25] the procedure of configuration options for MTE is based on the consequent one-to-one comparison of options (one of the mistaken is the essential one) and the selection of the best out of them. The graph interpretation of break-off condition, when $Q_i = Z_0$ (index "0" is the essential option), is presented at the Fig. 1. The preferable solutions lie in the field under the break-off slope.

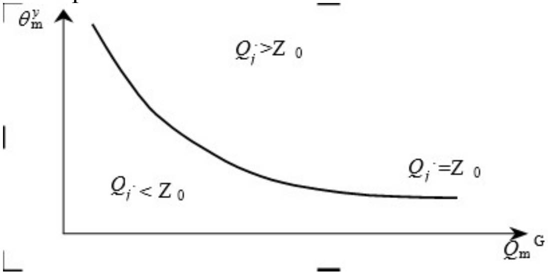

**Fig 1**. The graph interpretation of break-off condition for the selection of MTE options.

In Fig.1, the differentiation of unit costs for the designed equipment group is shown in accordance with the costs for the best version of its execution (production system) ($Z_0$). Naturally, the designer is interested in "super-efficient" options lying under the curve.

Let's consider the discrete model of PS, oriented on the selection of main and supporting subsystems in accordance with criterion (9) of minimum special annual costs:

$$\min_{\tilde{X}(\tilde{X}_1, \tilde{X}_2)} \left( \sum_{m}^{M} c_m^y \cdot b_m(\tilde{X}_1, \tilde{X}_2) + \sum_{p=1}^{P} c^{(P)}(\tilde{X}_2) \right),$$

where: M – number of different types (models) of equipment, taking into account technological processes of unit processing by the production program; m – number of an equipment group settled by MTE numeration ($m = \overline{1, M}$); $c_m^y$ – special annual costs for one unit of equipment by m - type: $c_m^y = \theta_m^y \cdot Q_m^G$; $\tilde{X} = (\tilde{X}_1, \tilde{X}_2)$ – vector of possible configurations: $\tilde{X}_1$ – essential one and $\tilde{X}_2$ – supporting subsystems; $b_m(\tilde{X}_1, \tilde{X}_2)$ – number of units OTE by m– type with configuration $\tilde{X} = (\tilde{X}_1, \tilde{X}_2)$; P – number of supporting subsystems in the current version $\tilde{X} = (\tilde{X}_1, \tilde{X}_2)$ (P = 5 : TWS, STP and SC, IDS, ASOMP, STSP and AP); $c^{(p)}(\tilde{X}_2)$ – balanced costs for p– subsystem with $\tilde{X}_2$ – configuration of supporting subsystems.

The vector of configuration $\tilde{x}^{(k)}$ for MTE provides the nomenclature content of used models of machinery:

$$\tilde{x}^{(k)} = (\alpha_1^{(k)}, \ldots, \alpha_m^{(k)}, \ldots, \alpha_M^{(k)}),$$

$$\alpha_m^{(k)} = \begin{cases} 1, \\ 0, \end{cases}$$

If m- model of machinery is included in the list of MTE, on the contrary.

k – serial number of MTE configuration with lexicographical streamline of zero vectors: (0,..., 0, 0), (0,..., 0, 1), (0, ..., 1, 0), (0, ..., 1, 1), ..., (1, ..., 1, 1) (this correlates with binary system of numbers 0, 1, 2, 3, ..., $2^M$).

There is the need to take into account only the set of admissible ones $\tilde{X}_1$: $\tilde{x}^{(k)} \in \tilde{X}_1$ out of all MTE configurations only when for any unit $A_i$ within the production program ($i = \overline{1, I}$), there is a route in the essential set of technological routes $\tilde{L}_i$, which is realized on the equipment of nomenclature $\tilde{x}^{(k)}$.

Let us consider the generation procedure for the set of possible MTE configurations.

Essential options for such a target are options of technological routes for $\tilde{L}_i$ processing of units for the production program:

$$\tilde{L}_i = \{< T_{i,m,r} >;\ m = \overline{1, M};\ r = \overline{1, R_i}\},$$

where: $T_{i,m,r}$ – time for processing of i- unit form-technological group of equipment in accordance with r-option of a route; $R_i$ – number of different options of routes for the processing of i-unit.

Let us consider that $T_{i,m,r} = 0$, if MTE of m-group does not participate in r-route of i-unit processing.

Let the first unit be produced in accordance with $r_1$ - technology ($r_1 = \overline{1, R_1}$), the second – $r_2$ - ($r_2 = \overline{1, R_2}$), ... , *I*– for $r_I$ ($r_I = \overline{1, R_I}$). Then components of a vector with possible MTE configuration in accordance with the technological process of this company are defined by formula:

$$\alpha_m^{(r_1, r_2, \dots, r_I)} = D\left(\sum_{i=1}^{I} D\left(T_{i,m,r_i}\right)\right),$$

where $D(x)$ – Dirac function: $D(x) = \begin{cases} 1 \text{ if } x>0, \\ 0 \text{ if } x=0. \end{cases}$

By varying the set ($r_1, r_2, \dots, r_I$), we will get vectors $\tilde{x}^{(r_1, r_2, \dots, r_I)}$ of possible MTE configurations. In a set $\tilde{X}_1$ we include different vectors $\tilde{x}^{(r_1, r_2, \dots, r_I)}$ with full permutation of the set ($r_1, r_2, \dots, r_I$). Number k of possible MTE configurations can be defined by the meaning $\min\{2^M; R_1 \cdot R_2 \cdot \dots \cdot R_I\}$.

The configuration (1,1,...,1) is included in any set of possible MTE configurations and is the only one for the case $R_1 = R_2 = \dots = R_I = 1$ (processing of units with fixed technological routes).

Vector of configuration $y^{(z)} = (\beta_1^{(z)}, \dots, \beta_p^{(z)}, \dots, \beta_P^{(z)})$ of supporting subsystems provides the option of realization for each subsystem in the selected z-option:

$$\beta_p^{(z)} = \begin{cases} 1 - mechanized\ option\ for\ p - subsystem\ implementation \\ 2 - automized\ option\ of\ p - subsystem\ implementation \end{cases}$$

Where z – serial number of configuration in the selected numeration of options for supporting subsystems ( $z = \overline{1, 2^P}$ ).

The set of configurations $\tilde{X}_1$ and $\tilde{X}_2$ is mutually independent because, for any MTE configuration and full range of supporting subsystems, there is possible for every subsystem in MTE to consider both mechanized and automatized implementation. Hence, the number of possible MTE configurations is equal to $2^P$.

However, in practice for every case, configurations of supporting systems are determined by the number z of possible options (usually, $z \le 10$): in real conditions for parts of supporting subsystems, there is an option of implementation that correlates with the technical level of production system output in accordance with a new program.

To sum up, let us highlight that the extremum in function(10) is considered on the set $\tilde{X}_1$ for possible MTE configurations, defined by a range of options for technological routes of units processing with special production programs, and on the special set $\tilde{X}_2$ for possible configurations of supporting subsystems.

At the preproject stage, the possible limitations for the synthesis task of PTS are

The limitation of MTE production power

$$\sum\nolimits_{i=1}^{I} N_i \sum\nolimits_{r=1}^{R} L_{i,R} \cdot T_{i,m,r} \le f_m^n\left(\tilde{X}_2\right) \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right) \cdot b_m\left(\tilde{X}_1, \tilde{X}_2\right) + \Delta\tilde{b}_{m,}$$

$m = \overline{1, M}$ ;

and the limitation for a unit processing only for one of the possible technological routes

$$\sum_{r=1}^{R_i} L_{i,r} = 1, \quad L_{i,r} \in \{0,1\}, \quad i = \overline{1, I}, \qquad (17)$$

where $N_i$ is the determined number of i- units in the considerable case; $L_{i,r}$ – zero variable, defining the choice of r-option for the technological route for i-units (within the frame of selected $\tilde{X}_1$ - MTE configuration);

$f_m^n\left(\tilde{X}_2\right)$ – the effective labor time for MTE of m-group with the configuration of supporting subsystems, described by vector $\tilde{X}_2$; $\lambda t_m$ – coefficient of technical usage for MTE of m-group; $\lambda_0(\tilde{X}_2)$ – coefficient of organizational usage of MTE with the configuration of supporting subsystems, described by vector $\tilde{X}_2$; $\Delta\tilde{b}_m$ – established by designer the total error of machinery capacity for MTE m-group.

The task (16) - (19) because of the form of targeted function and some limitations is the quadratic task of integer programming and belongs to NP type of complex tasks of discrete optimization. The exact solution for the task is possible only with the usage of approach with full check of all options for possible configurations $\tilde{X}_1$ – main equipment and $\tilde{X}_2$ – supporting subsystems. However, because of large dimension of this task ( $2^M \times 2^P$ ), the full check is not rational. That is why this is necessary to use approximate methods (for example, based on the paper [38, 39] which provide a possibility at the first stage to define the area of local optimum for a function (16), and then to form a range of competitive options for the solution of basic task. In this case, a designer makes the final choice of a solution (quasi-optimal option for PTS and OTS).

As a method of approximate solution for an optimization task, we suggest using consequent decomposition of a function [40] and to separately search for local optimum for PTS and OTS. The methodical base of such an approach is appointed above independence for a set of possible configurations, with $\tilde{X}_1$ – as the main one and $\tilde{X}_2$ – supporting subsystems for each special option of PS building.

The algorithm of complex analysis and synthesis of OTS is based on the decomposition of function (10) of the basic task for PS optimization and the outline of partial functioning

$$Z1(\tilde{x}_1) = \sum_{m}^{M} c_m^y \cdot b_m(\tilde{X}_1, \tilde{X}_2) \rightarrow \min,$$

as well as a search for the optimum of targeted function (16) for MTE configuration $\tilde{X}_1$, supporting the optimum of function (20).

The solution algorithm for tasks (16) - (19) of SCM optimization at the company include the following stages:

1. Synthesis of MTE configuration and PTS content calculation at the company for the selected basic configuration of supporting subsystems $\tilde{x}_2 = \tilde{x}_2^{(0)}$.

There is a solution for the task of generation for a set of admissible and synthesis of quasi-optimal option $\tilde{x}_1$ for PTS configuration, including the nomenclature and composition content of MTE and technological routes for units processing in accordance with the production program.

2. For the selected MTE configuration, $\tilde{x}_1$ is carried out for a full check of possible configurations and $\tilde{x}_2$ for supporting subsystems. The real labor fund for all types of processes for the main equipment is calculated for each configuration $\tilde{x}_2 \in \tilde{X}_2$ in accordance with that established by a designer $\lambda o(\tilde{X}_2)$ – coefficient of organizational usage of MTE and $f_m^n(\tilde{X}_2)$ – normative labor fund for MTE usage: $f_m^n\left(\tilde{X}_2\right) \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right)$.

In accordance with a limitation

$$\sum_{i=1}^{I} N_i \sum_{r=1}^{R} L_i, R \cdot T_{i,m,r} \leq f_m^n\left(\tilde{X}_2\right) \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right) \cdot b_m\left(\tilde{X}_1, \tilde{X}_2\right)$$

Number of MTE with all types including in $\tilde{x}_1$-configuration is defined:

$$\overline{b}_m(,) = \frac{\sum_{i=1}^{I} N_i \times L_{i,r} \times T_{i,m,r_i^0}}{f_m^n\left(\tilde{X}_2\right) \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right)}, m = \overline{1, M},$$

where $r_i^0 = r_i(\tilde{X}_1)$ – optimal route of i-unit with $\tilde{x}_1$-configuration of MTE.

The established number $b_m$ of equipment by m-type is defined by returns a number $\overline{b}_m$ up to the closest whole number: $b_m = ]\overline{b}_m[$.

3. The value for the targeted function (16) is defined for the selected set of main equipment and also for the supporting subsystems, thereby creating the structure $\tilde{X}_2$ of MTE configuration. If the value of the targeted function is lower than the record for the MTE configurations considered earlier, then the current value of the targeted function is taken into account as a new record.

The results of the formal synthesis of PTS are realized in terms of the solutions for the task. (20)

(16)–(20) form the basis for further human and machinery procedures in the frame of the PTS structure and synthesis improvement.

During the realization of the dialog procedure in the frame of results improvement for the formal synthesis of the PTS structure, the designer tries to achieve the following aims for the selected option:

- To use all the equipment in a maximum mode, shifting the sharing of details from the equipment with low capacity and excluding them to other used machines with higher coefficients;
- To sustain the equipment in the MTE set which can be implemented in the technological process of units processing planned in the studying option of production program and taking into account the factors of technical obsolescence;
- To provide the maximum possible coefficient of capacity for the equipment, which is preferable in MTE set.

The above-listed aims can be achieved by the process of sanctions [40] or, conversely, by reducing the prices for such equipment. Following this, task selection for optimal technological routes for each *i*-unit is carried out based on the new "prices."

If the absolute improvement of the record for the tasks of formal PTS synthesis happens, the calculated option of the PTS structure is used for the solution of the task for the synthesis of the supporting subsystem structure.

Thus, the volume synthesis of quasi-optimal option based on selected criterion of production and technological $\tilde{X}_1^0$, organizational and technical $\tilde{X}_2^0$ structures of a company is carried out in the process of solution for a task of structural and parametric optimization of company's system (21) for every competitive technical project, providing vector of perspective production program $(N_1, ..., N_i, ..., N_I)$.

At the same time the quasi-optimal option of MTE configuration is generated at the level of technological routes for units processing of production (22) program: $(b_1, ..., b_m, ..., b_M)$.

If $\left(b_1^{(n)}, \ldots, b_m^{(n)}, \ldots b_M^{(n)}\right)$ – set of used equipment in MTE synthesis, then the studied technical project includes the purchase, establishment, installation and further service of equipment, targeted by vector $\left(b_1 - b_1^{(n)}, \ldots, b_m - b_m^{(n)}, \ldots, b_M - b_M^{(n)}\right)$.

## III. Quality assessment of a solution in the optimization of SCM model of production and technological structure of a company

The model of production system, which is studied above, is oriented at the selection of production and technological structure and supporting systems of machinery building company based on the criterion of minimum annual costs.

To sum up mentioned above algorithm, the accuracy of received solution should be evaluated. In accordance with this target the decomposition of targeted function should be carried out for two variations.

The first one – costs for purchase, installation and use of equipment:

$$F_1(\tilde{X}_1) = \sum_{m=1}^{M} c_m^y \cdot b_m(\tilde{X}_1, \tilde{X}_2),$$

where $c_m^y$ – coefficients of special costs, estimated by one-time capital expenditures for purchase and installation of equipment and current costs for its usage.

The second one – indirect costs based on the number of main equipment. Thus, expenditures on equipment are targeted by linear function correlated with a number of equipment.

Options of technological routes should be selected in a way that costs $F_1$ are minimal. This provides the necessity to solve a task of discrete optimization (18)-(20).

The following definitions will be used based on a method of combinatorial SCM optimization: local and optimal solution inside of each technological group of equipment provides minimum special costs calculated based on machinery capacity; quasi-optimal solution of task for SCM optimization is a combination of local and optimal solutions for each technological group of equipment.

Let us formulate and prove the following theorem.

Theorem

Quasi-optimal solution is $\alpha$ *-optimal, where error* $\alpha$ *doesn't exceed the difference in costs, calculated for the combination of local and optimal solutions in nomenclature and number of equipment taking into account its capacity.*

Basic variables of suitable solution above were determined like $L_{i,r}$ (1, if for unites processing $i$ - group the route *r* is used; 0-in the controversial case). Variable groups $L_{i,r}$ have the following conditions

$$\sum_{r=1}^{R_i} L_{i,r} = 1, \qquad L_{i,r} \in \{0,1\},$$

Or one technological route out of $R_i$ possible ones should be selected for the unit processing of $i$ group. Costs of machinery capacity for $m$ - equipment for units processing of $i$ - group will be determined as $t_{i,m}$, and total timing costs – $t_m$. These variables are connected with c $L_{i,r}$ and between each other by the following links:

$$t_{i,m} = N_i \cdot \sum_{r=1}^{R_i} L_{i,r} \cdot T_{i,m,r} \quad ,$$

$$t_m = \sum_{i=1}^{I} t_{i,m} .$$

It can be written in the following mode:

$$b_m\left(\tilde{X}_1, \tilde{X}_2\right) = \left[ t_m / \left( f_m^n \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right)\right)\right],$$

where: [ ] – the whole part of a number; $f_m^n$ – effective timing fund of MTE $m$ -group; $\lambda t_m$ – coefficient of technical usage of MTE $m$ - group; (23) $\lambda o(\tilde{X}_2)$ – coefficient of organizational usage of PTS with $\tilde{X}_2$ configuration of supporting subsystems.

The optimization tasks includes the target to find the value $L_{i,r}$, supporting minimum $F_1$ functionality. Let us substitute the real costs $F_1$, calculated based on nomenclature and number of equipment, by special costs

$$F_1^{'} = \sum_{m=1}^{M_j} \frac{c_m^y}{f_m^n \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right)} \cdot t_m = \sum_{m=1}^{M} t_m$$

In the expression (28) the value $c_m'$ – is the special equipment capacity for *m* - equipment, that is why such calculation of special costs is the calculation of relative operations for equipment capacity. This is clear, that $F_1' \le F_1$.

These special costs $F_1'$ for the processing of all units in the volume of annual output program $N_i$ are equal to the sum of special costs for the processing of units for each group of MTE:

$$F_1' = \sum_{i=1}^{I} \sum_{m=1}^{M} c_m' \cdot t_{i,m} .$$

As technological routes for different units are independent, then the local and optimal (in accordance with special costs) route can be selected separately for each group by the simple check of $R_i$ options.

Then $\overline{L}_{i,r}$ – the local and optimal solution or $i$ -group is processed by the route, where $L_{i,r} = 1$. In this case, the minimum of special costs is defined by the expression: (24)

$$\overline{F}_1' = \sum_{m=1}^{M} c_m' \sum_{i=1}^{I} N_i \sum_{r=1}^{R_i} \overline{L}_{i,r} \times T_{i,m,r} .$$

In accordance with condition (24), the special costs $F_1'$, calculated by formula (29), will be higher than $\overline{F}_1'$ with any other selection of options or combination of $L_{i,r}$ values. This is particularly true for the optimal solution of tasks (18)–(20).

The real costs with the same combination of routes are not less than the special ones, and the real costs (25) $F_1^*$ in the optimal option are not less than $\overline{F}_1'$. Thus, $\overline{F}_1'$ is the bottom line of minimal costs.

$$\overline{F}_1' \le F_1^* \tag{31}$$

With the option of $\overline{L}_{i,r}$ routes (optimal ones in accordance with special costs), calculated by formulas (27)–(30), the $\overline{F}_1$ costs cannot be lower than the real costs $F_1^*$ in the optimal option. Consequently, $\overline{F}_1$ is the top line of minimal costs. Thus, the following double inequality exists:

$$\overline{F}_1' \le F_1^* \le \overline{F}_1 . \tag{32}$$

The combination of $\overline{L}_{i,r}$ solutions is the quasi-optimal solution for a combinatorial optimization task with a SCM structure, and $\alpha = \overline{F}_1 - \overline{F}_1'$ provides the error estimation.

Thus, the formulated theorem is proved.

*The quasi-optimal solution of the optimization task can be used in all cases when the targeted possible error* $\alpha$ *is provided.*

Thus, if the difference in costs between the quasi-optimal decision and the bottom line is approved for example, not less than 10% of costs with the quasi-optimal solution then such decision, which is equal to the combination of the local and optimal solutions, can be accepted as the final one for the selection of the PTS option. If the difference in costs (optimization error) is not approved, there will be a possible consequential improvement in a solution through single- and two-point variations when there is a change of one or two routes in the list of technological routes and the option search with error is conducted.

Let us consider the particular solution for this optimization task.

Ratio (27) is not linear in the system of expressions (20), (24)–(27). This can be changed by the linear inequality for the solution with minimal costs.

$$b_m\left(\tilde{X}_1, \tilde{X}_2\right) \ge \left[t_j / \left(f_m^n \cdot \lambda t_m \cdot \lambda o\left(\tilde{X}_2\right)\right)\right] \tag{33}$$

The task determined by limitations (20), (24)–(26), and (33), is a discrete programming task that can be solved by the branches and borders method with an effective check of options.

In PTS synthesis, the rejection from the requirement of the whole variations $L_{i,r}$ means the combination of technological routes, and the rejection from the whole variations $b_m$ is a shift from real to special costs. Within this task, the top line for the rejection of branches is the value $\overline{F}_1$, or real costs for the quasi-optimal solution. Additionally, the quasi-optimal solution provides the bottom line of the $\overline{F}_1'$ extremum, which cannot be improved. Therefore, the search can be stopped as soon as the whole solution is found when costs $F_1$ differ from $\overline{F}_1'$ which is less then the determined error $\alpha$ of the optimal solution.

Thus, the optimal solution to the PTS synthesis problem can be effectively obtained using the branch-and-bound method in combination with establishing the upper and lower boundaries of the minimum of the objective function for the quasi-optimal solution.

For the solution of such optimization task on a computer, there is a need to develop an algorithm of the developed method, which can find a solution for tasks with big data or change initial data.

Different combinations of solution improvement are possible — based on single-point variation, based on two-point variation, and with the branches and borders method.

Opportunities of production program enlargement with separate technological groups of equipment, program implementation with failure of some equipment, and the feasibility of capacity reservation can be additionally analyzed during PTS synthesis.

PTS synthesis can be performed based on the average values of coefficients $\lambda t_m$ and $\lambda_0\left(\tilde{X}_2\right)$. Initial data for PTS synthesis can be determined based on statistical data about the reliability of technological equipment and rules on effective fund timing related to the PS service. However, the accumulation of statistical data is a long-term process. As shown by the experience of researchers initial data can be determined based only on the use of methods with the aggregate building of universal or specialized machinery in accordance with nomenclature considering its serial output and the documentary indication of its reliability

# 3. Conclusions

Optimal supply chain design is vital to the success of industrial concerns now more than ever before. This paper reviews some principal research opportunities and challenges in the field of supply chain design. The growing area of enterprise-wide optimization and the increasing importance of energy and sustainability issues provide plentiful opportunities for supply chain design research. The effective grouping of technological routes for unit processing can be quantitatively assessed. Large numbers of units with different typologies are combined by group technological routes of processing; the higher the percentage of the same equipment's usage, the more effective the grouping. Thus, this study examined more options of grouping and differentiated technological routes for unit processing by settled nomenclature in the volume of output programs with combinatorial optimization.

The combinatorial optimization method can be used in the preliminary estimation of options for technical solutions related to the development or update of the production system of a company.

PS development entails the realization of three scenarios — enlargement, enlargement and change, and absolute change of nomenclature for unit processing.

The SCM optimization should be conducted based on the abovementioned method. If there is a large quantity of unused equipment and the capacity of other equipment is enough for unit processing in the determined technological route options, another approach can be applied. New data should be added to the initial table. Moreover, there should be one used technological route for details out of existing nomenclature, and new details should obtain the only option of a route based on existing types of equipment and required technology in accordance with the MTE of a company. For other details, the table is completed in accordance with previous rules. In the second and third cases, the table of options for PTS should be created from scratch. The difference from the first case is that the priority remains for the options of technological routes in which the equipment already installed is used (here the concept of priority is due to the fact that such route technology can be assigned to parts of this type).

Thus, taking into account that the methods of structural-parametric and combinatorial optimization of the production system have so far been studied fragmentarily, the systematic presentation of the approach proposed by the authors allows both to clarify the meaningful statement of the problem and possible approaches to its solution. In the future, the authors plan to develop detailed algorithms for structural, parametric and combinatorial optimization of production systems, their implementation and verification at a specific engineering enterprise.

## References

[1] Akhmetshin, E. M., Vasilev, V. L., Mironov, D. S., Zatsarinnaya, E. I., Romanova, M. V., & Yumashev, A. V. (2018). Internal control system in enterprise management: Analysis and interaction matrices. European Research Studies Journal, 21(2), 728-740.

[2] Al-Hashimy, H.N.H., Al Jubair, A.S., Jasim, E.T. (2019). The Effect of Accounting Information Systems (AIS) on Enterprise Resource Planning (ERP). Journal of Southwest Jiaotong University, 54(5). http://jsju.org/index.php/journal/article/view/394

[3] Ansoff I. Strategic management: trans. from English / edited by L.I. Evenko. M.: Economics 1989. 519 p.

[4] Apalkova T., Mishchenko A. Models of investment management in logistics in conditions of uncertainty and risk, Logistics and Supply Chain Management, №1 (78), pp. 82-97, 2017.

[5] Arslanova Z.E., Livshits V.N. Methods for assessing the effectiveness of investment projects // Investments in Russia. 1995. №. 4-5.

[6] Ayupova M.T. Mathematical models and project management methods for restructuring an industrial enterprise, Dis. abstract PhD in Economics, specialty 08.00.13. M.: Plekhanov Russian University of Economics, 24, 2008.

[7] Bauer R., Collar E., Tan V. Investment project management: IBM experience / Trans. from English M .: INFRA-M, 1995. 135 p.

[8] Bellman R., Dreyfus S. Applied problems of dynamic programming. M.: Publishing house of foreign literature. 1969. 460 p.

[9] Berens V., Havranek P. Guidelines for assessing the effectiveness of investments. M.: AOZT INTEREXPERT, INFRA-M, 1995. 295 p.

[10] Birman G., Schmidt S. Economic analysis of investment projects. M.: UNITY, 1997. 631 p.

[11] Blank A.I. Investment management. Kiev: MP ITEM LTD., 1995. 412 p.

[12] Brealey R., Myers S. Principles of corporate finance /Trans. from English M: Olympus Business, 1997. 1057 p.

[13] Bromwich M. Analysis of economic efficiency of investment / Trans. from English. M.: INFRA-M, 1996. 432 p.

[14] Chetyrkin E.M. Financial analysis of industrial investments. M.: Delo, 2002. 256 p.

[15] Idrisov A.B. et al. Strategic planning and analysis of investment performance. M.: Publishing House Eagle Owl, 1997. 272 p.

[16] Francis J.C Investments: Analysis and Management. MacGraw-Hill, 1991. 874 p.

[17] Gitman L.J., Jonk M.D. Fundamentals of investing / Trans. from English. M.: Business, 1997. 1008 p.

[18] Khalaf, M.F. (2019). Integrating Throuhput Accounting and Activity Based Costing in Industrial Companies. Journal of Southwest Jiaotong University, 54(6). http://jsju.org/index.php/journal/article/view/403

[19] Khalikov M.A. Discrete optimization of plans to increase the reliability of functional systems, Sat: Financial mathematics, M.: Lomonosov Moscow State University, pp. 281-295, 2001.

[20] Khalikov M.A. Economic and mathematical

modeling of sustainable development of engineering enterprises in a market economy, Dis. abstract Doctor of Economic sciences, specialty 08.00.13. M.: Plekhanov Russian University of Economics, 48, 2004.

[21] Kleiner G.B. Enterprise strategy, M.: Delo, 2008.

[22] Lipsits I.V., Kossov V.V. Investment project: methods of preparation and analysis / Educational reference manual. M.: Publishing House BEK, 1996. 304 p.

[23] Livshits V.N. Project analysis: a technique adopted by the World Bank // Economics and Mathematical Methods. 1994. V.30. № 3.

[24] Lvov D.S. and other Methodological problems of evaluating the effectiveness of investment projects // Economics and Mathematical Methods. 1995. V.31. Vol. 2. p. 5 - 19.

[25] Luenberger D., Yinyu Y. Linear and Nonlinear Programming, Springer Science + Bussiness Media, LLC, 2008.

[26] Mann A.S., Markowitz G.M. Sectoral economic and mathematical models. Analysis of production investments / Translation from English. M.: Progress, 1967. 424 p.

[27] Markina, I., & Diachkov, D. (2014). Factor approach to the determination of information strategy of companies. Journal of Contemporary Economics Issues, 2. https://doi.org/10.24194/21402

[28] Masse P. Criteria and methods for the optimal determination of investment / Trans.from French. M.: Statistics, 1971. 504 p.

[29] Hung, Tran Thanh. "Preventing human trafficking in Vietnam through economic empowerment programmes." International journal of criminology and sociology 9 (2020): 1-8.

[30] Minniti A., Turino F. Multi-product firms and business cycle dynamics. European Economic Review, Vol. 57. pp. 75-97, 2013.

[31] Pavlov N.N. et al. Investment project and business plan: purpose and structure // Investments in Russia. 1995. № 1. p. 23 - 27.

[32] Petrov V.Ya., Maslennikov A.N., Osipov L.A. Planning flexible manufacturing systems. Leningrad: Engineering, 1985. 182 p.

[33] Pleschinsky A.S. Optimization of enterprise investment projects in a market economy // Economics and Mathematical Methods. 1995. V.31. № 2. p.81 - 90.

[34] Shabalina U.M. The risk indicators in industrial and financial spheres of enterprises of the integrated group, Entrepreneur's Guide, №34, pp. 305-321, 2017, URL: http://www.rusacad.ru/docs/nauka/putevoditel_predpr34.pdf

[35] Vilensky P.L., Livshits V.N., Orlova E.R., Smolyak S.A. Assessment of the effectiveness of investment projects. M.: Delo, 1998. 248 p.

[36] Vilensky P.L., Smolyak S.A. Calculation of the investment project // Economics and mathematical methods. 1995. V.31. № 4. p. 31 - 35.

[37] Vitik, S. V., Koptyakova, S. V. and Balynskaya, N. R. (2016). Methodology for Assessing the Efficiency of Labor-Related Incentives at an Enterprise. International Business Management, 10, pp. 408-415.

[38] Voskresenskiy B.V., Manilovsky R.G. Production capacity of an engineering plant. M.: Mechanical Engineering, 1973. 336 p.

[39] Ward W., Deren B. The Economics of Project Analysis: a Practitioner's Guide. (EDI Technical materials): Washington, D.C., 1991. 318 p.

[40] Yanchuk, M. (2013). Application of theory of spiral dynamics in management of integrative development of economic systems. Journal of Contemporary Economics Issues, 3. https://doi.org/10.24194/31308