Using Dynamic Stability Strategy to Counteract Rapid Changing Demand Considering Deterioration and Partial Backordering

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***Abstract*—In a real business world, the production plan must respond to quick changing market and keep production line stable. An optimal stepwise production strategy is derived with stable production rate for some period, and then changing to another stable production rate to response market change for the next period. The cost factors we considered in the model include the production setup cost, the production-variation cost, the carrying cost, the deteriorating cost, the backordering and the lost sale shortage cost. For demand that changes linearly or exponentially with time, an inventory model with partial backordering is developed. Finally, sensitivity analysis is carried out to investigate how changing parameters affect this system.**

Keywords*—Stepwise production, Deterioration, Partial backordering*

1. Introduction

In recent years, the demand for local electric vehicles in Europe and the United States has been declining year by year. The main reason is that the competitiveness of imported electric vehicles has eroded the domestic demand market. In response to the increasing and decreasing market demand, Yang and Rand (1993) believed that the uncertainty risks brought by the three aspects of demand, supply, and manufacturing are the key to affecting the delivery date of the final product. Changes in customer demand are one of the sources of uncertainty in the supply chain. The reasons may be caused by customers ordering irregular quantities at irregular times, or consumption preferences changing over time. This often leads to demand forecast errors for companies. Yang and Rand (1993) proposed a product replenishment strategy where demand grows linearly over time. They proposed an algorithm to obtain the best solution by changing parameters within a certain range.

Regarding the demand uncertainty originating from the client, Dey et al. (2006) believe that consumer uncertainty is the uncertainty that cannot predict consumer preferences.

They considered the inventory of defective items sold in two stores (primary and secondary) over a limited time frame. Main stores receive goods with price differentials in large quantities but sell only those goods that are not defective, and demand for these increases periodically over time and decreases during periods of shortage, so that it returns to the initial value at the beginning of the next cycle. Therefore, in this store, shortages are allowed to occur and are completely overstocked. Additionally, at the beginning of the next cycle, retailers will purchase pure, non-defective products at a higher price to cover shortages and the large volume of differential products they typically sell. Defective units identified at the time of sale in the first store are continuously transferred to the adjacent second store from which, after rework, these secondary units are sold at a reduced price. Demand depends on selling price, which is inversely proportional to the defective rate. Study nonlinear optimization methods using gradients for mathematical planning and problem solving and illustrate models through numerical examples. Omar (2009) pointed out that the supply chain is related to the uncertainty of the demand side. He considered a supply and demand supply chain where sellers deliver products at a limited rate and ship products to buyers on a regular basis. The buyer, on the other hand, consumes the product at a linearly decreasing demand rate. Much of the previous work on this topic has assumed of a fixed demand rate, with the goal of determining the number of shipments and the size of those shipments to minimize total cost—assuming sellers and buyers collaborate and find a way to share the resulting benefits. This study shows how to arrive at an optimal strategy when shipment volumes are the same and illustrates this strategy with an example.

Shi et ai. (2011) believes that demand uncertainty is mainly affected by the different demand types of functional or innovative products. They proposed a closed supply chain system in which manufacturers have two pathways to meet market demand: manufacturing new products and recycling and remanufacturing into new products.

Remanufactured products are indistinguishable from brand new products and can be sold in the same market at the same price. Market demand is uncertain and sensitive to sales prices, while recycling volumes are also random and sensitive to the acquisition price of second-hand products. This study uses mathematical models with the goal of maximizing overall profits. Finally, through numerical examples, the impact of demand and revenue uncertainty on production plans, sales prices and second-hand product purchase prices is analyzed.

Mitra (2012) established a two-stage closed supply chain inventory system with deterministic and stochastic demand respectively and solves the following problems: (1) The cost of a closed supply chain must be higher than that of a traditional supply chain, (2) High recovery rate Whether it always translates into low demand changes to reduce expected costs, (3) explore the relationship between expected costs, demand and recovery.

In the past, researchers have modeled inventory batch sizes under trade credit financing by assuming that demand rates are constant. However, from a product life cycle perspective, demand only approaches constant during the maturity stage. During the growth phase of a product's life cycle (especially high-tech products), the demand function increases over time. To obtain robust and broadly general results, Teng et al. (2012) extended demand as a linear non-decreasing demand function over time. The results show that the basic theory obtained here is applicable to the growth and maturity stages of the product life cycle.

Research on deteriorating items has been studied by several authors in recent years. Ghare & Schrader (1963) were the first authors to consider on-going deterioration of inventory. Other authors such as Covert & Philip (1973), Dave (1979), Elsayed & Teresi (1983), Kang & Kim (1983), Mak (1982), Raafat et al. (1991) and Heng et al. (1991) assumed either instantaneous or finite production with different assumptions on the patterns of deterioration. Some of the models allowed for shortages, but none of them considered the effect of varying demand rate. Dave & Patel (1981), Sachan (1981) and Goswami & Chandhuri (1991) have developed models assuming linear increasing demand; all of them assumed linear trend in demand. The consideration of exponentially decreasing demand was first analyzed by Hollier & Mak (1983) who obtained optimal replenishment policies under both constant and variable replenishment intervals. Recently Aggarwal & Bahari-Kashani (1991) extended Hollier & Mak's model to allow flexible rates of production in each period. The model assumed fixed production interval. Both exponentially decreasing demand models did not allow for shortages. The declining market model developed by Wee (1995) allows for shortages. Table 1 listed some outstanding literature of rapid changing demand.

Table 1. Literature related to rapid changing demand

Authors	Demand increase	Demand decrease	Stepwise production
Yang & Rand, 1993.	V		
Yang et al., 1999.		V	V
Dey et al., 2006.	V		
Omar, M., 2009.		V	
Konstantaras et al., 2010.			V
Shi et al., 2011.	V		
Mitra, 2012.	V		
Teng et al., 2012.	V		V
Wang et al., 2016.	V		
Hong et al., 2016.	V		
Yu & Solvang, 2018.	V	V	
Deng et al., 2019.	V		
This study	V	V	V

Note: "V" means to focus.

This study develops the optimal stepwise production strategy to minimize the sum of the production setup cost, the production-variation cost, the carrying cost, the deteriorating cost, the backordering and the lost sale cost. The production rate can be adjusted periodically for the variable market demand to derive at the optimal stepwise production strategy.

2. Mathematical modeling and analysis

The mathematical model in this paper is developed on the basis of the following assumptions and notation:

(a) The production rate Qi(t) is an unknown staircase-like function:
$Q_i(t)=Q_i \quad t_{0i} \leq t \leq t_{0(i+1)} \quad i=1, 2, 3$…where t_{0i} is the i^{th} step starting time

(b) The demand rate d(t) is either a linear increasing function of time where
$d(t)=\alpha+\beta$ t; or an exponential increasing function of time where
$d(t)=\alpha e^{\beta t}$; α and β are non-negative constants.

(c) Lead-time is assumed to be negligible.

(d) A constant fraction, θ, of the on-hand inventory deteriorates per unit time.

(e) Unit cost of deterioration is C_2.

(f) There is no replacement or repair of deteriorated items at any time.

(g) There are production-variation cost $C_0+C_1(Q_i-Q_{i-1})$ where C_0 is production setup cost and C_1 is unit production-variation cost.

(h) Unit carrying cost per unit time is C_3.

(i) Production rate Q_i is constant for each cycle $i=1, 2, 3$…

(j) Only a fraction, R, of the demand during shortage is backordered from the next inventory cycle.

(k) A semi-continuous processing system, like automobiles, televisions and computers components and products, is considered to produce output that allows for some variety; products are highly similar but not identical. In specific time duration, the production rate (or capacity) of the semi-continuous process is highly uniform, while in other time duration the production rate may be adjusted higher due to increasing demand (Stepwise production). In that case, the production rate or capacity is adjusted according to the market demand. Hence, the production line of semi-continuous process operates perpetually and changes its production rate at the appropriate time.

(l) t_{0i} is the starting time of the i^{th} cycle for $i=1, 2, 3$…

(m) TC_i is the total cost per unit time in the i^{th} cycle

(n) The inventory returns to zero at $t_{0i}+T_{1i}$ and $t_{0i}+T_{1i}+T_{2i}$ in the i^{th} cycle.

(o) C_4 is the shortage cost per unit back-ordered per unit time.

(p) C_5 is the penalty cost of a lost sale including lost profit

(q) T_{1i} is the length from the starting time to the time when the inventory change to zero at the first time in the i^{th} production cycle.

(r) $I_{1i}(t_{1i})$ is the inventory level where t_{1i} is between 0 and T_{1i}.

(s) T_{2i} is the length between the time when the inventory is zero in the i^{th} production cycle.

(t) $I_{2i}(t_{2i})$ is the inventory level where t_{2i} is between 0 and T_{2i}.

(u) T_{3i} is the length from the time when the inventory is zero at the second time to the ending time in the i^{th} production cycle.
(v) $I_{3i}(t_{3i})$ is the inventory level where t_{3i} is between 0 and T_{3i}.

There is a shortage at the beginning T_{0i} of the i^{th} cycle. This shortage is cleared at time T_{1i} through production at a rate Q_i. Then excess production stars accumulating after meeting the current demands. On the other hand, the demand being an increasing function of time becomes gradually higher. As a result, the inventory falls to the zero level at time T_{2i} and shortage begins to develop again. The $(i+1)^{th}$ cycle begins at time T_{3i} at a higher production rate Q_{i+1}. The same process then continues in the $(i+1)^{th}$ cycle and so on. Given the above assumptions and notation, we now develop the inventory model. In the time duration T_{1i}, the variation of inventory level results from the production rate and the demand rate. Hence the inventory level's differential equation can be formulated as:

$$\frac{dI_{1i}(t_{1i})}{dt_{1i}} = Q_i - d(t_{1i} + t_{0i}),\ 0 \le t_{1i} \le T_{1i} \qquad (1)$$

Where $I_{1i}(t_{1i} = 0) = I_{3(i-1)}(t_{3(i-1)} = T_{3(i-1)})$ (2)

And $I_{1i}(t_{1i} = T_{1i}) = 0$ (3)

In the time duration T_{2i}, the variation of inventory level results from the production rate, the demand rate and the deteriorating rate of the positive stock. Hence the inventory level's differential equation can be formulated as:

$$\frac{dI_{2i}(t_{2i})}{dt_{2i}} + \theta I_{2i}(t_{2i}) = Q_i - d(t_{2i} + t_{0i} + T_{1i}),\quad 0 \le t_{2i} \le T_{2i} \qquad (4)$$

where $I_{2i}(t_{2i} = 0) = 0$ (5)

and $I_{2i}(t_{2i} = T_{2i}) = 0$ (6)

In the time duration T_{3i}, the variation of inventory level results from the production rate, the demand rate and the fraction of the demand during shortage that is backordered from the next inventory cycle. Hence the inventory level's differential equation can be formulated as:

$$\frac{dI_{3i}(t_{3i})}{dt_{3i}} = R(Q_i - d(t_{3i} + t_{0i} + T_{1i} + T_{2i})),\ 0 \le t_{3i} \le T_{3i} \qquad (7)$$

where $I_{3i}(t_{3i} = 0) = 0$ (8)

and $t_{0i} = \sum_{i=1}^{i-1}(T_{1i} + T_{2i} + T_{3i})$, $i=1, 2, 3\ldots$ (9)

The starting time $t_{01}$is set to zero. Since the starting inventory level is zero, the time period T_{11} is set to zero. By the boundary condition (3), the solution of the differential equation (1) can be solved as given:

$$I_{1i}(t_{1i}) = \int_{T_{1i}}^{t_{1i}} [Q_i - d(t_{1i} + t_{0i})]dt_{1i} \qquad 0 \le t_{1i} \le T_{1i}$$

(10)

By the boundary condition (5), the solution of the differential equation (4) is given in William & Richard (1965) as:

$$I_{2i}(t_{2i}) = (\exp(-\theta t_{2i}))\int_0^{t_{2i}} (\exp(\theta t_{2i}))[Q_i - d(t_{2i} + t_{0i} + T_{1i})]dt_{2i}$$

$,\ 0 \le t_{2i} \le T_{2i}$ (11)

By the boundary condition (8), the solution of the differential equation (7) can be expressed as:

$$I_{3i}(t_{3i}) = \int_0^{t_{3i}} R[Q_i - d(t_{3i} + t_{0i} + T_{1i} + T_{2i})]dt_{3i} \quad i = 1, 2, 3$$

$0 \le t_{3i} \le T_{3i}$ (12)

The relationship between $I_{1i}(t_{1i})$, $I_{2i}(t_{2i})$, $I_{3i}(t_{3i})$, $Q_i(t)$ and $d(t)$ can be depicted as Figure 1.

From the boundary conditions (2) and (6), the following relationship can be derived:

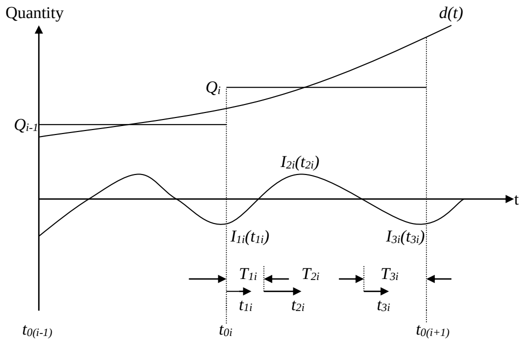

Figure 1. The relationships between production quantity, demand and inventory level with time

$$\int_{T_{1i}}^{0} [Q_i - d(t_{1i} + t_{0i})]dt_{1i} = I_{3(i-1)}(t_{3(i-1)}) \text{ when } t_{3(i-1)} = T_{3(i-1)} \qquad (13)$$

and

$$(\exp(-\theta T_{2i}))\int_0^{T_{2i}} (\exp(\theta t_{2i}))[Q_i - d(t_{2i} + t_{0i} + T_{1i})]dt_{2i} = 0$$

.... (14)

where $i = 1, 2, 3\ldots$

The total cost which include production-variation cost, deteriorating cost, carrying cost, partial back-ordering cost and lost sale cost can be expressed as:

$$TC_i = \frac{1}{T_{1i} + T_{2i} + T_{3i}}\{C_0 + C_1(Q_i - Q_{i-1}) + C_2[Q_i T_{2i} - \int_0^{T_{2i}} (\alpha + \beta(t_{2i} + t_{0i} + T_{1i}))dt_{2i}] + C_3\int_0^{T_{2i}} I_{2i}(t_{2i})dt_{2i} + C_4[\int_0^{T_{1i}} (-I_{1i}(t_{1i}))dt_{1i} + \int_0^{T_{3i}} (-I_{3i}(t_{3i}))dt_{3i}] + C_5(1 - R)[\int_0^{T_{3i}} (\alpha + \beta(t_{3i} + t_{0i} + T_{1i} + T_{2i}))dt_{3i} - Q_i T_{3i}]\}$$

.... (15)

where $i = 1, 2, 3\ldots$

The necessary conditions for the total cost function TC_i are as follows:

$$I_{2i}(t_{2i}) \geq 0 \quad 0 \leq t_{2i} \leq T_{2i} \tag{16}$$

$$I_{1i}(t_{1i}) \leq 0 \quad 0 \leq t_{1i} \leq T_{1i} \tag{17}$$

$$I_{3i}(t_{3i}) \leq 0 \quad 0 \leq t_{3i} \leq T_{3i} \tag{18}$$

$$Q_i > Q_{i-1} \tag{19}$$

$$Q_i \geq \frac{1}{T_{2i}} \int_0^{T_{2i}} (\alpha + \beta(t_{2i} + t_{0i} + T_{1i}))dt_{2i} \tag{20}$$

$$Q_i \leq \frac{1}{T_{3i}} \int_0^{T_{3i}} (\alpha + \beta(t_{3i} + t_{0i} + T_{1i} + T_{2i}))dt_{3i} \tag{21}$$

where $i = 1, 2, 3 \ldots$

3. Solution procedure

The stepwise production model can be solved by the following method. In the first step, we set the initial values $Q_0=0$, $T_{11}=0$, $t_{01}=0$ and $I_{11}(0 \leq t \leq T_{11}) = 0$. Substituting a range of Q_1 values, the respective value of T_{21} can be solved from (14). The corresponding values of T_{31} can be obtained by setting:

$$\frac{dTC_1}{dT_{31}} = 0 \tag{22}$$

For each set of (Q_1, T_{21}, T_{31}), TC_1 value can be calculated from (16). When the minimum value of TC_1 is found, the optimal values of (Q_1, T_{21}, T_{31}) will be determined. The value of $I_{31}(t_{31}=T_{31})$ can be found from (12). In the second step, the initial conditions are set at:

$$t_{02}=T_{21}+T_{31} \tag{23}$$

and

$$I_{12}(t_{12}=0)=I_{31}(t_{31}) \quad \text{when} \quad t_{31} = T_{31} \tag{24}$$

The values of T_{12} can be solved by substituting a range of Q_2 values from (14). The corresponding values of T_{22} can be obtained from (14). The values of T_{32} can be found by

$$\frac{dTC_2}{dT_{32}} = 0 \tag{25}$$

For each set $(Q_2, T_{12}, T_{22}, T_{32})$ values, TC_2 can be calculated from (15). When the minimum value of TC_2 is found, the optimal value of the set $(Q_2, T_{12}, T_{22}, T_{32})$ can be derived. The value of $I_{32}(t_{32})$ when $t_{32}=T_{32}$ can be found from (12). By the same procedures the subsequent steps can be solved.

4. Numerical example

Example 1

The preceding theory can be illustrated by the following numerical example. The demand rate is assumed to increase linearly, given by $d(t) = 100 + 10t$ and the production setup cost is $500, the unit production-variation cost is $10, the cost per deteriorated unit is $100, the unit carrying cost per month is $5, the shortage cost per unit back-ordered per month is $9, the penalty cost of a lost sale is $15. The units deteriorate at a constant rate with θ =20% and the system operates for infinite time horizon. By the above-mentioned procedures the optimal solutions of Q_i, T_{1i}, T_{2i}, T_{3i} and TC_i for the stepwise production cycles are derived in Table 1.

Table 1. The optimal solution of stepwise production cycles with d(t)=$\alpha + \beta t$

i	Q_i*	Q_i*-Q_{i-1}*	T_{1i}*	T_{2i}*	T_{3i}*	T_{1i}*+T_{2i}*+T_{3i}*	TC_i*
1	113	113	0.00	2.41	2.44	4.85	515.4
2	180	67	2.24	1.72	1.94	5.90	330.9
3	232	52	1.76	1.32	1.94	5.02	301.4
4	281	49	1.65	1.30	1.93	4.88	297.2
5	330	49	1.56	1.50	1.86	4.92	296.9
6	379	49	1.59	1.41	1.89	4.89	297.4

* Optimal solution

Example 2

The same parameters as in Example 1 are used except the demand rate is: $d(t) = 100\ e^{0.1\ t}$. The results of the analysis are shown in Table 2.

Table 2. The optimal solution of stepwise production with $d(t) = \alpha e^{\beta t}$

i	Q_i*	Q_i*-Q_{i-1}*	T_{1i}*	T_{2i}*	T_{3i}*	T_{1i}*+T_{2i}*+T_{3i}*	TC_i
1	114	114	0.00	2.39	1.33	3.72	674
2	184	70	0.69	3.00	0.41	4.10	609
3	244	60	0.50	1.15	0.73	2.38	624
4	309	65	0.59	0.95	0.69	2.23	689
5	386	77	0.46	1.18	0.52	2.16	790
6	474	88	0.39	1.11	0.52	2.02	927

* Optimal solution

5. Comments on the numerical examples

From Table 1, the following phenomena are observed:

(a) Steady state is achieved after the fourth production cycle.

(b) When the production cycle is at steady state, the values of $T_{1i}+T_{2i}+T_{3i}$ and Q_i-Q_{i-1} are constant.

(c) At steady state, Q_i*-Q_{i-1}* is almost identical to β $(T_{1i}$*+T_{2i}*+T_{3i}*). It is because the increase in production quantity is equal to the increase demand quantity in the subsequent cycles.

(d) At steady state, TC_i is constant. Since the deviation of demand rate, $d(t)$, from the production rate, Q_i , is the same in each cycle, the sum of the carrying cost, the deteriorating cost, the backordering and the lost sale cost is the same in each cycle time $(T_{1i}$*+T_{2i}*+T_{3i}*) for every i^{th} cycle in the steady state condition.

From Table 2, the following characteristics are observed:

(a) The greater the number of production cycles, the larger the values of TCi* and Qi*-Qi-1*, and the smaller the period (T1i*+T2i*+T3i*).

(b) The deviation of the exponential demand rate, d(t), from the stepwise production rate, Qi , becomes larger as i increases. This results in the increase of the carrying cost, the deteriorating cost, the backordering and the lost sale cost. Consequently, TCi increases monotonically. There is no steady state in this case.

6. Sensitivity analysis

For the numerical example given above, the optimal values of Q_i, T_{1i}, T_{2i}, T_{3i} and TC_i for a fixed set $S=\{\alpha, \beta, C_0, C_1, \theta, C_2, C_3, C_4, C_5, R\}$ of parameter values (denoted by Q_i^*, T_{1i}^*, T_{2i}^*, T_{3i}^* and TC_i^*, respectively) are found. Sensitivity analysis is done when only one of the parameters in the set S increases or decreases by some percentages and all other parameters remain unchanged. The study analyzes how the changes in the parameter values affect optimal values of Q_i, T_{1i}, T_{2i}, T_{3i} and TC_i. The results are shown in Table 3 for a linear increasing demand, and in Table 4 for an exponential increasing demand.

Table 3. Sensitivity analysis when d(t)= $\alpha + \beta t$

Parameter	% change in parameter	% change in Q_1^*	% change in T_{21}^*	% change in T_{31}^*	% change in TC_1^*
α	+50 -50	45.1 -46.0	7.1 -14.5	11.5 -12.7	19.3 -21.6
β	+50 -50	1.8 -4.4	-22.0 21.2	-9.8 29.1	17.9 -23.4
C_0	+50 -50	0 0	0 0	7.0 -7.8	9.8 -10.2
C_1	+50 -50	0 -1.8	0 -14.5	15.2 -14.8	21.8 -24.2
θ	+50 -50	-1.8 2.7	-17.0 26.6	8.6 -13.5	3.1 -5.6
C_2	+50 -50	-1.8 2.7	-14.5 21.2	7.0 -10.2	3.4 -5.8
C_3	+50 -50	-1.8 1.8	-14.5 14.1	5.3 -4.5	1.1 -0.6
C_4	+50 -50	0 -1.8	0 -14.5	-18.0 39.8	6.8 -11.3
C_5	+50 -50	0 0	0 0	-4.9 5.3	3.3 -3.5
R	+25 -25	0 0	0 0	-1.2 1.6	-2.7 3.1

7. Comments on the sensitivity analysis

From Table 3, the following observations are made:

(1) TC_1 is very sensitive to the changes in the parameter values C_1, α and β. When the value of α, β and C_1 is increased 50% independently, the total cost TC_i will increase 19.3%, 17.9% and 21.8% respectively.

(2) Reducing the values of θ and C_2 will increase the value of T_{21}^* and decrease the value of T_{31}^*.

(3) Decreasing the value of β or increasing the values of C_1 and α will increase the length of the cycle time.

(4) As C_4 increases, T_{31} will decrease, while T_{21} will remain unchanged.

(5) The value of Q_1^* is very sensitive to α.

(6) When the stock-out is completely backlogged (i.e. R=1), TC_1 will be reduced by about 2.7%.

From Table 4, the following observations are made:

(1) The value of TC_1 is very sensitive to α, β and C_1. When the value of α, β and C_1 is increased 50% independently, the total cost TC_i will increase 39.9%, 21.8% and 21.8% respectively.

(2) In order to increase the service level rate, defined as T_{11}^* divided by $(T_{11}^*+T_{21}^*)$, the best method is to decrease the values of θ and C_2.

(3) The value of cycle time is very sensitive to the value of β.

(4) The value of T_{31}^* is very sensitive to the parameters of C_5, C_1 or α.

(5) When the stock-out is completely backlogged (i.e. R=1), the value of TC_1 increase about 1.1% due to the increase of the backlogged shortage cost.

Table 4. Sensitivity analysis when $d(t) = \alpha e^{\beta t}$

Parameter	% change in parameter	% change in Q_1^*	% change in T_{21}^*	% change in T_{31}^*	% change in TC_1^*
α	+50 -50	21.3 -50.0	0 0	-8.3 21.1	39.9 -40.4
β	+50 -50	4.4 -4.4	-11.3 29.3	-24.8 48.9	21.8 -27.0
C_0	+50 -50	0 0	0 0	10.5 -12.0	9.8 -10.2
C_1	+50 -50	0 -0.9	0 -6.3	24.1 -24.8	21.8 -24.0
θ	+50 -50	-1.8 2.6	-15.1 22.6	18.0 -27.1	4.1 -6.9
C_2	+50 -50	-1.8 4.4	-12.6 29.3	14.3 -28.6	4.2 -7.2
C_3	+50 -50	0 0	0 0	1.5 -1.5	1.3 -0.1
C_4	+50 -50	0 0	0 0	-12.8 18.8	2.1 -2.7
C_5	+50 -50	0 -0.9	0 -6.3	-26.3 39.1	7.4 -10.2
R	+25 -25	0 0	0 0	-6.8 9.0	1.1 -1.0

8. Comments on the sensitivity analysis

This study develops an inventory model with partial backordering for a stepwise production system to respond quick changing market for deteriorating items. The demand is assumed to increase linearly or exponentially with time. The results of the sensitivity analysis are given in the preceding section. From the analysis, one can manipulate the controllable parameters to control the production behavior of the system. Such a system can help management improve both the consistency and quality of the personnel decisions.

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