Impact of Back up Quantity Contract on Two-level Supply Chain: A Simulation Approach

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***Abstract*— In this new era of Supply chain coordination contracts are offered and accepted according to the varying need and specification of the industry and business in discussion. The contract variations arise according to the circumstances and adaptability by both manufacturer and retailer. An important challenge faced by the contracts being offered is the adaptability and robustness when demand observed is different from the forecast. Because of stochastic and uncertain demand, retailer faces lost sales and eventually loses revenue within the same horizon. This paper discusses a back up quantity contract for a single season in which retailer orders for one-shot inventory ordering. Manufacturer retains some part of the ordered inventory as backup and provides the units at first stage. If stock at retailer lags behind the demand, he gets the backup quantity otherwise he pays some agreed upon nominal price to manufacturer in case that inventory is not at all required. We assumed the case of single manufacturer and single retailer. If units are not required, the risk of holding inventory lies with manufacturer and salvaged at zero. The strategy is suitable for businesses having short seasonal products and high demand variability. We used Monte Carlo simulation for analyzing lost sales and supply chain profit scenario and used worst case distribution and normal distribution to validate the Pareto improving contractual relationship.**

***Keywords*—** *Demand uncertainty, Two-level Supply chain, Monte Carlo Simulation, Backup agreement*

1. Introduction

Retailer is often faced with a situation requiring flexibility in responding to uncertain demand over a finite selling horizon owing to the presence of lead time. One avenue of research concerning this issue is the designing of contracting relationship that facilitates both manufacturer and retailer. Usually buyer/ retailer wants to place an order as early as possible for getting early delivery and resolving demand uncertainty but most of the time in short selling season second order placement or option to increase or decrease order quantity may not be a feasible choice for the manufacturer. This paper investigates one such contracting arrangement in a supply chain setting where risk of inventory holding by the buyer is mitigated up to the backup inventory. Manufacturer proposes a contract that allows the retailer to mitigate inventory stocking risk by allowing him a backup quantity. The retailer has to order aggressively for a short selling season as he does not get second opportunity to order. Now there is a trade off between resolving demand uncertainty and risk of overstocking. Nevertheless inventory backup agreement tries to find a Pareto improving solution to minimize both lost sales and the inventory hold up. Although the proposed strategy can be used with most of the contractual agreements but specifically for this paper we used it to get two-fold impact:

1. For uncertain demand we assumed that the buyer has only the educated guess about demand mean and demand variance which he infers from the forecast of the similar items data that already exist. So from buyer orders conservatively but does not get second opportunity to order. We tackled this issue by using *Scarf's formula* [1] for worst possible demand distribution and estimated the probability of getting out of stock with other *IGFR* distributions like truncated normal distribution as well [2]. We found that the probability to be lower for worst possible demand distribution with the proposed strategy.
2. The buyer risk of holding inventory is mitigated as the manufacturer holds a backup quantity usually 20% to 30% from the ordered quantity. Buyer pays a nominal agreed price which is less than the holding cost for the buyer and gets these units once the

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accurate forecast becomes available. If the accurate demand is more than the units held by the retailer, manufacturer transfers the whole backup quantity to the retailer. Now if some units remained unsold, retailer has to salvage them at his own expense. If this backup is not all required, manufacturer bears the cost.

The need for such strategy is needed for the producers of trendy and high fashion products who sell for a short season Usually the trendy products are continuously changing according to the need of the customers and sometimes are novel as well as in the case of a new blockbuster movies. In cases of apparels, producers go for quick response strategy from operational perspective. Quick response techniques allow them to closely match supply and demand and are well studied in the literature as well. The theme of this paper is different from quick response systems in two respects. Firstly quick response systems may not be feasible at all for some apparel industries because of their manufacturing facilities located at fat geographical positions. Outsourcing facilities may not be able to provide quality product when lead time is much shorter. Secondly buyer may not view second faster production mode as cost efficient one and may try to produce more during the first run of production.

Fisher and Raman (1996), [4] and [5] all studied quick response strategies from operational perspective and their results showed improved yield for the firms by better matching of supply and demand. However fast fashion systems for creating trendy and highly fashionable products received very little attention. Despite the intense recent interest in reducing lead time, many a firms are attempting to focus on making trendy products without reducing production lead times because of logistical and cultural difficulties as mentioned by [6].

.The most related work to the one presented in this paper is by [7]. She presented a model in which manufacturer uses two modes of production being slow mode and fast mode respectively. Manufacturer optimizes the initial production quantity so that there is no extra quantity; retailer also adopts the same criteria. The main initiation of the paper is development of faster but typically expensive production mode for second run of products closer to the selling season. Another paper by [4] for fashion buying backup agreements is a retrospective study for two period models. They defined prior probabilities for pure demand distributions and the states for the retailer, he may be observing at that instance. Although the manufacturer sustains holding cost for the backup units and collects the penalty cost from the retailer company.

This strategy allows the retailer to order aggressively for one-shot inventory and along with the backup agreement; we investigated worst case distribution for ordering. Using worst case distribution to order allows retailer to improve the probability of not being running out of stock and at the same time minimizing the hold up inventory. The model however provides the suitable ordering pattern for a retailer who only holds the educated guess about the demand mean and forecast error in the form of variance. This is of course true for seasonal products with a very short life cycle and whose demand is uncertain till real time data reveals the accurate demand information. Many of the businesses use different strategies for reducing lead time and quick response strategies. However many a businesses could not afford the second order placing and such quick response strategies because of different reasons like cheaper manufacturing facilities are located at a different geographical locations or fast mode of production is not at all feasible for the cost structure offered by the buyer.

The scenario based model is information centric with a minimum level of stock being the main decision factor for demand update. Our model is especially suitable for businesses having short selling seasons and high demand uncertainties like seasonal industries and perishable units whose demand may have a sudden surge causing backlog for the retailer. In case there is surge in demand retailer automatically gets limited units up to the level of minimum inventory available with the manufacturer. Moreover the buyer gets fully benefited by using worst case distribution ordering policy and probability of getting out of stock decreases considerably. As this policy benefits manufacturer as well, this strategy is centered on improving the performance of the dyad. The performance of the dyad here has mainly two goals:

1. The dyad coordination that can be achieved by the contract.
2. The possibility of arbitrarily allocating to each party the proportion of dyad's gain derived from the gain.

The flexibility in allocating gains through the adjustment in the contract parameters is a key to its applicability as there is no control by any one of the parities over the other. However the actual distribution depends upon the bargaining power of the firms at the moment of negotiation for the contract parameters.

2. Literature Review

Information plays a vital role in supply chain contracts especially when dealing with one season. The newsvendor type setting is applicable for most of the real world scenarios, for example for fashion apparels when buyer does not have replenishment opportunity and he must order before demand uncertainty is resolved. Short selling seasons are common for trendy and high fashion apparel industries and the catalog customer always faces the dilemma of balancing the stock outs and the inventory holdup. The classical newsboy problem is a simple stock control model with stochastic demand. Main objective is to minimize the cost of oversupply and cost of under-stock simultaneously. Classic newsboy uses the same distribution to generate daily demand and is independent of history of demand and supply. The single period newsvendor is reflective of many a real life situations and is often used to aid decision making in the fashion and sports industries.

Diverse issues must be addressed in the analysis and design of contracts, from the formulation of the models, which may differ considerably according to the manufacturing setting and the assumptions defined. There is lot of literature that deals with capacity reservation decisions and coordination issues. [8] treated the coordination of expansion of a manufacturer supplier via a capacity reservation contract in the high-tech industries. They considered a single supplier with many alternatives of customers but at a lower profit rate. They did not consider alternative market and thus considered the closed system where supplier is the sole source of material.

[9] considered implicitly that the supplier would build sufficient capacity to always satisfy the buyer's full order commitment. Erkoc and Wu (2005) and [11] relaxed the forced compliance assumption and evaluate the impact that this would have on the performance of both the parties.

Other forms of commitment contracts exit and are more stringent as they require buyer to commit a minimum quantity at the beginning of the planning horizon as detailed by [12]. The buyer has to buy that minimum quantity at the start of the horizon and permitted to adjust upwards at a price premium for this class of contracts. Similarly manufacturer's response for delivery is also a deciding parameter of the contract. The author is of the view that such commitments reduce variance in the order process to some extent. There is also some work from the industry practices when supplier offers discount for a prior commitment as mentioned in [13], but the main focus is on dollar volume where retailer commits to buy a class of products with minimum dollar volume commitment and in return he gets price discount.

Bassok (1997a), [15] and [16] analyzed minimum purchase agreement with cumulated purchases over a multi-period horizon exceeding previously specified quantity.

There are also numbers of papers which deal with uncertain demand and forecast updates. [17] used Bayesian approach for estimating demand. [18] developed a myopic strategy using a parameterized adaptive demand process. [19] developed a dynamic programming framework for rolling horizon decision making with forecast update but bearing some additional cost. They developed a stochastic production problem requiring forecast window and optimal production quantity in each period. On the contrary, our strategy is applicable for a short single season style and fashionable products whose forecasted demand is nothing more than an educated guess by experts. Above mentioned research does not account for a single season with highly uncertain demand. Moreover [4], is quite similar to the strategy proposed as their strategy is based on backup agreement but they assume prior probabilities based on historical data and update the demand forecast on its basis. They also assume returned quantities from one horizon to other. We never assumed a rolling horizon and holding of quantity from one horizon to other. We provided Monte Carlo simulation results, [20] for back up utilization with IGFR distributions. For one-shot inventory model we define the back up utilization rather than dynamic rolling horizon model.

In marketing and economics wholesale price has been modeled to depict influence on supply contracts. For instance [21] considered the role of pricing and service commitments in achieving better channel coordination when retailer has better information about the market demand. Our model assumes fixed price and the main decision is the one shot ordering quantity that decreases the probability of lost sales and at the same time increases the chance inventory availability when required.

Customers' stochastic demand when realized has drastic impact on overall supply chain profit. Retailer would prefer revised ordering from forecasting to the epoch when customers' demand is realized or when more accurate forecast is available. We analyzed through simulation the impact of back up quantity for trendy and highly fashion items and showed that back up can be a profitable option for manufacturer as it increases service level and profits simultaneously especially when buyer ordering pattern is based on estimated mean and variance.

3. Model Setup

At the time of contracting, manufacturer reserves capacity before the demand is realized. Being a catalogue customer of style products, retailer ordering quantity is generally less that the capacity reserved.. This is a well known fixed price contract inefficiency scenario. The chaos created by uncertain demand is minimized by allowing minimum units holding to retailer. Manufacturer offers such back up for mitigating retailer's inventory risk. The minimum stock level δ is known to the manufacturer and the retailer and at first epoch, retailer informs manufacturer about the estimated demand forecast which follows worst cast distribution when retailer only has the knowledge about the mean and variance of the demand. Customer demand follows i.i.d. Retailer has to satisfy maximum demand by one-shot inventory decision and for that manufacturer mitigates the risk by offering a backup quantity. Retailer pays an agreed nominal cost for the backup and will get it in full when $q-d<\delta$. Once units acquired, retailer cannot return the overstock to the manufacturer and has to salvage at his own expense. On the other hand if backup quantity is never required, the loss is for manufacturer and it is salvaged at zero.

In this information centric model, there is single manufacturer and single retailer setting and both are aware of δ and after the first epoch if q-d is less than δ, retailer is permitted to get additional units up to δ. In case manufacturer is unable to fulfill the demand fully, he replenishes the partial order up to the backup quantity. Manufacturer only has forced compliance to fulfill the retailer first ordering.

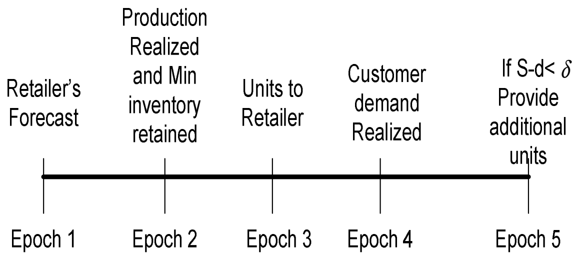

Figure 1. Decision Epochs for the Model

The model is based on better information flow for improving the efficiency of the supply chain**.** Although δ can be better estimated using infinite perturbation analysis [22], we chose the value arbitrarily for different simulation runs in order to identify the impact.

3.1 Basic setting

The manufacturer charges whole sale price *w* and reserves capacity before demand is realized. . The production cost is c; retailer charges a retail price *P* and units can be salvaged for *v***.** The Optimal quantity is *q*; cost of under stocking is given by $C_u = P - w$ and the cost of overstocking is $C_o = w - \upsilon$. The main difference being that manufacturer does not hold the backup quantity for next season as it becomes obsolete. Retailer's objective function is

$$\Pi_R = (P-\upsilon)S(q_1) - (w-\upsilon)q_1 + (P-\upsilon)E(q_2)$$
$$-wE(q_2) + \upsilon q_2$$

Manufacturer maximizes his profit function which is:

$$\Pi_m = (w-c)q_1 - cq_2 + wE(q_2)$$

Where

$$S(q_1) = \int_0^{q_1} (q_1 - x)\, fxdx \text{ and}$$

$$E(q_2) = \int_{q_1}^{q_2} fxdx$$

$E(q_2)$ is an indicator function for expected back up quantity utilized by the retailer. We used Monte Carlo simulation to observe the ordering pattern impact on probability of lost sales and supply chain profits. For one-shot inventory decision dynamic program may not pose a better strategy for two important reasons:

Demand is highly uncertain for short lived; trendy items and buyer can not update demand forecast for shelf-constraint items.

2nd mode of faster production practically may not be feasible for such small reorders. The ordering pattern of retailer follows **worst case distribution** given by [1] as he only assumes demand mean and variance.

$$Q^* = \mu + \frac{\sigma}{2}\left[\left(\frac{m}{d}\right)^{\frac{1}{2}} - \left(\frac{d}{m}\right)^{\frac{1}{2}}\right]$$

Where

P = (1+m) w > w Retail price

υ = (1-d) w < w Salvage value for buyer

We can show that for fixed price contracts, manufacturer offers more capacity than what buyer orders under worst case distribution.. This is essentially a case in one period ordering decision. Let the manufacturer decides production level in the absence of base contract as

$$\Pi_m = (w-c)(k - \int_0^k Fxdx) - c_k k$$

k is the capacity reserved at the time of negotiation and c_k is the capacity cost per unit. From FOC we can get optimal capacity level as:

$$k^* = F^{-1}\left(1 - \frac{c_k k}{w-c}\right)$$

We analyzed truncated normal distribution as well for comparative static for better insights.

Lemma 1:

The cost function will remain optimal if for the given contract

$$0 \le E[q_2]w \le wq_2$$

See appendix for Proof.

Assuming the manufacturer cost is the same for every extra unit.

Following proposition can be concluded from lemma 1:

Proposition 1 *Under the proposed strategy, expected cost of additional units is same as for second production run with no extra cost for lead time reduction. The cost of these additional units never exceeds the cost for expedited second production run.*

Gain from additional units is more when compared with fast production mode. The overall supply chain cost is lower than the fast production mode. It is always profitable for the manufacturer to produce in one shot than to reserve capacity or using fast mode of production.

Important thing in proposition 1 is about the willingness of manufacturer to produce in one go as cost for him is also lower than using fast production mode or by reserving capacity. According to lemma 3 of [7], manufacturer prefers to produce maximum of demand or the quantity ordered by the retailer. In our case retailer does not infer the risk of inventory holding when orders according to the proposed strategy, so manufacturer is better off by producing the ordered quantity.

Lemma 2:

As long as the additional units required $E[q_2]$ are positive; profit for both the parties will be maximizing and is maximum when $E[q_2] \le \delta$.

See appendix for Proof.

Following holds true according to Lemma 2:

Proposition 2 *The proposed strategy is Pareto optimal when buyer orders aggressively under the policy assuming demand follows i.i.d.*

Following is always true:

The profit for both the manufacturer and the retailer increases.

The overall supply chain profit increases.

The Likelihood of backordering decreases with increase in upper bound of gain for both the parties.

Whenever the units transferred to the retailer are more than the mean demand, both parties make more profit than the traditional setting.

From Lemma 2 we can infer that under one-shot inventory decision, retailer and the manufacturer both gets benefited when retailer orders using the proposed strategy. However it is important to note that from retailer‘s point of view ordering more than what proposed strategy suggests may not be feasible even for retailer as he has to take backup as a whole and not partially. Point 4 shows the interesting fact that whenever *IGFR* distribution represents the demand distribution, the proposed policy becomes Pareto optimal even when the quantity transferred is more than the mean of demand. For analysis and simplicity we assume that retailer can salvage units at half price.

4. Comparative Static and Analysis

We simulated the proposed model and compared it with traditional wholesale setting when retailer uses truncated normal distribution for ordering**.** We assumed that salvage units carry some value for retailer. Retailer orders under the backup agreement using worst case distribution for one-shot inventory decision. For numerical study, retailer’s demand mean is 2500 units with standard deviation of 150. Here we assumed higher demand variance because of uncertainty. We used Herbert Scarf (1958) order quantity closed form formula. The lesser the demand variance, the more confidence buyer puts on the estimated mean and orders more conservatively. On the other hand manufacturer reserves capacity and allocates resources according to the ordered quantity. Data collected from one such manufacturing facility suggests that to meet forced compliance, they deploy up to 5% more resources. Manufacturer informs the retailer about the total backup quantity which may be a little more than what is ordered. As we try to make lost sales as few as possible, these extra units are mostly helpful in making more profit.

Manufacturer’s production is modeled as normally distributed with mean 2825 and standard deviation of 50 units. Customer’s demand is i.i.d with mean 2500. Monte Carlo simulation was conducted for 10,000 runs. We perturbed the values for δ from 5% to 30% for both the models and final results are summarized in table 1.

Table 1. Cost and Profit values for different δ

Min Inventory Level '%'	Overall Supply chain Cost '$'	Overall Supply chain Profit '$'	%age increase in Cost	%age increase in Profit
Proposed				
5	4193	21038	10.57	7.76
10	4151	20680	9.66	5.93
20	4075	19764	7.98	2.22
30	4016	19263	6.62	-0.73
Existing				
5	3750	19405		
10	3750	19453		
20	3750	19325		
30	3750	19403		

Note: The values in the table are approximate figures as for every simulation run the values may change marginally.

For us the most plausible value for δ was 5% to 20% and it depends upon the demand variance and production rate as well.

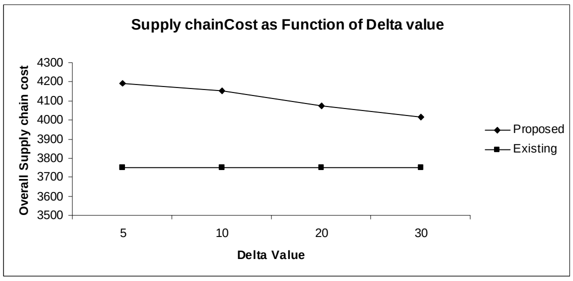

Figure 2. Overall supply chain cost as a function of δ value

From figure 2 it can be easily inferred that δ values are very important in knowing the point above which new setting is optimal. Simulation results confer that 5% and above till 20% δ makes the proposed setting optimal as it causes a substantial increase in profit as depicted in table 1. However firms should determine the optimal backup quantity based on some historical data.

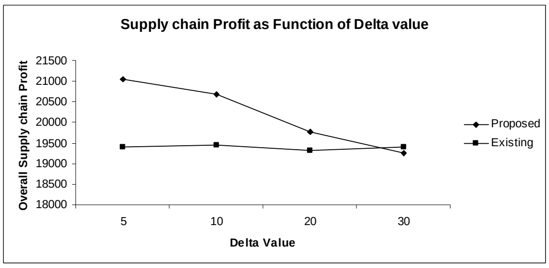

Figure 3. Overall supply chain Profit as a function of δ value

4.1 Impact of Increasing demand variance

For uncertain demand, demand variance plays an important part and especially when data is not enough to ascertain the form of the demand. Usually buyer has to often work with guess-estimates of the demand mean and the forecast error or the standard deviation. For analysis purpose we also investigated the effect of ordering with demand by truncated normal distribution and compared the likelihood of backordering for both cases

We summarized results for different demand standard deviations in Table 2

Table 2. Effect of Different Demand STDs on supply chain profit and Cost

Quantity Ordered	Standard Deviation of Demand	Demand Rate	Proposed Model Total Cost	Proposed Model Total Profit	Existing Model Total Cost	Existing Model Total Profit
2700	200	2500	4152	20704	3750	19460
2700	150	2500	4151	20617	3750	19400
2700	100	2500	4149	20499	3750	19323

The increase in overall supply chain profit is 2.5% or above for all the feasible cases which suggests that for the given policy minimum inventory level is a deciding factor rather than production rates alone. So perturbation in inventory level retained is significant for the optimality of the given strategy.

We also want to check the likelihood of profit increase for the model. This gives insight to the contracting parties about the real scenario they will be facing under the given assumptions and their chances of getting higher supply chain profits. The key performance indicators for the contract efficiency are reduction in the likelihood of backordering, increase in the bound of achievable gains and utilization of the backup quantity.

Observation 1:

It can easily be verified from figures 4 and 5 and achievable profit range increases for the proposed setting and more likelihood of profit is from 25000 to 29000 monetary units. Whereas for the traditional setting, the upper bound are around 23000 monetary units. Profit for proposed model is normally distributed in the above said range. It shows the Pareto optimal scenario when the buyer requires additional units.

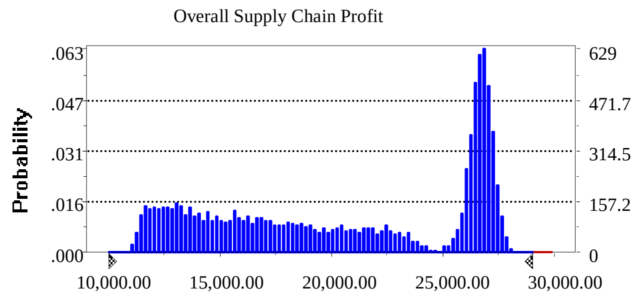

Figure 4. Probability of overall profit Lower and Upper Bound (Proposed Model)

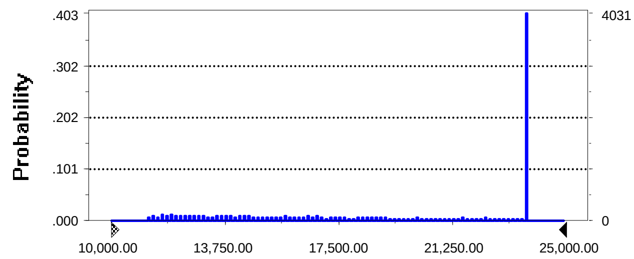

Figure 5. Probability of overall profit Lower and Upper Bound (Traditional setting)

Now if we compare the proposed model with the monopolistic setting under stochastic demand, we can see an interesting fact that our model is near optimal for the supply chain. The monopolistic profit is normally distributed from 25000 to 30000 with the mean of around 22500 which is comparable to our model mean of around 20620 monetary units.

Observation 2:

We observed that the likelihood of overstock decreased considerably when proposed model was used with worst case distribution ordering. The probability of backup utilization was 69% as compared to normally distributed ordering policy with 62%. Both the policies reduced likelihood of backordering significantly when compared to traditional setting which was around 55%.

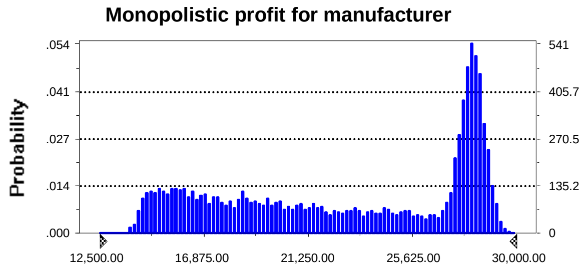

Figure 6. Monopolistic profits for manufacturer (lower and upper bounds)

Table 3. Effect of Different Demand STDs on Retailer's and Manufacturer's Profits

Production Rate(STD)	Retailer's Profit (Proposed Model)	Manufacturer's Profit (Proposed Model)	Retailer's Profit (Existing Model)	Manufacturer's Profit (Existing Model)
2825(50)	7212	13691	6819	12640
2825(50)	7174	13443	6792	12607
2825(50)	7123	13376	6758	12565

5. Concluding Remarks

We have examined the implications surrounding the decision of one time ordering by the buyer when demand is uncertain because of trendier and highly fashionable product which becomes obsolete in a single season. Our model captures the key trade-off between the risk of overstock and running out of inventory during the season. An early commitment by the buyer ensures that he will receive the ordered quantity and eliminates all the risks of the manufacturer. However this early commitment limits the buyer to respond latter if he receives better forecast. We characterized the back up agreement with an ordering policy given by *Scarf* and found a balance between overstock and backordering. The given scenario is of practical importance and can easily be implemented in industries having short product life cycles and high demand variability. For such short cycle trendier products it is not always feasible to go for fast mode of production. We found the Pareto optimal region for the proposed setting. The proposed setting reduces the likelihood of lost sales and at the same time improves supply chain gains by better utilization of backup quantity. We further examined the model with truncated normally distributed demand and found that both the ordering policies with backup precedes the traditional setting in terms of profit gain and reducing likelihood of lost sales and overstocking. The explicit nature of the analysis enables us to find the critical values for contract parameters like additional units, minimum inventory value, and overall supply chain cost and profit. Future research would be to extend this work for a scenario when there are high and low market segments and buyer has to order first for the low price segment and afterwards has to manage the salvaged units for the high price segment. Also production decision with multiple competing retailers would open a new avenue of research with such trendier products.

Appendix

Proof of Lemma 1:

Let $wq_1 + E[q_2]w \geq wq_1$ For q= q_1

We have

$$E[q_2]w \geq 0$$

Now we assume

$wq_1 + E[q_2]w \leq wq$ For q=q

We further assume that $wq = wq_1 + wq_2$

which implies:

$$wq_1 + E[q_2]w \leq wq_1 + wq_2$$

$$E[q_2]w \leq wq_2$$

This completes the proof.

As manufacturer cost is same for both contracts, the strategy will be optimal for the given finite upper bound for cost functions of the given new strategy. The given strategy remains optimal till the expected cost of additional units exceeds the cost for these units had they been provided for one time production mode.

Proof of Lemma 2:

Case 1: When q=q

$$(P-\upsilon)S(q_1) - (w-\upsilon)q_1 + (P-\upsilon)E(q_2) - wE(q_2) + \upsilon q_2$$
$$\geq (P-\upsilon)S(q_1) - (w-\upsilon)q_1$$

We have

$$0 \leq PE(q_2)$$

$$E[q_2] = \begin{bmatrix} q-\xi < 0 & \delta or \delta - \Delta \\ & 0 \\ q-\xi > \delta & \end{bmatrix}$$

So

$$E[q_2] = \delta$$

Or

$$E[q_2] = \delta - \Delta$$

Where Δ is the difference between the units produced and the demand realized

We have

$$0 \leq E[sy] \leq \delta$$

Case 2: When q= q_1

$$(P-\upsilon)S(q_1) - (w-\upsilon)q_1 + (P-\upsilon)E(q_2) - (w+c_n)E(q_2) + \upsilon q_2$$
$$\leq (P-\upsilon)S(q_1) - (w-\upsilon)q_1$$

Now we assume that q= $q_1 + \delta$

We can show that

$$(P-\upsilon)S(q_1) - (w-\upsilon)q_1 + (P-\upsilon)E(q_2)$$
$$\leq (P-\upsilon)S(q) + \upsilon q$$

$$(P-\upsilon)S(q_1) - (w-\upsilon)q_1 + (P-\upsilon)E(q_2)$$
$$\leq (P-\upsilon)S(q_1+\delta) + \upsilon q$$

And we can easily infer that

$$E[q_2] \leq \delta$$

This completes the Proof.

So as in case 1 the additional units are bounded above by minimum level of inventory retained which maximizes the revenue by having no salvage units. When difference of demand and units produced exceeds minimum inventory level retained, the overall profit of the strategy precedes the traditional contract setting.

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