Cooperative OEM Supply Chains with Multiple EOQ Delivery and Profit Sharing

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***Abstract*—This study develops a mathematical inventory model of a cooperative manufacturer-retailer supply chain for deteriorating items which experience continuous decrease in unit price. This inventory model, from the perspective of OEM (original equipment manufacturer) supplier, proposes an efficient EOQ (economic order quantity) replenishment policy for a single supplier-buyer partnership in which the retailer conducts just-in-time through a realization of multiple deliveries. In the system, the contract in the two-echelon supply chain aims to benefit both players through a profit-sharing mechanism. This study maximizes the total profit of the manufacturer using a heuristic search algorithm. A numerical example and sensitivity analysis illustrate the model and its application.**

Keywords*—Supply chain; OEM; Deterioration; Just-in-time; Profit sharing*

1. Introduction

Due to rapid technological innovation and global competitiveness, the selling price and the demand rate of hi-tech products usually decline significantly with time. The hi-tech products have the following characteristics: There are shorter product life cycle time, quicker responsive time, increasing need for globalization and massive customization. Moreover, the material purchase cost and the product market price are decreasing at a continuous rate. In some hi-tech industries such as computers and communication consumer's products, some component cost and product prices are declining at about 1% per week [1]. This implies that purchasing or selling one-week earlier or later will result in about 1% loss.

Ref. [2] was the first author to incorporate the concepts of inventory theory and economic price theory. Thereafter, several other authors investigated inventory models that took into account the interaction between the economic order quantity and pricing policies. Ref. [3] assumed compound increasing price and setup cost with inflation in a finite horizon. In addition, Ref. [4] explored two typical cases: supplier dominance with large production lot and shipment sizes, and buyer dominance with small frequent shipments. By using an EOQ-type inventory-modeling framework, Ref. [5] analyzed the effect of offering a lower price during stock out to compensate for customer waiting time while simultaneously solving both the optimal prices and lengths of in-stock and stock out periods. Ref. [6] considered a market demand affected by its price and the product had a deterioration rate following Weibull distribution. They proposed a compensation model developed to coordinate this two-echelon supply chain system to increase the product lifecycle and profits of the individuals as well as the system. Ref. [7] assumed that product deterioration is time-sensitive and developed an optimal integrated inventory policy for time-sensitive deteriorating products by taking into account a strategic alliance for a three-echelon supply chain. Ref. [8] extended EOQ models for deteriorating items and partial backlogging when demand is quadratic in time. They relaxed their assumptions of equal replenishment cycles and constant shortage lengths and showed that the optimal replenishment schedule exists uniquely, proved that the total relevant cost is a convex function of the number of replenishments and then proposed an algorithm to find the optimal replenishment number and schedule.

Deterioration, in general, is defined as decay, damage, spoilage, dryness, vaporization, etc., of products. Ongoing deteriorating inventory has been studied by several researchers in recent decades. Ref. [9] were the first authors to consider the ongoing deterioration of inventory, and they developed an EOQ model for items with an exponentially decaying inventory. Ref. [10] proposed a deteriorating production-inventory model with a Weibull distribution and permissible shortage. Ref. [11] proposed an exponentially

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deteriorating model that considered prices and production levels. An exponentially deteriorating production-inventory model with permissible shortage was presented by [12]. Researchers such as [13] and [14] assumed either instantaneous or finite production rates with various assumptions regarding the patterns of deterioration. Ref. [15] presented a review of the advances of the deteriorating inventory literature since the early 1990s and then classified it into three categories. Ref. [16] gave a comprehensive literature review of models for inventory control with deteriorating items that have been published since the review of [15].

This study focuses on a fashion hi-tech product, where the product price and inventory usually decline with time. We propose an optimal production/replenishment model for a cooperative two-echelon supply chain, where the retailer is the price-leader and the manufacturer is the follower. The retailer in turn shares his profits with the manufacturer to compensate for any losses the manufacturer might have incurred. The objective is to maximize the total profit of the supply chain. A search algorithm has also been developed to obtain optimal life cycle time. The rest of this paper is organized as follows: Section 2 introduces a profit-sharing model in the two-echelon cooperated supply chain. Section 3 presents a solution procedure to find the optimal value. Section 4 presents a numerical example to illustrate the proposed model. Section 5 utilizes a sensitivity analysis of the proposed model. Section 6 we have a discussion for the above example. Finally, Section 7 comprises the conclusion.

2. Model Development

Figure 1 shows how both parties in the two-echelon supply chain behave in a strategic cooperative supply chain. In order to reduce carrying inventory, an EOQ policy is utilized for replenishment and to meet the retailer's requests, JIT deliveries and multiple shipments is used. At the end of the manufacturer's non-profit period, the retailer will implement the profit-sharing policy to make up for the manufacturer's losses.

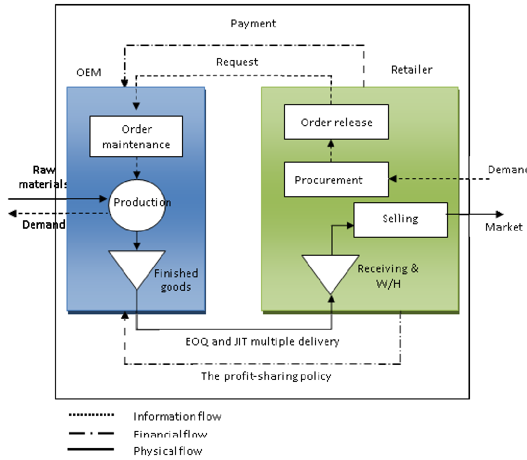

Figure 1. The behavior of two-echelon cooperated supply chain

THE following assumptions are used in the model development:

(1) A single product is considered.
(2) No shortages are allowed.
(3) There is no constraint in space, capacity or capital.
(4) Production rate can be changed.
(5) The first time interval in the two-echelon supply chain system, t_0, is given.
(6) For the retailer is a price-leader, he has a deal with the manufacturer to reduce the purchasing price with response to the selling price decrease.
(7) The multiple EOQ delivery is used with JIT policy between manufacturer and retailer.
(8) The rate of replenishment in the retailer is instantaneous.
(9) No replacement of deteriorated items is allowed.
(10) The deterioration rate is modeled by Weibull distribution. The two-parameter Weibull density function from [17] is
(11) The retailer shares his profit in the manufacturer's non-profit period.

The following notation is used:

L_R The unit holding cost per unit time for retailer

θ_R The deterioration cost per unit time for retailer

S_R The order cost per order for retailer

$\pi_i^R(t)$ The unit selling price of retailer at the time t during the i^{th} period, $\pi_i^R(t) = \pi_1^R \cdot e^{-\mu_R t}$, where μ_R is the decline rate of the selling price. π_i^R is the unit selling price of retailer at the i^{th} period beginning and π_1^R is given. (see Figure 3)

$D_i(t)$ The demands function at time t during the i^{th} period, $D_i(\mathrm{t}) = A - B \cdot \pi_1^R \cdot e^{-\mu_R t}$ where A is the scale parameter and B is the sensitive parameter.

η The manufacturing cost per unit (including material cost) for manufacturer

L_M The unit holding cost per unit time for manufacturer

θ_M The deterioration cost per unit for manufacturer

S_M The setup cost per setup for manufacturer

$\pi_i^M(t)$ The unit OEM fee of the manufacturer at time t during the i^{th} period, $\pi_i^M(t) = \pi_1^M \cdot e^{-\mu_M t}$, where μ_M is the decline rate of manufacturer's OEM fee and $\mu_M \le \mu_R$ (the proof shown in Appendix A). π_i^M is the unit OEM fee at the i^{th} period beginning and π_1^M is given. (see Figure 3)

ω The ratio of the retailer's profit share

q_i The orders quantity for the i^{th} period

p_i The production rate of the manufacturer during the i^{th} period

t_0 The first time interval in the two-echelon supply chain system

t_i The time interval in the i^{th} period

Figure 2 depicts the behavior of inventory system along time for the both players.

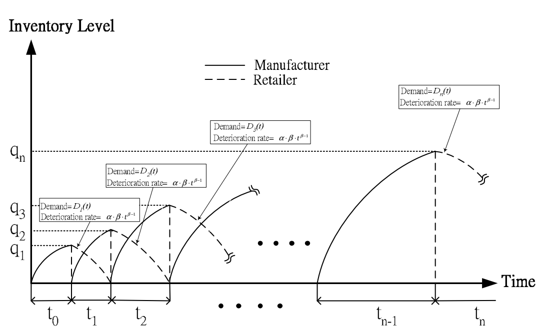

Figure 2. Inventory levels of the Manufacturer and the Retailer.

2.1 Modeling Retailer Behavior

The replenishment lot-size and selling price are reviewed periodically by the retailer at a sequence time t_i, i=1, 2, 3,.., n. At the beginning of each period, a decision is made on the selling price and the lot size q_i delivered by the manufacturer. Within each period, the change in the retailer's inventory level during an infinitesimal time, dt, is a function of the deterioration rate $\alpha\beta t^{\beta-1}$, the demand rate $D_i(t)$, and the inventory level of the replenishment period $I_i^R(t)$.

The demand rate, $D_i(t)$, is assumed to be a linear function of the unit selling price, $\pi_i^R(t)$. Furthermore, $\pi_i^R(t)$ exponentially declines with time t, where $\pi_i^R(t) = \pi_i^R \cdot e^{-\mu_R t}$, $i = 1,2,3,..,n,\ \ 0 \le t \le t_i$. It is straightforward to derive that the original retailer's selling price for i^{th} replenishment period as π_1^R is given and $\pi_i^R = \pi_1^R e^{-\mu_R \sum_{k=1}^{i-1} t_k}$, $i = 2,3,...,n$. Consequently, the demand rate of i^{th} replenishment period is: $D_i = D_i(\mathrm{t}) = A - B\pi_i^R e^{-\mu_R t},\ \ 0 \le t \le t_i$.

The inventory level over the ith period [0, ti] is formulated as

$$\frac{dI_i^R(t)}{dt} = -\left(A - B\pi_i^R e^{-\mu_R \cdot t}\right) - \alpha \cdot \beta \cdot t^{\beta-1} I_i^R(t),\ \ 0 \le t \le t_i,\ \ i = 1,2,3,..,n \qquad (1)$$

From the above differential equation, after adjusting for the constant of integration with various boundary conditions: the differential equation become:

$$I_i^R(t) = \left(-A + B\pi_i^R\right)\left((t - t_i)(1 - \alpha t^\beta) + \frac{\alpha(t^{\beta+1} - t_i^{\beta+1})}{\beta+1}\right) - \pi_i^R \cdot B \frac{\mu_R(t^2 - t_i^2)}{2},$$
$$0 \le t \le t_i,\ \ i = 1,2,3,..,n \qquad (2)$$

From (2) and , the ordering lot size of the retailer can be derived as

$$q_i = \left(A - B\pi_i^R\right)\left(t_i + \frac{\alpha t_i^{\beta+1}}{\beta+1}\right) + \frac{B\mu_R \pi_i^R t_i^2}{2},\ \ i = 1,2,3,..,n \qquad (3)$$

The revenue during the i^{th} period (denoted by Rv_i^R) is

$$\begin{aligned} Rv_i^R &= \int_0^{t_i} \pi_i^R(t)\cdot D_i(t)\cdot dt \\ &= \int_0^{t_i} \pi_i^R \cdot e^{-\mu_R\cdot t}\cdot\left(A - B\cdot\pi_i^R\cdot e^{-\mu_R\cdot t}\right)dt \\ &= \pi_i^R\int_0^{t_i}\left(A\cdot e^{-\mu_R\cdot t} - B\cdot\pi_i^R\cdot e^{-2\mu_R\cdot t}\right)dt \\ &= \left(\frac{\pi_i^R}{2\mu_r}\right)\left(-2A\cdot e^{-\mu_R\cdot t_i} + B\cdot\pi_i^R\cdot e^{-2\mu_R\cdot t_i} + 2A - B\cdot\pi_i^R\right) \end{aligned} \tag{4}$$

The ordering or replenishment cost during the i^{th} period is Or_R. The holding cost during the i^{th} period (denoted by Hd_i^R) is

$$\begin{aligned} Hd_i^R &= \int_0^{t_i} L_R\cdot I_i^R(t)\cdot dt = L_R\int_0^{t_i} I_i^R(t)dt \\ &= L_R\left(\left(A - B\pi_i^R\right)\left(\frac{t_i^2}{2} + \frac{\alpha\beta t_i^{\beta+2}}{(\beta+1)(\beta+2)}\right) + \frac{B\cdot\mu_R\cdot\pi_i^R\cdot t_i^3}{3}\right) \end{aligned} \tag{5}$$

The unit OEM fee of the manufacturer at time t during the i^{th} period, $\pi_i^M(t)$, exponentially declines with time, t, as depicted in Figure 3. Where π_1^M is given and $\pi_i^M(t) = \pi_i^M\cdot e^{-\mu_M\, t}$, $i = 1,2,3,..,n,\ \ 0 \le t \le t_i$, the original retailer's selling price for i^{th} replenishment period is $\pi_i^M = \pi_1^M e^{-\mu_M\sum_{k=1}^{i-1} t_k}$, $i = 2,3,...,n$. Thus, the purchase cost for the i^{th} period (denoted by Pu_i^R) is

$$Pu_i^R = \pi_i^M\cdot q_i,\ i = 1,2,3,..,n \tag{6}$$

The retailer's profit per unit time during each replenishing period, $\Pr_i^R$, is constructed by the following formula:
Profit =Revenue- Ordering cost - Holding cost - Deteriorating cost - Purchase cost.

By computing these terms, $\Pr_i^R$ can be obtained as:

$$\Pr_i^R(t_i) = \left(Rv_i^R - Or_R - Hd_i^R - De_i^R - Pu_i^R\right)\cdot\frac{1}{t_i} \tag{7}$$

To maximize the retailer's net profit by taking the first derivative of $\Pr_i^R(t_i)$ with respect to t_i setting the result to zero, one has $\partial\Pr_i^R/\partial t_i = 0$. The optimal time interval t_i^* can be solved by Maple. Since $\Pr_i^R$ is a very complicated function due to high-power expression of the exponential function, a graphical representation showing the concave of the $\Pr_i^R$ function is given in Figure 3. By taking the second derivative, one has $\partial^2\Pr_i^R/\partial t_i^{\,2} < 0$ (eg. $\partial^2\Pr_1^R/\partial t_1^{\,2} =$ -150467<0).

The above analysis proves that the function $\Pr_i^R(t_i)$ is strictly concave.

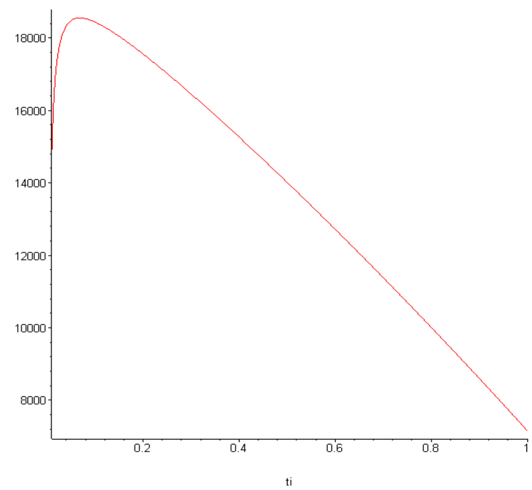

Figure 3. Graphical representation of a concavity $\Pr_1^R$ where i = 1, $\mu_M = \mu_R = 0.4$)

2.2 Modeling Manufacturer Behavior

Assumed the first time interval in the two-echelon supply chain system, t_0, is given, the inventory level over the i^{th} period [0, t_i] is formulated as

$$\frac{dI_i^M(t)}{dt} = p_i - \alpha\cdot\beta\cdot t_i^{\beta-1}\cdot I_i^M(t),\ \ 0 \le t \le t_i,\ \ i = 0,1,2,..,n \tag{8}$$

From the above differential equation, after adjusting for the constant of integration with various boundary conditions: $I_i^M(0) = 0,\ I_i^M(t_i) = q_{i+1},\ \ i = 0,1,2,..,n,$ the differential equation become:

$$I_i^M(t) = p_i\left(t_i + \frac{\alpha t_i^{\beta+1}}{\beta+1}\right)\left(1 - \alpha t^\beta\right),\ \ 0 \le t \le t_i \tag{9}$$

From (9) and $I_i^M(t_i) = q_{i+1}$, the production rate of the manufacturer can be derived as

$$p_i = \frac{q_{i+1}}{\left(1 - \alpha t_i^\beta\right)\left(t_i + \dfrac{\alpha t_i^{\beta+1}}{\beta+1}\right)},\ \ i = 0,1,2,..,n \tag{10}$$

The revenue at the i^{th} period end is

$$Rv_i^M = \pi_i^M\cdot q_{i+1},\ \ i = 0,1,2,..,n \tag{11}$$

The setup cost during the whole product life cycle is O_M.

The manufacturing cost during the i^{th} period (denoted by Pd_i^M) is

$$Pd_i^M = \eta \cdot (q_{i+1} + \Delta q_{i+1}), \quad i = 0,1,2,..,n \qquad (12)$$

where Δq_{i+1} is the deteriorating quantity during the production cycle t_i. From (3), one has

$$\Delta q_{i+1} = p_i \cdot t_i - q_{i+1}, \quad i = 0,1,..,n \qquad (13)$$

The holding cost during the i^{th} period is

$$Hd_i^M = \int_0^{t_i} L_M \cdot I_i^M(t) \cdot dt = L_M \cdot P_i \left(t_i^2 - \left(\frac{\alpha t_i^{\beta+1}}{\beta+1} \right)^2 \right), \quad i = 0,1,..,n \qquad (14)$$

The manufacturer's profit per unit time during each production period, Pr_i^M, is constructed by the following formula:

Profit = Revenue- Setup cost - Manufacturing cost - Holding cost- Deteriorating cost.

By computing these terms, Pr_i^M can be obtained as:

$$\mathrm{Pr}_i^M(t_i) = \left(Rv_i^M - S_M - Pd_i^M - Hd_i^M - De_i^M \right) \cdot \frac{1}{t_i}, \quad i = 0,1,..,n \qquad (15)$$

2.3 The Profit-Sharing Policy

The profit-sharing policy depends on how much profits the retailer shares with the manufacturer by operating on the parameter ω. Therefore, $\omega \cdot \mathrm{Pr}_i^{R^*}$ is the manufacturer's extra profit gained from the retailer during the i^{th} period. Let $\mathrm{Pr}_i^{M'}$ be the sum of $\mathrm{Pr}_i^{M'}$ and $\omega \cdot \mathrm{Pr}_i^{R^*}$.

The optimization problem can be formulated as

$$\text{Maximize} \quad \mathrm{TP_M}(t_i, n) = \sum_{i=1}^{n} \mathrm{Pr}_i^{M'} \qquad (16)$$

$$\text{Subject to} : \quad \mathrm{Pr}_i^{M'} = \mathrm{Pr}_i^M + \omega \cdot \mathrm{Pr}_i^{R^*} \qquad (16a)$$

$$0 \le \omega \le 1 \qquad (16b)$$

3. Solution Procedure

The concave property of the $\mathrm{TP_M}$ function is examined. The following simple but efficient search algorithm is developed to derive the optimal value of t_i, q_i, p_i and n.

Search Algorithm

Step 1: start by $i=1$.

Step 2: Find the optimal period time in the i^{th} period, t_i^*, by Eq. (7).

Step 3: To computed the unit selling price $\pi_i^R(t_i^*)$ and OEM fee $\pi_i^M(t_i^*)$ at the time t_i^*.

Step 4: Substitute t_i^* and π_i^R into Eq. (3) to computed the optimal ordering lot size q_i^*.

Step 5: To compute the production rate p_i^* by Eq. (10).

Step 6: Substitute (t_i^*, q_i^*, p_i^*) into Equation (15) to compute the manufacturer's profits Pr_i^M.

Step 7: Check Pr_i^{R*} is less than zero? If yes, go to Step 9. Otherwise, go to Step 8.

Step 8: Check Pr_i^M is less than zero? If yes, use the ratio ω to share the retailer's profit to manufacturer. Otherwise, set $i=i+1$ and repeat step 2 to step 8.

Step 9: End.

Therefore, the corresponding product life cycle $\sum_{i=1}^{n} t_i$, the total profit of the manufacturer $\sum_{i=1}^{n} \mathrm{Pr}_i^M$ and the retailer $\sum_{i=1}^{n} \mathrm{Pr}_i^R$ can be derived respectively.

4. Numerical Example

The proposed model is illustrated using a numerical example extended from Yu et al. (2009). The related data from the retailer are as follows: $\pi_1^R =$ \$10/unit, $L_R =$ \$1.3/unit/year, $\theta_R =$ \$4/unit, $S_R =$ \$50/order, $\mu_R =$ 0.4, $A = 5000$, $B = 100$, α=0.15, β=1.5, $\omega = 0.05$. The related data from the manufacturer are as follows: $t_0 =$ 0.0861 year, $\eta =$ \$4.5/unit, $\pi_0^M =$ \$6/unit, $L_M =$ \$0.9/unit/year, $\theta_M =$ \$4/unit, $S_M =$ \$60/setup, $\mu_M = 0.4$.

As the profit share ratio is ω = 0.3, the optimal values with the profit-sharing policy are summarized in Table 1. It is derived as $\left\{ n^*, \sum q_i^*, \sum t_i^*, \sum \mathrm{Pr}_i^{R*}, \sum \mathrm{Pr}_i^M \right\}$ = {22 times, 6,672 units, 1.568 years, \$237,641, and \$80,314}. As the profit share ratio increases to ω = 0.5, the optimal values with the profit-sharing policy are summarized in Table 2. It is derived as $\left\{n^*, \sum q_i^*, \sum t_i^*, \sum \mathrm{Pr}_i^{R*}, \sum \mathrm{Pr}_i^{M}\right\}$ = {25 times, 7,704 units, 1.798 years, \$240,640, and \$92,451}. Consequently, as the profit share ratio increases, the performance including both the total profit and the product life cycle for manufacturer and retailer is improved. Furthermore, the profits with using the profit-sharing policy are more than that without using the profit-sharing policy as shown in Table 1 and Table 2. The related optimal solution is $\left\{n^*, \sum q_i^*, \sum t_i^*, \sum \mathrm{Pr}_i^{R*}, \sum \mathrm{Pr}_i^{M}\right\}$ = {17 times, 5013 units, 1.193 years, \$206,062, and \$72,583}.

Insert Table 1 and Table 2 here

5. Sensitivity Analysis

The optimal values of n, q_i, t_i, Pr_i^R and Pr_i^M for a fixed set of parameters Φ={ π_0^M, π_1^R, η, L_M, L_R, θ_R, θ_M, S_M, S_R, μ_M, μ_R, A, B, α, β, t_0, ω} are denoted respectively by n^*, $\sum q_i^*$, $\sum t_i^*$, Pr_i^{R*} and Pr_i^M. The changes in n^*, $\sum q_i^*$, $\sum t_i^*$, Pr_i^{R*} and Pr_i^M are considered when the parameters in the set Φ vary. The impact of the parameters in Φ on the manufacturer's total profit can be concluded as follows:

(1) The parameters π_0^M, θ_R, μ_R, A, α, ω are positive correlation with the change of the manufacturer's total profit. The change of the manufacturer's total profit is most sensitive to π_0^M and *A*. When they are decreased or increased by 17%, the percentage of the change tends to be over 65% as shown in the Table 3. It is slightly sensitive to the parameters μ_R and L_R.

(2) The change of the manufacturer's profit is least sensitive to the parameters θ_R and α, because the deteriorating cost ratio is very small compared to each player's cost. When these parameters increase by 20%, the value of change increases by less than 0.5%.

(3) The parameters π_1^R, η, L_M, L_R, S_M, S_R, μ_M, B, β have negative correlation with the change of the manufacturer's total profit. Most of them are the manufacturer's parameters. η is the most sensitive one, when the unit production decreases by 20%, the manufacturer's profit will tend to increase 13 times. But when the unit manufacturing cost increases more than 5%, there will be no profit for the manufacturer. The least sensitive parameters are θ_M andβ.

(4) When the decline rate of manufacturer's selling price μ_M decreases, the manufacturer's profit tends to increase. Conversely, when μ_M increases, the manufacturer's profit tends to decrease. The percentage of profit change is over 23% based on ±20% change of μ_M.

(5) The profit-sharing policy has much effect on the product lifecycle and the manufacturer's profits. Even though the value of ω is small, both of them increase fast as shown in Table 4.

(6) When the ratio of the retailer's profit share ω increases, the total profits for either one tend to increase. But when ω increases to 0.6, the total profits for the retailer tend to decrease due to too much profits shared to the manufacturer as shown in Table 4.

Insert Table 3 and Table 4 here

6. Discussion

The following conclusions are drawn from the above analysis and numerical example:

(1) Though the decline rate of the selling price is the same as that of the OEM fee, the manufacturer's profits decrease more quickly (see Table 1).

(2) Without using the profit-sharing policy, the product life cycle (*PLC*= $\sum_{i=1}^{n} t_i^*$) is 1.193 years. But when the profit-sharing policy is applied, the product life cycle extends to 1.568 years (an increase of 31.4%) as shown in the Table 1.

(3) After 1.568 years, the manufacturer will stop making profits even though the retailer continues to earn profits. The 1.568 years is the product life cycle (PLC) of the private brand item. Only when the retailer offers the larger profit-share ratio or OEM fee, the manufacturer does not follow their cost structure. As shown in the Table 1(ω=0.3) and Table 2(ω=0.5), the PLC increases from

1.568 years to 1.798 years (14.67%).

(4) In Table 1, with the profit-sharing policy (ω=0.3), the manufacturer's profit increases from $72,583 to $80,314 (10.65%) and the retailer's profit increases from $206,062 to $237,641 (15.32%). In Table 2, with the profit-sharing policy (ω=0.5), the manufacturer's profit increases to $92,451 (27.37%) and the retailer's profit increases to $240,640 (16.78%). It is obvious that the profit-sharing policy is better for the both players. Moreover, the cooperated policy is more beneficial to the retailer because the manufacturer always following the retailer's EOQ decision, the large ratio of the profit sharing should be considered to the manufacturer.

(5) When we compare the different profit sharing between μ_M and μ_R, we see that when $\mu_M = \mu_R$, the total joint profit, *TP*, is the smallest (see Table 5).

Insert Table 5 here

(6) By comparing the two policies, the benefits of profit-sharing policy are obvious as shown in Table 6 and Figure 4.

Insert Table 6 here

(7) Moreover, we provide insights on how to reduce the unit manufacturing cost, the selling price, the original pricing and the retailer's ordering cost in order to increase the product lifecycle and the manufacturer's and the retailer's profit.

Table 1. The numerical results without/with applying the profit-sharing policy ($\mu_M = \mu_R = 0.4$, ω=0.3)

Period	t_i	$\sum_{i=1}^{n} t_i^*$	q_i	$\Pr_i^{R^*}$	TP_R^*	$\Pr_i^M$	TP_M
(the following periods not using the profit-sharing policy)							
0	0.0861	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
1	0.0670	0.0670	269	14541	14541	6788	6788
2	0.0675	0.1345	272	14228	28769	8318	15106
3	0.0680	0.2025	276	13916	42685	7743	22849
4	0.0685	0.2710	280	13605	56290	7170	30019
5	0.0685	0.3395	282	13295	69585	6547	36566
6	0.0690	0.4085	285	12990	82575	6038	42604
7	0.0695	0.4780	289	12686	95261	5482	48086
8	0.0695	0.5475	291	12384	107645	4889	52975
9	0.0700	0.6175	294	12087	119732	4385	57360
10	0.0705	0.6880	298	11792	131524	2840	60200
11	0.0710	0.7590	302	11500	143024	3316	63516
12	0.0715	0.8305	305	11210	154234	2790	66306
13	0.0715	0.9020	307	10923	165157	2248	68554
14	0.0720	0.9740	310	10641	175798	1757	70311
15	0.0725	1.0465	314	10362	186160	1253	71564
16	0.0730	1.1195	318	10087	196247	755	72319
17	0.0735	1.1930	321	9815	206062	264	72583
(the following periods using the profit-sharing policy)							
18	0.0740	1.2670	325	6682	212744	2644	75227
19	0.0745	1.3415	328	6496	219240	2087	77314
20	0.0750	1.4165	332	6313	225553	1538	78852
21	0.0755	1.4920	335	6133	231686	997	79849
22	0.0760	1.5680	339	5955	237641	465	80314

Table 3. Sensitivity analysis of ρ_0^{M}

π_0^{M}	n_i^*	$\sum_{i=1}^{n} t_i^*$	$\sum_{i=1}^{n} q_i^*$	TP_R^*	TP_M
(without the profit-sharing policy)					
5.0	11	0.769	3,179	182,575	25,146
6.0	17	1.568	5,013	206,062	72,583
7.0	22	1.553	6,603	185,459	139,058
(with the profit-sharing policy)					
5.0	1	1.355	5,732	255,835	40,806
6.0	22	1.798	6,672	237,641	80,314
7.0	26	1.858	7,973	203,567	142,751

Table 2. The numerical results without/with applying the profit-sharing policy ($\mu_M = \mu_R = 0.4$, ω=0.5)

Period	t_i^*	$\sum_{i=1}^{n} t_i^*$	q_i	$\mathrm{Pr}_i^{\mathrm{R}^*}$	TP_R^*	Pr_i^M	TP_M
(the following periods not using the profit-sharing policy)							
0	0.0861	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
1	0.0670	0.0670	269	14541	14541	6788	6788
2	0.0675	0.1345	272	14228	28769	8318	15106
3	0.0680	0.2025	276	13916	42685	7743	22849
4	0.0685	0.2710	280	13605	56290	7170	30019
5	0.0685	0.3395	282	13295	69585	6547	36566
6	0.0690	0.4085	285	12990	82575	6038	42604
7	0.0695	0.4780	289	12686	95261	5482	48086
8	0.0695	0.5475	291	12384	107645	4889	52975
9	0.0700	0.6175	294	12087	119732	4385	57360
10	0.0705	0.6880	298	11792	131524	2840	60200
11	0.0710	0.7590	302	11500	143024	3316	63516
12	0.0715	0.8305	305	11210	154234	2790	66306
13	0.0715	0.9020	307	10923	165157	2248	68554
14	0.0720	0.9740	310	10641	175798	1757	70311
15	0.0725	1.0465	314	10362	186160	1253	71564
16	0.7300	1.1195	318	10087	196247	755	72319
17	0.0735	1.1930	321	9815	206062	264	72583
(the following periods using the profit-sharing policy)							
18	0.0740	1.2670	325	4773	210835	4553	77136
19	0.0745	1.3415	328	4640	215475	3943	81079
20	0.0750	1.4165	332	4509	219984	3342	84421
21	0.0755	1.4920	335	4381	224365	2750	87171
22	0.0760	1.5680	339	4254	228619	2167	89338
23	0.0765	1.6445	342	4129	232748	1593	90931
24	0.0765	1.7210	343	4006	236754	1044	91975
25	0.0770	1.7980	347	3886	240640	476	92451

Table 4. Sensitivity analysis of ω

ω	n_i^*	$\sum_{i=1}^{n} t_i^*$	$\sum_{i=1}^{n} q_i^*$	TP_R^*	TP_M
0.3	22	1.568	6,672	237,641	80,314
0.4	23	1.678	7,123	239,032	86,766
0.5	25	1.798	7,704	240,640	92,451
0.6	27	1.985	8,267	239,975	100,963
0.7	28	2.032	8,768	233,348	108,885

Table 5. Comparison of total joint profit for different relation of μ_M and μ_R (ω=0)

μ_M, μ_R	n_i^*	$\sum_{i=1}^{n} t_i^*$	$\sum_{i=1}^{n} q_i^*$	TP_R^*	TP_M	TP
$\mu_M < \mu_R$	31	2.245	9.741	203,731	146,025	349,756
$\mu_M = \mu_R$	17	1.193	5,013	206,062	72,583	278,645
$\mu_M < \mu_R$	15	1.243	5,116	236,287	63,150	299,437

Note: $TP_R^* = \sum_{i=1}^{n} \mathrm{Pr}_i^{R^*}$, $TP_M = \sum_{i=1}^{n} \mathrm{Pr}_i^{M'}$ and $TP = TP_R^* + TP_M$

Table 6. The benefits of applying the profit-sharing policy ($\mu_M = \mu_R$)

ω	n_i^*	$\sum_{i=1}^{n} t_i^*$	$\sum_{i=1}^{n} q_i^*$	TP_R^*	TP_M	TP	PJPC(%)
ω=0.0	17	1.193	5,013	206,062	72,583	278,645	NA
ω=0.3	22	1.568	6,672	237,641	80,314	317,955	14.11
ω=0.5	25	1.798	7,704	240,640	92,451	333,091	19.54
ω=0.7	30	1.908	8,854	243,236	102,767	346,003	24.17

Note: PJPC: Percentage of joint profit change = $\left\{ \mathrm{TP}(t_i, n) - \underset{\omega=0}{\mathrm{TP}}(t_i, n) \right\} / \underset{\omega=0}{\mathrm{TP}}(t_i, n)$

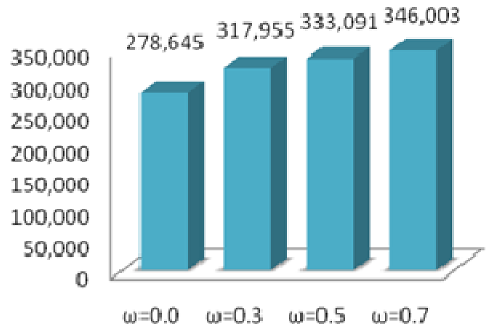

Figure 4. Comparison of total joint profit for different sharing ratio

7. Conclusions

This study develops an inventory replenishment model for a two-echelon cooperative supply chain with a varying deterioration rate and continuously decreasing selling price. We propose an optimal replenishment model with a dynamic production rate and multiple EOQ deliveries to the retailer. Through a numerical example and sensitivity analysis, we provide some insights for the cooperative supply chain. The strategy is shown to extend the product life cycle and increases profits for both parties. For a profit sharing ratio of 0.3, the life cycle is increased from 1.193 years to 1.568 years, and the manufacturer's profits are increased from $72,583 to $80,314, a 10.65% in profits from the non-cooperated policy. Moreover, we found that when the retailer's selling price increases, the market demand decreases accordingly. Consequently, the manufacturer's profits tend to decrease. The proposed model has the potential for application in a multi-echelon supply chain inventory system. Our model can be extended to other inventory systems. For example, one extension is to consider a dynamic production decision with *n*-echelon supply chain. Multi-item versions of the model would also be worth studying, as would models that consider constraints of space capacity and budgets.

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Appendix A

The unit selling price of retailer should be greater than the unit OEM fee of the manufacturer at the time t during the i^{th} period.

$$\pi_i^M(t) \leq \pi_i^R(t) \tag{A.1}$$

Assumed that the product lifecycle ends at time T_{end} and as $n=k$, one has

$$\pi_k^M(T_{end}) = \pi_0^M \exp(-\mu_M \cdot T_{end}) \quad \text{and}$$
$$\pi_k^R(T_{end}) = \pi_1^R \exp(-\mu_R \cdot T_{end}) \tag{A.2}$$

Substitute (A.2) into (A.1), we can show that

$$\frac{\pi_0^M}{\pi_1^R} \exp\left((\mu_R - \mu_M)T_{end}\right) \leq 1 \tag{A.3}$$

Therefore, the upper bound of the product lifecycle can be derived as:

$$T_{end} \leq \frac{1}{\mu_R - \mu_M} \cdot \ln\left(\frac{\pi_1^R}{\pi_0^M}\right) \tag{A.4}$$

For the assumption of $\pi_0^M \leq \pi_1^R$ and the end time is none zero, $0 < T_{end}$, we can show that

$$\mu_M < \mu_R \tag{A.5}$$