A Multi-objective Stochastic Programming Model for Order Quantity Allocation under Supply Uncertainty

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***Abstract*— As the source of the whole supply chain, purchasing and supply management plays a vital role in the survival and development of the enterprises. However, managers may meet diverse uncertainty due to different kinds of risks in supply chain. Sourcing decisions under uncertainty, especially the supplier selection and order quantity allocation, is of great significance for the managers. This paper considers the order quantity allocation problem from the perspective of manufacturers under supply uncertainty conditions. By taking the constraints of interval of purchasing quantity and minimum production batch into account, a multi-objective mixed-integer stochastic programming model considering uncertainty in both supply timing and quantity is presented. The model is converted into a linear programming model by transforming the stochastic constraints into deterministic equivalents,. An improved two-phase heuristic approach is proposed and its feasibility and efficiency is illustrated through a numerical example. Further, another numerical instance is conducted to evaluate the effects of the weight of each objective and uncertainty degree on the optimal ordering policy and to obtain some managerial insights for the decision-making of the manufacturers.**

Keywords*—Supply chain risk management, supply uncertainty, supplier selection, order quantity allocation, mixed-integer programming*

1. Introduction

Nowadays, supply chains are subject to different kinds of operational risks and natural risks. Manufacturers have to deal with the supply uncertainty, which might be due to uncertain capacity, quality or even strategic problems at the suppliers or other natural and manmade disasters. Assembly manufacturing requires high demand for the punctuality of the supply process. Selecting the most appropriate suppliers from supply base and allocating orders among the selected ones significantly reduce the total cost and improve comprehensive competitiveness of the company [14]. In uncertain environment, managers have to make a trade-off between low-price yet high-risk suppliers and relatively high-price yet reliable suppliers. Therefore, in current supply chain, the supplier selection and order quantity allocation problem is a complex stochastic problem [17].

It has seen a growing attention given in the field of decisions on the selection of suppliers with different capacity, price, service and quality level, and on assigning order quantity among them in recent years. Gurnani et al. considered the optimal order policies with both stochastic demand and uncertain delivery very early and then presented two heuristic strategies. They showed that the optimal inventory policy was order-up-to with identical order-up-to levels and if the inventory level was below a critical threshold, it was optimal to diversify and order from the joint supplier [8]. In the case of multiple sourcing, given multiple criteria and suppliers' capacity constraints, Ghodsypour et al. constructed a mixed-integer non-linear programming model, which took into account the total cost of logistics and then proposed an algorithm to solve it[6]. An EOQ model with numerous suppliers with random capacities in a continuous-review system was presented in Ref.[4] and computational results showed that when the optimal policy was applied, the number of unfulfilled order units from all suppliers must all be the same. They further obtained some characterization and properties for the uniform and exponential capacity cases. Based on Ghodsypour's research, Ekici pointed out two issues with their assumptions and discussed two different capacitated supplier settings. Then, he proposed a model that provided the same or a better solution [3]. However, the literatures above only considered the order policy in the case of single-material while this paper considers multiple kinds of materials.

Gurnani et al. modeled the procurement problem as a Nash game where the buyer had to allocate his order between two suppliers with different price and deliver ratio. Furthermore, they modeled the case in an asymmetric information scenario and showed that buyer benefits from the information asymmetry[9]. Considering a supply chain with one supplier and one

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retailer with random yield and uncertain demand, He et al. proposed several risk sharing contracts and compared the results of these contracts. Finally, the numerical experiments showed that yield uncertainty might enhance the supply chain performance and decrease the double marginalization effect[10], which was different from previous research. When considering a decentralized assembly system under uncertain demand and random supply yield, Guler et al. proposed two contracts and showed that suppliers coordinated the chain under forced compliance[7]. After that, another academic proposed a single-period ordering and uncertain delivery planning model so as to find a coordination mechanism that allows the producer-supplier system to perform as a centralized one[19]. A multi-objective supplier allocation model was proposed to help make decision about supplier selection under uncertainty environment and provide proactive mitigation strategies against disruptions [2]. However, the literature above focus more on the coordination of the supply chain without considering the issue of the kitting of the materials and maximize the profit of the manufacturer. Besides, Zhang et al. studied the supplier selection and order quantity allocation problem under uncertain demand, quantity discounts and fixed selection costs conditions. They proposed an optimal algorithm based on Bender's decomposition and conducted numerical experiments to show its efficiency and obtained some managerial insights[18]. The optimal allocation problem across the suppliers given the risk of supplier failures and contingency planning in the decision process was considered by Ruiz-Torres et al.[16]. A multi-objective model in which cost, quality and tardiness is was minimized under stochastic demand and price-dependent demand conditions was considered[5].

In this paper, we discuss issues related to multiple suppliers, random delivery, single-product assembled with multiple parts. Our problem is similar to that of Ghodsypour and O'Brien[6] since both of us discuss multiple sourcing with multiple criteria and with suppliers' capacity constraints, but the suppliers' capacity is stochastic in our model and the type number of material is over one. Another closely related study is by Bilsel and Ravindran[2]. A multi-objective chance constrained programming model was presented for supplier allocation under uncertainty, but we consider the tardiness time, which is a fuzzy value. A multiple objective mixed-integer stochastic programming model considering the purchasing quota interval and minimum production batch is constructed to select appropriate suppliers and to allocate the purchasing quota among them. To reach the satisfactory solution, a two stage heuristic algorithm is proposed here. Finally, we illustrate the effect of uncertainty degree and weight of each objective on optimal order allocation solution using a numerical example.

The paper is organized as follows. In Section 2, we present the basic assumptions, parameters and decision variables of the model and develop the multi-objective order allocation model with multiple suppliers, random delivery and multiple materials. Section 3 is devoted to transferring the chance-constrained constraints in the model into linear ones and to developing a two-phase approach to find the optimal order solution. The efficiency of the model and algorithm is illustrated in section 4 through a numerical experiment. Section 5 analyzes the effect of the degree of supply uncertainty and the value of weight of each objective on the optimal order allocation quantities using another instance. Section 6 concludes the paper and discusses extensions.

2. Order Quantity Allocation Model

2.1 Assumptions and Notations

The following assumptions are used in the model development:

- There is only one type of final product which is assembled by multiple materials.
- Each material could be obtained from multiple suppliers with different price, capacity and quality level.
- The manufacturer has some information about the uncertain capacity based on historical data.
- The quantity of materials purchased just meets the production demand for once.
- The quantity constraints and quality constraints must be fulfilled before regular production.

The following notation is used:

- Index

m : The m th material
n : The n th supplier

- Parameters

M : The number of the type of material
D : The quantity of demand
N_m : The number of supplier of m th material
p_{mn} : Unit price of m at supplier n
U_{mn} : Capacity at supplier n for material m
A_{mn} : Available quantity at supplier n for m
ε : Minimum ratio of production batch
S_m : Unit tardiness cost of material m
P : Total purchasing budget for all materials
t_{mn} : Tardiness time at supplier n for m
T_0 : Length of planning horizon
r_{mn} : Perfect rate at supplier n for material m
K_{mn} : Minimum order quantity at supplier n for m
η : Minimum accepted perfect rate
H_m : Unit holding cost for material m
α_{mn} : Percentage of deliver at supplier n for m
y_m : 1, if m meet the requirement of ε

- Decision variables

x_{mn} : 1, if q_{mn} more than 0 and 0 otherwise
q_{mn} : Order quantity allocated to supplier n for m

2.1 Formulation

In this model, it is considered that the manufacturer would like to choose the most appropriate suppliers and to allocate order quantity among them, whose price, perfect rate and reliability level are different. It is a multiple criteria decision-making problem and three objectives are considered in the model. The first objective is to minimize the total cost, which consists of purchasing cost, holding cost and tardiness cost. The second objective is to maximize the quantity of perfect materials. And the last one is to minimize the tardiness degree of the whole materials. Firstly, the purchasing cost can be formulated as:

$$C_{purchasing} = \sum_{m=1}^{M}\sum_{n=1}^{N_m} p_{mn} q_{mn} x_{mn} \tag{1}$$

Holding cost is composed of two terms due to manufacturer's requirement of minimum production batch. The tardiness time t_{mn} is a triangular fuzzy number, whose value is related to the tardiness quantity and supplier's capacity, that is $t_{mn} = (Q_{mn}^{min}, Q_{mn}^{mid}, Q_{mn}^{max})$, where $Q_{mn}^{min} = K_{mn}$, $Q_{mn}^{max} = q_{mn}$, $Q_{mn}^{mid} = (K_{mn} + q_{mn})/2$, and the membership function of t_{mn} is

$$\mu(t_{mn}) = \begin{cases} \lfloor T_0 (A_{mn} - Q_{mn}^{min}) / Q_{mn}^{mid} - Q_{mn}^{min} \rfloor, & Q_{mn}^{min} \le A_{mn} \le Q_{mn}^{mid}; \\ \lfloor T_0 (Q_{mn}^{max} - A_{mn}) / Q_{mn}^{max} - Q_{mn}^{mid} \rfloor, & Q_{mn}^{mid} \le A_{mn} \le Q_{mn}^{max}; \\ 0, & others \end{cases} \tag{2}$$

where $A_{mn}=q_{mn}\alpha_{mn}$,. Let $t_m=max\{t_{m1},t_{m2},\ldots,t_{mn}\}$, which indicates the maximum tardiness time for material m. Therefore, holding cost

$$C_{holding} = \sum_{m=1}^{M} H_m \{ y_m [t_m \sum_{n=1}^{N_m} q_{mn}\alpha_{mn} + \sum_{n=1}^{N_m} q_{mn}(1-\alpha_{mn})(t_m - t_{mn})] + (1-y_m)\sum_{n=1}^{N_m} q_{mn}(1-\alpha_{mn})(t_m - t_{mn})\} \tag{3}$$

and tardiness cost:

$$C_{tardiness} = \sum_{m=1}^{M} S_m [t_{mn} \sum_{n=1}^{N_m} q_{mn}(1-\alpha_{mn})] \tag{4}$$

The supplier selection and order quantity allocation model formulation is as follows:

$$min\ z_1 = C_{purchasing} + C_{holding} + C_{tardiness} \tag{5}$$

$$max\ z_2 = \sum_{m=1}^{M}\sum_{n=1}^{N_m} q_{mn} r_{mn} \tag{6}$$

$$min\ z_3 = \sum_{m=1}^{M}\sum_{n=1}^{N_m} q_{mn} t_{mn} \tag{7}$$

$$s.t. \quad \sum_{m=1}^{M}\sum_{n=1}^{N_m} p_{mn} q_{mn} \le P \tag{8}$$

$$\sum_{n=1}^{N_m} q_{mn} = D, \forall m \tag{9}$$

$$\sum_{n=1}^{N_m} U_{mn} \ge D, \forall m \tag{10}$$

$$U_{mn} \ge q_{mn} \ge A_{mn}, \forall m,n \tag{11}$$

$$\sum_{n=1}^{N_m} q_{mn} r_{mn} \ge r \tag{12}$$

$$Pr\{\sum_{n=1}^{N_m} q_{mn}\alpha_{mn} \ge \varepsilon D\} \ge \theta, \forall m \tag{13}$$

$$Pr\{\sum_{n=1}^{N_m} q_{mn} r_{mn} \le \eta D\} \ge \varphi, \forall m \tag{14}$$

$$\sum_{m=1}^{M} x_{mn} = 1, \forall n \tag{15}$$

$$\varepsilon, \eta, \theta, \varphi, \alpha_{mn} \in [0,1] \tag{16}$$

$$x_{mn}, y_m \in \{0,1\}, \forall m,n \tag{17}$$

$$q_{mn}, U_{mn} \in Z^+, \forall m,n \tag{18}$$

The first objective function in Eq.(5) minimizes the total cost. The second objective function in Eq.(6) maximizes the total quality of purchased products. The third objective function in Eq.(7) minimizes the total tardiness degree of purchased products. Constraint (8) limits the capital budget of the total cost. Constraints in (9) ensure that the total quantity assigned to each supplier equal to the total demand. Constraints in (10) restrict the total supply capacity of suppliers to meet the demand of each material. Constraints in (11) gurantee that the capacity of each supplier cover the ordered quantity. Constraints in (12) represent manufacturers' quality requirements of the materials. Constraints in (13) mean the chance constraints for the suppliers to meet the demand of minimum production batch. Constraints in (14) represent the chance constraints that suppliers meet the quality requirement. Constraints (15) prohibit assigning the same supplier to more than one type of material. Constraints in (16), (17) and (18) force decimal, binary, non-negative and integral requirements on the variables.

3. Model Linearization and Methodology

3.1 Linearization of the Model

This section addresses the linearization of the chance-constraints in the model. Constraints in (13) and (14) contain random variable α_{mn}, denoting the uncertain delivery ratio of supplier n for material m. We assume that α_{mn} follows the normal distribution $N(\mu_{mn}^{\alpha}, \sigma_{mn}^{\alpha\,2})$. From traditional perspective, chance constraints are usually transformed into deterministic equivalents [13]. Firstly, constraints (13) can be transformed into:

$$Pr\{\sum_{n=1}^{N_m} q_{mn}\alpha_{mn} x_{mn} \ge \varepsilon D\} \ge \theta, \forall m \tag{19}$$

Let $g(\alpha_{mn}) = \varepsilon D - \sum_{n=1}^{N_m} q_{mn}\alpha_{mn} x_{mn}$, and the expectation and variance of $g(\alpha_{mn})$ is $E(g(\alpha_{mn})) = E(\varepsilon D) - \sum_{n=1}^{N_m} q_{mn} x_{mn} E(\alpha_{mn})$ and

$V(g(\alpha_{mn}))=V(\varepsilon D)+\sum_{n=1}^{N_n} q_{mn}^2 x_{mn}^2 V(\alpha_{mn})$. Then, let

$$\lambda=\frac{\varepsilon D-\sum_{n=1}^{N_n} q_{mn}\alpha_{mn}x_{mn}-(E(\varepsilon D)-\sum_{n=1}^{N_n} q_{mn}x_{mn}E(\alpha_{mn}))}{\sqrt{V(\varepsilon D)+\sum_{n=1}^{N_n} q_{mn}^2 x_{mn}^2 V(\alpha_{mn})}}$$

we can easily know λ follows standard normal distribution $N(0,1)$, and the probability density function of λ is: $\phi(\lambda)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\lambda} e^{-\frac{t^2}{2}}dt$ and Eq.(13) becomes

$$Pr\left\{\lambda \geq -\frac{E(\varepsilon D)-\sum_{n=1}^{N_n} q_{mn}x_{mn}E(\alpha_{mn})}{\sqrt{V(\varepsilon D)+\sum_{n=1}^{N_n} q_{mn}^2 x_{mn}^2 V(\alpha_{mn})}}\right\} \geq \theta \qquad (20)$$

Therefore,

$$\phi^{-1}(\theta) \leq -\frac{E(\varepsilon D)-\sum_{n=1}^{N_n} q_{mn}x_{mn}E(\alpha_{mn})}{\sqrt{V(\varepsilon D)+\sum_{n=1}^{N_n} q_{mn}^2 x_{mn}^2 V(\alpha_{mn})}} \qquad (21)$$

That is, deterministic equivalent to the chance constraints in (13) is

$$\sum_{n=1}^{N_n} q_{mn}x_{mn}E(\alpha_{mn})-\phi^{-1}(\theta)\sqrt{V(\varepsilon D)+\sum_{n=1}^{N_n} q_{mn}^2 x_{mn}^2 V(\alpha_{mn})} \geq E(\varepsilon D) \qquad (22)$$

However, Eq.(21) is non-linear. Let $D'=\varepsilon D$, $E(q_{mn}\alpha_{mn})=\mu_{mn}$, we have

$$\sum_{n=1}^{N_n}\mu_{mn}x_{mn}-\phi^{-1}(\theta)\sqrt{\sum_{n=1}^{N_n}\sigma_{mn}^2 x_{mn}^2} \geq D' \Leftrightarrow$$
$$(\sum_{n=1}^{N_n}\mu_{mn}x_{mn})^2-2D'\sum_{n=1}^{N_n}\mu_{mn}x_{mn}+(D')^2$$
$$\geq (\phi^{-1}(\theta))^2\sum_{n=1}^{N_n}\sigma_{mn}x_{mn} \qquad (23)$$

By using the method presented by Bilsel[13], we can linearize the Eq.(23) , and we will not go enter into detail here.The transformation process of Eq.(14) is the same as Eq.(13). Then, the model above is transformed into a deterministic multiple objective linear mixed integer programming model.

3.1 A Two-phase Approach

For small-scale problem, Lingo or some other mathematical programming software can be used to solve it directly. However, it may not achieve optimal or satisfactory solution within effective time for large-scale problems. There are several approaches to solve multi-objective linear programming problem[15]. Bilsel et al. further pointed out that the result of the non-preemptive goal programming was the best[2]. Based on their research, we develop a heuristic algorithm for the model. The algorithm can be described as two stages. The first stage is to reduce the scale of feasible solution space and the second stage is to search the optimal solution in the feasible solution space. Assume the case that the manufacturer plans to purchase three types of materials and each kind of material has three candidate suppliers. The specific steps are as follows:

- First Stage: Reduce the scale of the initial feasible solution according to the characteristics of the model.

Step 1: Find all initial feasible solutions that meet the constraints in Eq.(9) and the initial feasible solution space is denoted by I^0. Each feasible solution can be formalized as $S_i=(s_{i1},s_{i2},s_{i3})$, where s_{i1} represents the order allocation solution in the i th feasible solution, namely $s_{i1}=(q_{i11},q_{i12},q_{i13})$

Step 2: Set the minimum procurement batch for each type of material, which is denoted by ls_1, ls_2 and ls_3 respectively. The value of ls_i is related to specific material. Let μ_{U_i} denote the average supply capacity of each type of material, $\sigma_{U_i}^2$ denote corresponding variance and p_i the average price of material i. Let $ml=p_i\sigma_{U_i}^2/\mu_{U_i}$, which indicates the availability and supply stability of material i.Obviously, the smaller this value is, the smaller the corresponding procurement lot size will be. Then, we have feasible solution space I^1

Step 3: Sort the objectives of the model according to decision maker's preference, denote by l_j, $j=1,2,3$. The priority of the objective is associated with the weight of each objective that we will discuss in stage 2. Perform the constraints in turn that correspond to the order of the objective, we have new and smaller-scale feasible solution space I^2.

Step 4: Conduct the constraints that correspond to the chance constraints in Eq.(13) and (14) and we might achieve new feasible solution space and further denoted by I^3. Then, we recognize it as the final feasible solution space which satisfies all constraints in the model

- Stage 2: Find the optimal solution in the feasible solution space using the improved non-preemptive goal programming method. The specific steps are as follows

Step 1: Consider each objective function as the optimization goal and ignore other objectives under space I^1 and work out the corresponding optimal value, denoted as the ideal value of z_1^* , z_2^* and z_3^* respectively

Step 2: Assign the weight for each objective that denoted by w_1, w_2, and w_3,. The value of each weight is consistent with the order of each objective in step 3 of stage 1

Step 3: Let $z=\frac{w_1(z_1-z_1^*)}{(z_1^{max}-z_1^*)}+\frac{w_2(z_2^*-z_2)}{(z_2^*-z_2^{min})}+\frac{w_3(z_3-z_3^*)}{(z_3^{max}-z_3^*)}$ and find the best solution that keeps the value of z the smallest in space I^2 and denote it as $S^*=(s_1^*,s_2^*,s_3^*)$. Then, S^* is the optimal solution that we are searching for.

4. Numerical Example

To evaluate the performance of proposed model and algorithm, the following computational experiment is conducted on randomly generated instances. Assume that the manufacturer would purchase three types of materials and each of them owns three candidate suppliers. We consider different settings for the three kinds of materials with different price, quality and capacity level. For convenience, we set the same level of available rate for the same material and denote them by α_1, α_2 and α_3 and assume that they follow normal distribution $N(0.9,0.4)$, $N(0.7,0.12)$ and $N(0.8,0.08)$ respectively. The basic data of the model is shown in Table 1.

Table 1. Stochastic capacity data of each supplier

m	n	p_{mn}	U_{mn}	r_{mn}	K_{mn}	H_m	S_m	α_{mn}	ls_m
1	1	1.08	70	0.93	0.4	0.2	0.24	1.00	5
	2	1.12	80	0.94	0.4	0.2	0.24	0.70	5
	3	1.20	90	0.94	0.4	0.2	0.24	0.81	5
2	1	7.13	40	0.98	0.2	1.6	6.56	0.30	1
	2	8.20	50	0.98	0.2	1.6	6.56	0.62	1
	3	9.08	100	0.99	0.2	1.6	6.56	0.44	1
3	1	4.45	50	0.94	0.3	0.9	1.84	0.55	2
	2	4.56	60	0.96	0.3	0.9	1.84	1.00	2
	3	4.78	70	0.97	0.3	0.9	1.84	0.46	2

Follow the steps of the algorithm indicated in Section 3.2, we reduce the scale of the feasible solution space from the initial 60 million to 7 million and finally to 300 thousands. Then, the ideal values illustrated in step 1 of the stage 2, that is $z_1^*=4184.81$, $z_2^*=432.91$ and $z_3^*=650.22$ We consider different kinds of weight combination scenarios and reach corresponding solutions as shown in Table 2.

Table 2. Optimal order quantity allocation solution under different weight

(w_1,w_2,w_3)	(s_1^*, s_2^*, s_3^*)		
	$(q_{11}^*,q_{12}^*,q_{13}^*)$	$(q_{21}^*,q_{22}^*,q_{23}^*)$	$(q_{31}^*,q_{32}^*,q_{33}^*)$
(1.0,0.0,0.0)	(70,40,40)	(20,50,80)	(40,60,50)
(0.0,1.0,0.0)	(0,60,90)	(0,50,100)	(20,60,70)
(0.0,0.0,1.0)	(30,50,70)	(25,50,75)	(45,60,45)
(0.8,0.1,0.1)	(30,55,65)	(21,50,79)	(42,60,48)
(0.2,0.7,0.1)	(0,65,85)	(21,50,79)	(28,60,62)
(0.3,0.1,0.6)	(30,55,65)	(20,50,80)	(42,60,48)

The first three lines in table 2 show the optimal solutions that only consider the single objective respectively in turn. For example, if the decision-maker only care about total cost, the optimal order allocation solution of material 1 is(70,40,40), which means purchase 70 units material 1 from supplier 1, 40 units from supplier 2 and 40 units from supplier 3. The optimal solution for material 2 is (20,50,80) while (40,60,50) for material 3. Obviously, there is a big difference between material 1 and material 2 in price and unit tardiness cost, which leads to the order solution for these materials that material 1 gives preference to suppliers with low price and material 2 gives priority to suppliers with high available capacity. Line 4 to 6 respectively assigns greater weight to objective 1, 2, and 3 and it shows that the solution shown in line 4 is closely to the solution in line 1 and so as the line 5 and 6. It reveals that the optimal solution for different weight level at the same uncertain degree is different and the greater the objective weight is, the closer the corresponding order allocation solution to the solution regarding the objective as the single objective in the model.

5. Discussion

In this section, we will evaluate the effect of the degree of uncertainty and different weight level for each objective on the optimal solution. We consider two different settings as shown in Table 3 where the two materials has quite different characteristic. Further, we give the procurement lot-sizing a fixed and minimum value, that is, 1, which could provide distinct results for us. The optimal order allocation solutions under different conditions are shown in Table 4-6.

Table 3. Stochastic supplier capacity data

m	n	p_{mn}	U_{mn}	r_{mn}	K_{mn}	H_m	S_m
1	1	1.08	90	0.93	0.4	0.2	0.24
	2	1.20	110	0.94	0.4	0.2	0.24
2	1	7.13	60	0.98	0.2	1.6	6.56
	2	9.08	120	0.99	0.2	1.6	6.56

Let $W=\{W_1,W_2,\ldots W_n\}$ represent the set of different weights combinations, where $W_i=(w_{i1},w_{i2},w_{i3})$, and let $S_{i,j,k}=(Q_1,Q_2)_{i,j,k}=(q_{11_{ijk}},q_{12_{ijk}}q_{21_{ijk}},q_{22_{ijk}})$, which represents the material allocation solution under the level of W_i and the available deliver proportions for material 1 and material 2 are $\alpha_1=j$ and $\alpha_2=k$ respectively. From the data analysis of Table 4-6, we can obtain:

- When $\alpha_1=\alpha_2=1.0$, $\forall W_i(i=1,2,\ldots,n)$, we have $S_{ijk}=S_{i+1,j,k}$ (i=1,2,…n-1). That is to say, when the availability of all the materials can be guaranteed, the material allocation solution is always fixed no matter the weight of each objective is, and the specific material allocation solution is connected with its price and quality level. It reveals that the order allocation solution of the manufacture is fixed and has nothing to do with the weights combination when the ordered material can be delivered on time
- When $\alpha_1=\alpha_2\neq 0$, if $W_i\neq W_{i'}(i,i'=1,2,\ldots,n)$, then $S_{ijk}\neq S_{i';j,k}(i,i'=1,2,\ldots n)$. That is to say, when the available to deliver proportions of materials are the same but not equal to 1, the allocation solutions under different weight combinations are different. However, as shown in Figure 1, order allocation solutions under different weight combinations have similar features: the product quotas of the suppliers with a lower price who are matched with material 1 which has a low price but also a low quality level increase at first and then decrease with the increase of the degree of uncertainty; the product quotas of the suppliers with a lower price in material 2 which has a high price but also a high quality level decrease gradually and tend to be stable. It reveals that the subjective decisions of decision maker can affect the order allocation solution when the level of the uncertainty degree is fixed.

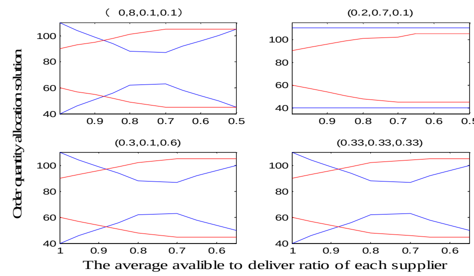

Figure 1. The optimal solution under different conditions

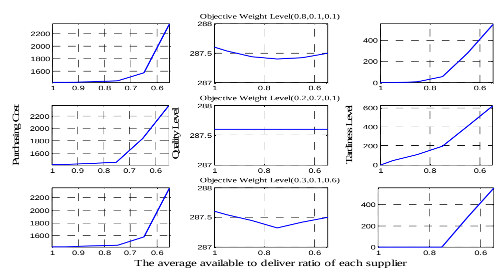

Figure 2. The impact of uncertainty and weight on different objective

Table 4. Comparison of solutions under different weights and $\alpha_1=\alpha_2$

(w_1,w_2,w_3)	$\alpha_1(\alpha_2)$	(q^*_{11},q^*_{12})	(q^*_{21},q^*_{22})	z_1	z_2	z_3
(0.8,0.1,0.1)	1.00	(40,110)	(60,90)	1420.20	287.60	0
	0.95	(46,104)	(57, 93)	1425.33	287.54	0
	0.85	(56,94)	(52, 98)	1434.73	287.44	6.76
	0.75	(60,90)	(45,105)	1448.24	287.40	55.80
	0.65	(58,92)	(45,105)	1576.60	287.42	279.34
(0.2,0.7,0.1)	0.95	(40,110)	(57,93)	1426.60	287.60	41.80

	0.85	(40,110)	(51,99)	1441.67	287.60	112.20
	0.75	(40,110)	(45,105)	1461.41	287.60	195.80
	0.65	(40,110)	(45,105)	1841.18	287.60	405.70
(0.3,0.1,0.6)	0.95	(46,104)	(57,93)	1425.33	287.54	0
	0.85	(56,94)	(51,99)	1435.83	287.44	0
	0.75	(68,82)	(45,105)	1446.09	287.32	0
	0.65	(58,92)	(45,105)	1576.60	287.42	279.34

Table 5. Comparison of solutions under different weights andα_1 > α_2

(w_1,w_2,w_3)	α_1	α_2	(q^*_{11},q^*_{12})	(q^*_{21},q^*_{22})	z_1	z_2	z_3
(0.8,0.1,0.1)	0.95	0.85	(46,104)	(52, 98)	1435.93	287.54	6.76
	0.85	0.75	(56,94)	(45,105)	1447.53	287.44	0
	0.75	0.65	(68,82)	(45,105)	1568.11	287.32	139.50
	0.65	0.55	(58,92)	(45,105)	2313.91	287.42	420.34
(0.2,0.7,0.1)	0.95	0.85	(40,110)	(51,99)	1438.30	287.60	41.80
	0.85	0.75	(40,110)	(45,105)	1453.37	287.60	112.20
	0.75	0.65	(40,110)	(45,105)	1595.33	287.60	335.30
	0.65	0.55	(40,110)	(45,105)	2345.54	287.60	546.70
(0.3,0.1,0.6)	0.95	0.85	(46,104)	(51,99)	1437.03	287.54	0
	0.85	0.75	(56,94)	(45,105)	1447.53	287.44	0
	0.75	0.65	(68,82)	(45,105)	1568.11	287.32	139.50
	0.65	0.55	(58,92)	(45,105)	2313.91	287.42	420.34

Table 6. Comparison of solutions under different weights and α_1 < α_2

(w_1,w_2,w_3)	α_1	α_2	(q^*_{11},q^*_{12})	(q^*_{21},q^*_{22})	z_1	z_2	z_3
(0.8,0.1,0.1)	0.85	0.95	(56,94)	(58,92)	1422.97	287.44	6.96
	0.75	0.85	(68,82)	(52,98)	1433.29	287.32	6.76
	0.65	0.75	(58, 92)	(45,105)	1454.59	287.42	139.84
	0.55	0.65	(50,100)	(45,105)	1849.86	287.50	412.50
(0.2,0.7,0.1)	0.85	0.95	(40,110)	(57,93)	1429.97	287.60	112.20
	0.75	0.85	(40,110)	(51,99)	1449.71	287.60	195.80
	0.65	0.75	(40,110)	(45,105)	1525.73	287.60	266.20
	0.55	0.65	(40,110)	(45,105)	1864.62	287.60	482.70
(0.3,0.1,0.6)	0.85	0.95	(56, 94)	(57,93)	1424.13	287.44	0
	0.75	0.85	(68, 82)	(51,99)	1434.39	287.32	0
	0.65	0.75	(58,92)	(45,105)	1454.59	287.42	139.84
	0.55	0.65	(50,100)	(45,105)	1849.86	287.50	412.50

- $\forall W_i(i=1,2,...,n)$, fix α_1, $\alpha_1 \neq 1.0$, then the order allocation solution of Material 1 is fixed, and when $\alpha_2 \leq \alpha_1$,the allocation solution of Material 2 tend to give large quota to the supplier with a high supply capacity gradually with the increasing of α_2; when $\alpha_2 > \alpha_1$, the allocation solutions of Material 2 gradually tend to give large quota to the supplier with small supply capacity but a low price; similarly, fix α_2 and α_2, $\neq$ 1.0, ditto for the allocation solution law of Material 1. It reveals that when the uncertainty degree of a material is fixed, its allocation solution is fixed and unaffected by the uncertainty degree of other materials and the weight of objective
- Generally, as the uncertainty degree increases, the allocation solutions with different weight combinations show the same characteristics. As shown in Figure 2, with the increasing of the uncertainty degree, the total cost increase gradually, and the rate of increasing changes from weak to strong, the level of material quality decreases at first and then increases, but the level of materials tardiness increase gradually
- With different weight combinations, as the degree of uncertainty increases, the allocation solutions of different materials gradually tend to be stable. Define that material tends to be stable after the uncertainty degree α_i, and the rate of material tending to be stable is associated with the weight level. For example, when the decision-maker thinks a lot of procurement cost, the materials with a higher price achieve stability first; the materials with a lower price and a lower quality

achieve stability first, while the decision-maker thinks highly of quality level. It reveals that under different level of weight combination, each kind of material has a fixed allocation solution with small risk and the solution can be adopted when the uncertainty degree of materials is almost impossible to estimate.

6 Conclusion and Future Extensions

Risk management is an inherent part of the procurement process, especially those products with complicated structure and assembled by large amount of components. Effect of disruptions on supply chains can be crucial. This paper studies a stochastic multi-objective supplier selection and order quantity allocation problem with random supply capacity and uncertain tardiness time. We derive the deterministic equivalents for capacity and demand chance constraints under the normality assumption. After linearization of the nonlinear deterministic equivalents, a two-phase algorithm is proposed to solve the problem. The model presented in this paper is valuable for manufacturers in the supply chains. Since the numerical results show that specific material allocation solution is related to multiple factors, such as objective weight, uncertainty degree of supply, price of each kind of material and so on, it can be used as guidelines in industrial implementations. Future research can be down to extend the model to incorporate other features, such as discount mechanism, multi-period decision-making and different uncertain delivery scenarios considering the production mode. Besides, other algorithms, like genetic algorithm or particle swarm optimization algorithm will also be considered to achieve the satisfactory solutions.

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